Neutrino masses and unitarity violation from higher $SU(2)$ representations

Alessio Giarnetti, $^{a,b}$ Juan Herrero-García, $^c$ Simone Marciano $^*,^{a,b}$ Davide Meloni$^{a,b}$ and Drona Vatsyayan$^c$

$^a$Dipartimento di Matematica e Fisica, Universitá di Roma Tre, Via della Vasca Navale 84, 00146, Roma, Italy
$^b$INFN Sezione di Roma Tre, Via della Vasca Navale 84, 00146, Roma, Italy
$^c$Departamento de Física Teórica, Universidad de Valencia and IFIC, Universidad de Valencia-CSIC, C/ Catedrático José Beltrán, 2 | E-46980 Paterna, Spain

E-mail: alessio.giarnetti@uniroma3.it, juan.herrero@ific.uv.es, simone.marciano@uniroma3.it, davide.meloni@uniroma3.it, drona.vatsyayan@ific.uv.es

The Weinberg operator is the unique dimension-5 operator that generates Majorana masses for neutrinos after electroweak symmetry breaking. Thus, it seems natural that the way to have small neutrino masses is that they are Majorana particles, with their masses obtained at tree level suppressed by a heavy scale, i.e. the seesaw mechanism. We consider new dimension-5 operators with additional scalars that transform under higher representations $\mathcal{R}$ under $SU(2)$, up to $\mathcal{R} \leq 5$. We analyse the constraints on the VEVs of the new scalar multiplets from the $\rho$ parameter, as well as the generation of dimension-6 operators and non-unitarity constraints.
1. Introduction

Arguably, one of the most well-founded approaches for explaining the smallness of neutrino masses involves the use of seesaws [1–3]. In these scenarios, neutrinos are considered Majorana particles, and their masses are suppressed through interactions with a new heavy particle. These mechanisms necessitate the existence of heavy mediators, which can be either scalar or fermion particles. This study explores an extension of the Standard Model (SM) by introducing new scalar multiplets capable of attaining a small vacuum expectation value (VEV) and new fermion mediators that can be either Majorana or vector-like (VL) particles. Our focus is on models featuring at most SU(2) quintuplet representations for the scalars/fermions. We exclude the consideration of scalar mediators, as this would lead to the type-II seesaw. Indeed, without ad hoc hierarchies in the Yukawa couplings, the contribution to neutrino masses in such cases would be predominantly driven by operators involving the SM Higgs doublet. To focus on new mechanisms, our attention is directed solely towards models where at least the VEV of a new scalar is present in the final expression for neutrino masses.

2. Neutrino masses from the Effective Field Theory

Let us start considering the SM as an Effective Field Theory (EFT) valid at the electroweak scale. The only dimension-5 operator with the SM Higgs doublet ($H$) and the lepton doublets ($L$) is the Weinberg operator, which reads

$$O_5^{(0)} = \frac{c_5^{(0)}}{2\Lambda} LLHH + \text{H.c.},$$

where $\Lambda$ is the scale of new physics. At tree level, the UV completions of the Weinberg operator manifest as the conventional seesaws. Following Electroweak Symmetry Breaking (EWSB), a neutrino mass term arises from $O_5^{(0)}$, given by $m_\nu^{(0)} = c_5^{(0)} v_H^2 / \Lambda$, where $v_H = 246$ GeV represents the Vacuum Expectation Value (VEV) of the SM Higgs field. This offers an explanation for the tiny neutrino masses, suppressed by the high scale of lepton number violation.

Despite the elegance inherent in this scenario, dealing with $O(1)$ couplings requires an exceedingly high scale, $\Lambda \lesssim 10^{14}$ GeV, to replicate the atmospheric mass scale, $m_\nu^{(0)} \gtrsim O(0.05)$ eV. Consequently, probing the new physics responsible for lepton number violation becomes extremely difficult. In this study, we investigate the prospect of overcoming this challenge by positing the existence of new low-energy SU(2) scalar multiplets $\Phi_i$. The upper bounds on the masses of these new scalars are determined by considerations of perturbativity and unitarity, stemming from the fact that their mass is proportional to $\lambda v_H^2$. This implies an augmentation of the SM with new low-energy degrees of freedom, $O$ (TeV), necessitating their inclusion in the construction of a new SM EFT. In particular, as we will show, at dimension-5 new Weinberg-like operators may be constructed, so that

$$\mathcal{L}_5 = \frac{c_5^{(0)}}{2\Lambda} LLHH + \frac{c_5^{(1)}}{2\Lambda} LLH\Phi_i + \frac{c_5^{(2)}}{2\Lambda} LL\Phi_i\Phi_i + \frac{c_5^{(3)}}{2\Lambda} LL\Phi_i\Phi_j + \text{H.c.}.$$  

