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Wavepacket Localization, Evolution and Decoherence in Neutrino Oscillations

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I will present some of the results obtained regarding the emergence of decoherence in neutrino oscillations. In our model all the particles, including the source and detector, are treated dynamically and evolved consistently with Quantum Field Theory; decoherence can emerge naturally given the time evolution of the initial state and the final state considered.

We have shown that some of the assumptions commonly used in the literature, such as the covariance of the wavepackets, are inconsistent. We found that a crucial ingredient for decoherence is the localization in space-time of the neutrino creation and detection: in Nature, such a measurement is usually carried out by environmental interactions, however it could also be approximated by considering localized wavefunctions in the final state. On the other hand, if the environmental interactions are not present (for example, if the decay happens in vacuum), the final position of the daughter particles will not be measured, i.e. they will be described by plane waves instead: in this case the neutrino is not localized either, and we don't have decoherence.

A consequence of the time-evolution is that a Gaussian wavepacket will gradually spread: I will show that such an effect could in principle affect decoherence; moreover it would depend on the absolute mass scale of the neutrino, not on the Δm^2, which could offer a possible way to probe such a parameter by studying the neutrino oscillations.

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1. Introduction

Neutrinos are created in a superposition of mass eigenstates: as they propagate, each eigenstate gains a phase proportional to its energy; the difference between these phases gives rise to neutrino oscillations.

In realistic scenarios, neutrinos must be described using localized wavepackets with finite spatial dimensions. Different mass eigenstates, however, will travel at different velocities and, while propagating, they will separate. If this separation exceeds the wavepacket's dimension, it leads to the suppression of oscillations, a phenomenon referred to as quantum decoherence.

Although the possibility of quantum decoherence was postulated nearly half a century ago, a comprehensive theoretical understanding of this phenomenon remains elusive. Several models can be found in literature, employing different approaches, but they all must rely on substantial assumptions. For instance, crucial parameters related to decoherence, such as the shape and size of the neutrino wavepacket, must be introduced by hand and are frequently estimated using order-of-magnitude arguments, which can lead to very different estimations of the coherence length.

Furthermore, experimental observation of decoherence presents a considerable challenge. Numerous factors, such as baseline uncertainty and finite energy resolution of detectors, can suppress the oscillations as well, and often it is not possible to know whether the dampening is due to decoherence or other factors.

Nevertheless, decoherence may hold significance for the next generation of neutrino experiments, as they aim to measure oscillation parameters with unprecedented precision: for example, JUNO will measure most of them to the sub-percent precision [1]. In such a scenario deocherence does not have to destroy completely the oscillations to be relevant, even a very small change of the amplitude could still impact the final results significantly.

2. The Model

To gain a deeper insight into how decoherence can impact oscillation probabilities we are working on a model, using a Quantum Field Theory approach. We started by considering very simplified cases, increasing gradually the complexity working towards more realistic scenarios [3, 4]. We obtain nonetheless interesting results, for instance, we have shown that some of the assumptions currently often used in literature, such as the covariance of the neutrino wavepacket, are inconsistent [5].

In order to simplify the calculations, we have used several assumptions, which however could affect the final result only quantitatively, without changing qualitatively the oscillation behaviour. In particular, we work in 1+1 dimension, using only scalar fields. We also considered only two neutrino flavors and assumed maximal mixing. Neutrinos are created and detected using 2-body decays, namely

$$S_H \to S_L + \nu_i \qquad D_L + \nu_i \to D_H$$

where $S_{H(L)}$ is the source particle before (after) the decay, in the same way, $D_{L(H)}$ is the detector particle before (after) the absorption, while ν_i represent one particular neutrino flavor.

At $t = 0$ the system is described by an initial state $|\Omega(0)\rangle$, which contains all the information on the source and (if present) detector particle. The state is evolved using the time-evolution operator

$|\Omega(t)\rangle = e^{-iHt}|\Omega(0)\rangle$, where H is the Hamiltonian, which contains all the information on the neutrino creation, propagation and detection. The transition amplitude is computed by projecting $|\Omega(t)\rangle$ into a final state $|F\rangle$

$$A(t) = \langle F|\Omega(t)\rangle \qquad P(t) = |A(t)|^2$$

It is important to point out that environmental interactions are incredibly difficult to include in a simple model: for this reason, they are not taken into account yet. Nevertheless, as we will see later, these interactions play a pivotal role in the emergence of decoherence. In the next section, we will discuss how to approximate their impact on the system.

