PROCEEDINGS OF SCIENCE

Simultaneous determination of the charm mixing and *CP*-violating parameters together with the CKM angle γ

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Recently, experiments have been testing the neutral D meson system with increasing precision. Since $D^0 - \overline{D^0}$ mixing occurs through Flavour Changing Neutral Currents, Standard Model amplitudes are absent at the tree level and suppressed at the loop level. Therefore, heavy New Physics coupled to the up-type quarks could manifest itself by modifying the small charm mixing and *CP*-violating parameters. We present a strategy to combine observables from the D and B meson systems within a Bayesian framework to determine simultaneously the charm mixing and *CP*-violating parameters and the CKM angle γ. We check consistency between γ estimates obtained separating the beauty measurements according to the charge of the B mesons. We obtain an updated determination of the dispersive and absorptive charm mixing parameters and the angle γ.

The European Physical Society Conference on High Energy Physics (EPS-HEP2023)
21-25 August 2023
Hamburg, Germany

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1. Introduction

Charm mixing occurs through Flavour Changing Neutral Currents (FCNC) that are absent at the tree level in the Standard Model (SM) and suppressed by the Glashow-Iliopoulous-Maiani (GIM) mechanism and by the hierarchical structure of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Thus, heavy New Physics (NP) may enter $D^0 - \overline{D^0}$ mixing, altering its physical properties as the amount of CP violation. The latter, in fact, is expected to be very small in the SM, since it is suppressed by the fourth power of the Cabibbo angle and, so, could be used as an interesting benchmark of the SM. In these proceedings, we discuss a strategy to estimate the charm mixing and CP-violating parameters by combining the most precise available charm measurement within a Bayesian framework, including beauty observables. As already discussed in previous work by the LHCb Collaboration [1], this method allows to determine simultaneously the CKM angle γ, besides improving the precision of the charm observables. Our analysis extends the one by the LHCb Collaboration, including all relevant observables in the charm sector from other experiments. Therefore, we obtain the most updated estimates of charm mixing parameters.

2. Parametrization of the charm system

The time evolution of a linear combination of D^0 and $\overline{D^0}$ mesons follows the Schrödinger equation, with a 2×2 non-hermitian Hamiltonian which can be written in terms of its dispersive and absorptive components as

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma}. \tag{1}$$

The matrix elements of $\mathbf{H}$ enter the observables through the so-called mixing parameters

$$\phi_{12} = \arg\left[\frac{M_{12}}{\Gamma_{12}}\right],\ x_{12} = 2\frac{|M_{12}|}{\Gamma},\ y_{12} = \frac{|\Gamma_{12}|}{\Gamma}, \tag{2}$$

with Γ being the average decay width of the Hamiltonian eigenstates. The phase ϕ_{12} in Eq. (2) governs CP violation in pure mixing, while the other two parameters, x_{12} and y_{12}, are CP-conserving and approximate the differences between the masses and decay widths of the two Hamiltonian eigenstates in units of Γ, up to second order in the small ϕ_{12}.
In the SM, it is possible to decompose the absorptive and dispersive parts of H_{12} as

$$\xi_{12}^{SM} = \frac{(\lambda_{uc}^s - \lambda_{uc}^d)^2}{4}\xi_2 + \frac{(\lambda_{uc}^s - \lambda_{uc}^d)\lambda_{uc}^b}{2}\xi_1 + \frac{(\lambda_{uc}^b)^2}{4}\xi_0,\ \xi = M, \Gamma. \tag{3}$$