As long as the new scalars take VEVs, $\langle \phi_i \rangle \equiv v_i \neq 0$, Majorana neutrino masses are then proportional to $v_H^2 / \Lambda, v_H v_i / \Lambda, v_i^2 / \Lambda$ and $v_i v_j / \Lambda$ for $O_5^{(0)}, O_5^{(1)}, O_5^{(2)}$ and $O_5^{(3)}$, respectively.
Neutrino masses and unitarity violation from higher SU(2) representations

Upper limits for the VEVs of the newly introduced scalars can be derived by requiring that the violation of the SM custodial symmetry is within the deviation of the $\rho$ parameter from one, using the experimental value $\rho = 1.00040 \pm 0.00024$ [4]. The resulting upper bound is typically quite modest, approximately $v_i \lesssim O(1) \text{GeV}$. This constitutes a crucial aspect of our investigation: the inherent suppression in the VEVs, particularly if they originate from the SM Higgs, aligns with our objective of obtaining minute neutrino masses without necessitating excessively large scales for lepton-number violation, i.e., $\Lambda$ is effectively suppressed. Consequently, our focus will center on models that specifically do not generate $O_5^{(0)}$, but rather exclusively produce $O_5^{(1)}$, or $O_5^{(2)}$ and/or $O_5^{(3)}$. For instance, if $O_5^{(2)}$ is generated, compared to the SM Weinberg operator, we have

$$\frac{\Lambda}{c_5^{(2)}} \sim \frac{v_i^2}{v_H^2} \frac{\Lambda}{c_5^{(0)}} \lesssim 10^{-4} \frac{\Lambda}{c_5^{(0)}}.$$  \hfill (3)

Together with the presence of the new scalars at the EW scale and with Electroweak Precision Tests, this makes these scenarios more testable than the usual high-scale seesaws.

3. UV completions

In this section, we will examine potential tree-level UV completions for the dimension-5 operators outlined in equation 2. Given the extensive exploration of the Two Higgs Doublet Model in existing literature (e.g., [5, 6]), we will exclude models featuring additional doublets. Moreover, we will disregard models that replicate conventional seesaws. Given the contraction $LL \sim 2 \otimes 2 = 3 \oplus 1$, the plausible scalar mediator options are limited to either a triplet or a singlet. However, the latter would not facilitate processes capable of yielding neutrino masses, leaving the scalar triplet as the sole viable choice. Nonetheless, this would entail a type-II seesaw. Hence, we will exclusively consider fermion mediators, encompassing both Majorana and VL particles.

3.1 UV completions with one new scalar multiplet

Let us now discuss the possible tree-level UV completions with a new scalar $\Phi_1$ transforming as a representation $R$ under SU(2) with hypercharge $Y_{\Phi_1}$ and with a Majorana mediator $\Sigma$. The Yukawa and fermion mass terms of the Lagrangian can be written as

$$\mathcal{L} \supset -\bar{\nu}_H H^\dagger \Sigma - \bar{\nu}_1 \Phi_1 \Sigma - \frac{1}{2} \Sigma^\dagger M_\Sigma \Sigma + \text{ H.c.},$$  \hfill (4)

where $y_H$ and $y_1$ are the Yukawa matrices associated with the SM higgs doublet $H$ and the new scalar field $\Phi_1$, respectively. Based on group theoretical considerations, it becomes evident that the new scalar multiplet must possess a non-zero hypercharge of $Y = -1/2$. Consequently, to accommodate a neutral component capable of acquiring a VEV, the new scalar must transform as $R \sim (2, 4)$. Given the established criteria, the most relevant selection is $\Phi_1 \sim (4, -1/2)$. Extending this analysis to the fermionic mediator, we find $\Sigma = (5, 0)$. This model is denoted as $A_1$. Now, let’s turn our attention to a vector-like fermion mediator $F$ with a non-zero hypercharge $Y_F \neq 0$. Due to its inherent nature, we can differentiate between the left and right chiral components, allowing for distinct couplings with the SM Higgs doublet and the new scalar. The relevant terms of the Lagrangian read

$$\mathcal{L} \supset -\bar{\nu}_H H F_R^\dagger - \bar{\nu}_1 \Phi_1 F_L^\dagger - \bar{F} M_\phi F + \text{ H.c.},$$  \hfill (5)
where the vector-like fermion mediator is \( \mathcal{F} = \mathcal{F}_R + \mathcal{F}_L \). It is interesting to notice that in this case the usual SM Weinberg operator is not allowed. From the \( y_H \) Yukawa term it follows that \( Y_F = -1 \), while the rest of the terms fix \( Y_{\Phi_i} = -3/2 \). We term this model as \( A_2 \).