The transition probability is computed using a tree-level approximation, which is justified as long as t (*i.e.* the duration of the experiment) is considerably shorter than the lifetime of the source particle.

Before delving into further details, let's introduce some notation that will be used throughout this paper:

- $G(x - x_0, \sigma)$ will indicate a Gaussian function centered on x_0 and with width σ
- $|A, p\rangle$ will indicate a state containing only the particle A, in a momentum eigenstate with eigenvalue p
- $|A, p; B, q\rangle$ will indicate a state containing two particles, A a and B, both in a momentum eigenstate, with p and q indicating the respective eigenvalues

What is the wavepacket of the neutrino produced by the decay of S_H? Let us consider an initial state $|\Omega_w(0)\rangle$ containing only S_H

$$|\Omega_w(0)\rangle = \int \mathrm{d}p G(p - p_0, \sigma_H)|S_H, p\rangle$$

After a time t, the system is described by the following wavepacket (in the momentum space) [6]:

$$\begin{aligned}\psi_i(k,q) &= \langle S_L, k, \nu_i, q|e^{-iHt}|\Omega_w(0)\rangle = \int \mathrm{d}p G(p - p_0, \sigma_H)\langle S_L, k, \nu_i, q|e^{-iHt}|S_H, p\rangle \\ &\propto \int \mathrm{d}p G(p - p_0, \sigma_H)\delta(k + q - p)F(k, q) \end{aligned} \tag{1}$$

where, in the last step, we have dropped a multiplicative factor that would depend very weakly on the momenta and would not affect our calculation. The δ function will get rid of the integral over p, while $F(k, q)$ will enforce the on-shell condition:

$$F(k,q) = \frac{e^{-i\mathcal{E}_1 t} - e^{-i\mathcal{E}_0 t}}{\mathcal{E}_1 - \mathcal{E}_0}$$

where $\mathcal{E}_0$, $\mathcal{E}_1$ are the energies of the system before and after the emission of the neutrino. Notably, it is peaked when $\mathcal{E}_0 = \mathcal{E}_1$ and the suppression of the off-shell contributions is stronger when t increases. $F(k, q)$ can be expressed as

$$F(k,q) = \int_0^t \mathrm{d}t_1 e^{-i(\mathcal{E}_0 t_1 + \mathcal{E}_1(t - t_1))}$$

where t_1 can be identified as the neutrino creation time; the physical interpretation of this equation is that the time-evolved state is the coherent sum over all the possible creation times: this is not surprising, because so far we have not introduced anything in our model that can measure when the neutrino is produced. The distribution of the neutrino momentum can be found by computing $\int \mathrm{d}k|\psi(k,q)|^2$: Two noteworthy properties should be highlighted [6]:

- The neutrino is localized in the momentum space, however $\sigma_\nu \neq \sigma_H$. The neutrino wavepacket indeed is a Gaussian with standard deviation equal to

$$\sigma_\nu = \sigma_H \frac{v_H - v_L}{v_{\nu,i} - v_L} \simeq \sigma_H(v_H - v_L) \simeq \sigma_H \frac{E_\nu}{M_H}$$

  where v_H, v_L and $v_{\nu,i}$ are the group velocities of S_H, S_L and ν_i, respectively, E_ν the neutrino energy and M_H the mass of the source particle. The last steps rely on the assumptions of ultrarelativistic neutrinos and non-relativistic source particles. This happens because the on-shell condition affects the energy, not directly the momentum, and if the source particle is non-relativistic, a change of its momentum would translate into a much smaller change of its energy.

- In the coordinate space, however, the dimension of the neutrino wavepacket is proportional to t, which means that the neutrino is, for all practical purposes, completely delocalized, because the time-evolved state is the coherent sum over all the possible creation times.

3. Emergence of Decoherence

3.1 Vacuum

Let us consider the case where the neutrino is created and detected in vacuum; our initial state would be

$$|\Omega_\nu(0)\rangle = \int \mathrm{d}p\mathrm{d}w G(p - p_0, \sigma_S)G(w - w_0, \sigma_D)e^{-iLw}|S_H, p; D_L, w\rangle$$

Since both the decay and the detection happen in the vacuum, the final states will not be measured and our time-evolved initial state will be projected onto

$$|F_\nu\rangle = |S_L, l; D_H, k\rangle$$

In this case, it can be shown that there is no decoherence due to the separation of the wavepackets [4], even though oscillations can be attenuated for other reasons, such as when one mass eigenstate is kinematically suppressed. The time-evolved initial state is the coherent sum over all the possible creation and detection times; consequently, even if the mass eigenstates created at time $\bar{t}_1$ are completely separated when they arrive at the detector, they can still interfere with the eigenstates created at time $\bar{t}_1 \pm \epsilon$ and the oscillations are still present.