where $\lambda_{kl}^j = V_{kj}^* V_{lj}$ and $\xi_n \approx O(\varepsilon^n)$, with $V_{(k,l)j}$ being CKM matrix elements and ε the U-spin breaking parameter. In the so-called approximate universality scenario, CP violation in the interference between decay with and without dispersive and absorptive mixing can be described through two universal weak phases $\phi_2^{M,\Gamma}$ [2]. The latter are the relative phases between M_{12} or Γ_{12} and the corresponding first (dominant) contribution in the right-hand side of Eq. (3). Furthermore, the difference between ϕ_2^M and ϕ_2^Γ is exactly ϕ_{12}, meaning that all the CP violation in the charm sector is entirely determined by knowledge of these two universal weak phases. The latter can be estimated in the SM from their definitions, neglecting the third (smallest) contribution in Eq. (3): $\phi_{12}^{SM} \sim (\phi_2^M)^{SM} \sim (\phi_2^\Gamma)^{SM} \sim (2.2 \times 10^{-3}) \times [0.3/\varepsilon]$. Therefore, at the current level of experimental precision, the neutral D meson system can be fully described by determining four parameters x_{12}, y_{12}, ϕ_2^M and ϕ_2^Γ.

3. Charm observables

The charm observables involve both time-dependent and time-integrated measurements of D mesons reconstructed from Cabibbo Favoured (CF)/Doubly Cabibbo Suppressed (DCS) final states, such as $f = K^-\pi^+$, or CP eigenstates like $f_{CP} = \pi^+\pi^-$ (see [3] and all the references therein for a complete set of measurements by the LHCb Collaboration).

One of the most frequently used observables for two-body final states is the time-dependent measurement of the ratios between the DCS and the CF rates $R^\pm_{t_j}$, given by

$$R^\pm_{t_j} = \frac{\Gamma_{t_j}(D^0/\overline{D^0} \to \overline{f}/f)}{\Gamma_{t_j}(D^0/\overline{D^0} \to f/\overline{f})} = r^2_{D[f]}(1 \pm A_D) + \langle\tau\rangle_j \sqrt{r^2_{D[f]}(1 \pm A_D)}\, d^\pm_f + \langle\tau^2\rangle_j d'^{\pm}_f, \tag{4}$$

to second order in the small mixing and CP-violating parameters. Here, the decay time $\tau = \Gamma t$ is partitioned into bins, indicated as j. The parameter $r_{D[f]}$ is the ratio of the magnitudes of the decay amplitudes in the CP-conserving limit, while A_D is the difference between the direct CP asymmetries of the CF and DCS processes. The coefficients $d^\pm_f$, $d'^{\pm}_f$ depend on x_{12}, y_{12}, $\phi_2^{M,\Gamma}$ and can be extracted experimentally by fitting the data simultaneously with $r_{D[f]}(1 \pm A_D)$ and we use them as observables in our combination.

By integrating both sides of Eq. (4) over time, we get the expressions for the ratios of the branching fractions measured for the mode $K^0_S K^-\pi^+$ [4, 5], which we included as well.

A similar study is performed for three-body final states, such as $K^0_S\pi^+\pi^-$. The main difference compared to the previous case is the phase space dependence of the decay amplitudes. However, it is possible to partition the two-dimensional Dalitz plot into bins, which we denote as $\pm i$, and once again obtain the ratios

$$R^\pm_{it_j} = \frac{\Gamma_{\mp it_j}(D^0/\overline{D^0} \to f_{CP})}{\Gamma_{\pm it_j}(D^0/\overline{D^0} \to f_{CP})} \approx r_i + \langle\tau\rangle_j \sqrt{r_i}[s_i(1+r_i)(x_{f_{CP}} \pm \Delta x_{f_{CP}}) - c_i(1-r_i)(y_{f_{CP}} \pm \Delta y_{f_{CP}})] + \langle\tau^2\rangle_j[\cdots], \tag{5}$$

where we omitted the coefficients of $\langle\tau^2\rangle_j$ for simplicity. Here, r_i is the integral over the i-th bin of the ratio of the squared magnitudes of the decay amplitudes, while c_i and s_i are the real and imaginary parts of their interference. Then, from Eq. (5), it is possible to obtain the observables

$$x_{f_{CP}} = x_{12}\cos\phi_2^M,\ y_{f_{CP}} = y_{12}\cos\phi_2^\Gamma,\ \Delta x_{f_{CP}} = -y_{12}\sin\phi_2^\Gamma,\ \Delta y_{f_{CP}} = x_{12}\sin\phi_2^M. \tag{6}$$