### 3.2 UV completions with two new scalar multiplets

With two additional scalars \( \Phi_1 \) and \( \Phi_2 \), and with a Majorana fermion mediator \( \Sigma \), we have

\[
\mathcal{L} \supset -\Sigma y_1 \Phi_1^T L - \Sigma y_2 L \Phi_2^T - \frac{1}{2} \Sigma^T M_\Sigma \Sigma + \text{H.c.},
\]

(6)

where \( y_1, y_2 \) are Yukawa couplings. The Lagrangian structure implies that \( Y(\Phi_1) = Y(\Phi_2) = 1/2 \). To facilitate the acquisition of a VEV for \( \phi_{1,2} \), it is imperative that the new multiplets’ \( SU(2) \) dimensions be even. Consequently, we express \( \phi_1 = (2X_1, 1/2) \) and \( \phi_2 = (2X_2, 1/2) \), where \( (X_1, X_2) > 1 \). For the sake of simplicity, we additionally assume \( X_1 \neq X_2 \) to avoid scenarios involving particles with identical quantum numbers, which would result in the same physical configuration. Imposing the invariance under the \( SU(2) \otimes U(1) \) symmetry group of the Weinberg operator \( (\phi_1 L)(\phi_2 L) \), we can derive the possible choices for \( X_1 \) and \( X_2 \). The contractions are:

\[
(\phi_1 L) = 2X_1 \otimes 2 = 2X_1 - 1 \oplus 2X_1 + 1, \quad (\phi_2 L) = 2X_2 \otimes 2 = 2X_2 - 1 \oplus 2X_2 + 1. \tag{7}
\]

Therefore, the condition \( X_2 = X_1 + 1 \) holds (assuming that \( \Phi_2 \) is the scalar in the highest representation). From this it follows that \( N_2 = N_1 + 2 \), where \( N_i = 2X_i \) is the (even) dimension of the representation. Let us now discuss the VL mediator case, where the mediator can couple to both new scalar multiplets. To avoid the previous cases, we will focus on the Lagrangian

\[
\mathcal{L} \supset -\Sigma y_1 \Phi_1 \mathcal{F}_R - \Sigma y_2 \Phi_2 \mathcal{F}_L^c - \mathcal{F} M_F \mathcal{F} + \text{H.c.},
\]

(8)

It can be seen that the previously stipulated constraint, demanding that the hypercharges of the two new multiplets be \( 1/2 \), is no longer necessary. It suffices that their sum equals \(-1\), hence, one can also have \( SU(2) \) odd representations. Once again, to generate \( (\phi_1 L)(\phi_2 L) \), it is imperative for the two \( SU(2) \) scalar fields to represent either two consecutive even or odd representations. As a result, an entirely new collection of feasible models emerges, identified in the subsequent discussion as \( B_i \).

### 4. List of genuine models

In Table 1, we summarize genuine models containing at most quintuplets, detailing transformation properties as \( \Phi_i = (N_j, Y_i) \), with \( N_j = 2Y_i + 1 \) representing the \( SU(2) \) representation’s dimension for

<table>
<thead>
<tr>
<th>Model</th>
<th>New Scalar Multiplets</th>
<th>Fermion Mediator</th>
<th>Operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>( \Phi_1 = 4S_{1/2} )</td>
<td>( \Sigma = 5F )</td>
<td>( O_5^{(2)} )</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( \Phi_1 = 4S_{3/2} )</td>
<td>( \mathcal{F} = 3F )</td>
<td>( O_5^{(1)} )</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>( \Phi_1 = 4S_{1/2}, \Phi_2 = 4S_{3/2} )</td>
<td>( \mathcal{F} = 5F )</td>
<td>( O_5^{(3)} )</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>( \Phi_1 = 3S_{0}, \Phi_2 = 5S_{1} )</td>
<td>( \mathcal{F} = 4F_{1/2} )</td>
<td>( O_5^{(3)} )</td>
</tr>
<tr>
<td>( B_3 )</td>
<td>( \Phi_1 = 5S_{-2}, \Phi_2 = 5S_{1} )</td>
<td>( \mathcal{F} = 4F_{1/2} )</td>
<td>( O_5^{(3)} )</td>
</tr>
<tr>
<td>( B_4 )</td>
<td>( \Phi_1 = 5S_{-1}, \Phi_2 = 5S_{0} )</td>
<td>( \mathcal{F} = 4F_{1/2} )</td>
<td>( O_5^{(3)} )</td>
</tr>
</tbody>
</table>