3.2 Localized Final State

In experiments, however, the neutrino creation and absorption processes never happen in vacuum, which means that the particles generated can interact with the environment. These interactions lead to the measurement of neutrino production and detection [2]. This means that, if the environmental interactions were taken into account in our model, the sum over all the possible production and detection time would now be incoherent. A straightforward way to approximate the impact of environmental interactions is to consider localized final states [6]. For instance, if we know at time t, the position and momentum of S_L (within a certain accuracy) the location where the neutrino can be created would be constrained by kinematics.

Now that the final states are localized, there's no need for the presence of a detector to introduce the baseline. Therefore, let's focus solely on neutrino creation. The initial state can be expressed as:

$$|\Omega_l(0)\rangle = \int \mathrm{d}p G(p - p_0, \sigma_I)|S_H, p\rangle$$

while in the final state is defined as:

$$|F_l\rangle = \int \mathrm{d}q\mathrm{d}k G(q - q_0, \sigma_\nu)e^{-iqL_\nu} G(k - k_0, \sigma_F)e^{-ikL_F}|S_H, k; \nu_i, q\rangle$$

For simplicity, we will take σ_ν and σ_F equal to the dimension of the wavepackets that emerge in the time-evolved state, namely [6]

$$\sigma_\nu = \sigma_I(v_H - v_L) \qquad \sigma_F = \sigma_I$$

$|F_l\rangle$ depends on two parameters, L_F and L_ν, the latter can be identified with the baseline. Since the neutrino velocity is known, L_ν will also constrain the time when the neutrino can be created: in the computation of the transition amplitude the coherent integral over t_1 remains, but different times now contribute differently, being weighted by a Gaussian centered on $\hat{t}_1$, which depends on L_ν. We can write $L_F = \hat{L}_F + \delta L$, where $\hat{L}_F$ is the position where we would expect classically S_L to be at time t if it was created at $\hat{t}_1$. If we take $\delta L = 0$ the transition probability is (up to a renormalization factor) [6]

$$P(\delta L = 0) \propto \left(\frac{1 + e^{-\delta^2/2}}{2}\right)^2 - \mathrm{Sin}^2\left(\frac{\Delta m^2 L}{4E}\right) e^{-\delta^2/2} \qquad \delta^2 = \frac{2L^2}{3L_{coh}^2} \qquad L_{coh} = \frac{2E^2\sigma_{\nu,x}}{\Delta m^2} \qquad \sigma_{\nu,x} = \frac{1}{\sigma_\nu}$$

If we want to average over all the possible δL, the integral must be incoherent, because each δL corresponds to a different final state. We have

$$P = \int \mathrm{d}\delta L P(\delta L) = \int \mathrm{d}\delta L |A(\delta L)|^2 \qquad \delta^2 \to \delta_L^2 = \frac{L^2}{3L_{coh}^2} + \frac{(\Delta m^2 \sigma_{\nu,x})^2}{3(4E)^2}$$

The additional term corresponds to the uncertainty on the baseline due to the finite size of the neutrino wavepacket [6].

3.3 Spread of the Wavepacket

It is well known that the dimension of a Gaussian wavepacket will increase with time because each p-component will travel at a different velocity. When we study decoherence, it is often impossible to obtain analytical results without expressing the energy as a Taylor series: if this expansion is used, however, the spread of the wavepacket cannot be observed if terms up to at least the second order of the expansion are not taken into account:

$$E(p) \simeq E_0 + v(p - p_0) + v'(p - p_0)^2 + \dots \qquad v' = \frac{m_i^2}{E^3}$$

If the second order term is included, the spread of the wavepacket will affect the decoherence: for instance, when $L \to \infty$, the separation of the wavepackets does not kill completely the oscillation, but decoherence "saturate" to an asymptotical value

$$\delta^2 \to \frac{L_{sp}^2}{L_{coh}^2} \qquad L_{sp} = \frac{1}{v'\sigma_v^2}$$

It is worth noticing that such an effect depends on the neutrino mass, not on Δm^2: if it is possible to observe it, it could give us a way to probe the absolute neutrino mass scale directly from oscillation experiments.

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