Table 1: List of genuine models.
Neutrino masses and unitarity violation from higher SU(2) representations

Figure 1: Allowed regions obtained using the 95% CL limit on $\Delta \rho$ for the new scalar multiplets VEVs in the $B_i$ models outlined in Table 1. Two different structures emerge: hyperbolae for $B_{1,3}$ and ellipses for $B_{2,4}$.

weak isospin $I_j$. The sets labeled $A_i$ and $B_i$ involve adding one and two new scalars, respectively. The fermion mediator $\Sigma (F)$ is Majorana (Vector-like). The custodial symmetry breaking parameter $\Delta \rho$ depends on the quantum numbers of the scalar fields and their VEVs, allowing us to constrain the latter using the measured $\rho$ value (within the 95% C.L.), as shown in Fig. 1. The hierarchy between the SM Higgs VEV and the new scalars’ VEVs, $v_i \ll v_H$, is crucial in lowering the mass of the mediators of the new Weinberg-like operators around the TeV scale for the models in Table 1, making them testable in the next-generation experiments. Additionally, small VEVs can be achieved without fine-tuning in models with linear terms in the scalar potential, such as $\lambda H^3 \phi_i$ or $\mu H^2 \phi_i$. All models in Table 1, except for $B_{3,4}$, incorporate such terms.

5. Deviation of the PMNS matrix from unitarity

When integrating out the heavy mediators, dimension-6 operators are generated [7]. They do not violate the lepton number like the dimension-5 effective operators, but, when canonically normalized neutrino kinetic terms are formulated, they result in a non-unitary PMNS matrix denoted as $U \equiv (1 - \epsilon) U_L$. On general grounds, considering both the Higgs doublet and the new multiplet $\phi_i$, the dimension-6 operators (lepton number conserving) read

$$O_6^{(0)} = \left(T_\alpha \tilde{H}^\dagger \tilde{H} \right) i \tilde{\phi}^\dagger L_\beta, \quad O_6^{(1)} = \left(T_\alpha \tilde{\phi}_i^\dagger \right) i \tilde{\phi}^\dagger L_\beta,$$

with couplings $c_6^{(0)} = \left(y_H M_F^2 y_H^\dagger \right)_{\alpha \beta}$ and $c_6^{(1)} = \left(y_i M_F^2 y_i^\dagger \right)_{\alpha \beta}$. Generally, these dimension-6 operators are strongly suppressed due to their similar structure to the dimension-5 operators responsible for neutrino masses. Consequently, unless there is a mechanism decoupling the dimension-5 and 6 Wilson coefficients, allowing for a low-scale seesaw, the associated phenomenological signatures are constrained. In the case of Majorana mediators (Model $A_1$), an inverse seesaw scheme can be formulated [8–10]. Here, the interplay of larger Yukawas and smaller scales

\footnote{A similar list can be found in Ref. [3]. However, the models involving two quintuplets are completely novel, to the best of our knowledge. Further, we also discuss the constraints on the scalar multiplets, and the implications for non-unitarity via a study of the dimension 6 operators.}
may lead to notable contributions to dimension-6 operators. In the VL models (Models $A_2, B_i$), the product of Yukawas violates lepton number (LN) and contributions to dimension-6 operators, depending on either $y_{1,2}$ or $y_H$, can be substantial. For Model $A_2$, since the neutrino masses can be obtained by adopting a hierarchy such as $y_1v_1 \ll y_Hv_H$, there could be significant impacts stemming from $c_6^{(0)}$, with $c_6^{(1)} \ll c_6^{(0)}$. These hierarchical versions of vector-like fields resemble the equivalent of an inverse seesaw involving hypercharge-less fermions. The crucial aspect lies in the fact that the product of Yukawas and VEVs violates LN, whereas each of them individually does not, contributing to $D = 6$ operators. The same reasoning applies for the $B_i$ models. This leads to a very rich phenomenology: $Z$ lepton flavor violation, universality violation, non-unitarity PMNS...

6. Conclusions

We provide a catalogue of genuine models featuring additional scalar multiplets transforming with representations up to $R \sim 5$ under $SU(2)$, capable of generating a neutrino mass term through the seesaw mechanism. Notably, these models do not rely on invoking lepton-number-violating scales close to the GUT scale. These scenarios lead to a very rich phenomenology of the new scalars and due to the dimension-6 induced operators.

Acknowledgments

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References


Notice that the alternative hierarchy, requiring $y_H \ll 0.01y_1$, seems less natural.