The last class of charm observables relevant to this work rely on the so-called exponential approximation, which is a first-order expansion of the decay rates in the charm mixing and CP-violating parameters. The exponential approximation can be used to get simple expressions for the time-dependent CP-conserving quantities $\rho_f^{f_{CP}}(t)$ and the CP asymmetries $A_{f_{CP}}(t)$ as [6, 7]

$$\rho_f^{f_{CP}}(t) = \frac{d\Gamma(D^0 \to f_{CP})/dt + d\Gamma(\overline{D^0} \to f_{CP})/dt}{d\Gamma(D^0 \to f)/dt + d\Gamma(\overline{D^0} \to \overline{f})/dt} \propto 1 - \tau\left[\eta_{f_{CP}} y_{f_{CP}} - r_{D[f]}\left(y_{12}\cos\Delta_f - x_{12}\sin\Delta_f\right)\right], \tag{7}$$

and

$$A_{f_{CP}}(t) = \frac{d\Gamma(D^0 \to f_{CP})/dt - d\Gamma(\overline{D^0} \to f_{CP})/dt}{d\Gamma(D^0 \to f_{CP})/dt + d\Gamma(\overline{D^0} \to f_{CP})/dt} = a_{f_{CP}} + \tau\eta_{f_{CP}}\left[-x_{12}\sin\phi_2^M + a_{f_{CP}} y_{12}\right], \tag{8}$$

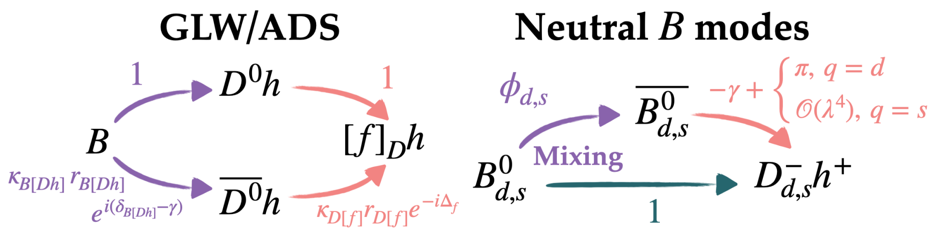

Figure 1: GLWS/ADS method to measure γ from the relative weak phase of the two tree-level amplitudes for cascade decays: $B \to D^0$ and $B \to \overline{D^0}$ (left figure). γ extraction from the CP-violating phase of the interference between mixing and decay of neutral B decays to charmed mesons (right figure).

where Δ_f in Eq. (7) is the strong phase of the CF decay amplitudes, while $\eta_{f_{CP}}$ is the CP eigenvalue of f_{CP}, and $a_{f_{CP}}$ in Eq. (8) is the direct CP asymmetry. Then, the slopes in Eqs. (7) and (8) are experimentally determined with linear fits of the data and are used as observables for our analysis. Other than these measurements, we added to our combination the time-integrated expression of Eq. (8) [8], asymmetries obtained from quantum correlated $D^0 - \overline{D^0}$ pairs [9], and other inputs for the charm decay amplitudes [10–13].

4. Beauty observables and the CKM angle γ

We now consider decays in which a B meson goes into a hadron state h (e.g. K, π) and a superposition of D^0 and $\overline{D^0}$, which subsequently decays to one of the final states f introduced before. These observables improve the charm part of the combination and, at the same time, allow to measure the CKM angle $\gamma = \arg[-\lambda^b_{uc}\lambda^{d*}_{uc}]$ from the relative weak phase between two interfering amplitudes, as shown schematically in Fig. 1.
Following the LHCb analysis [1], the time-integrated decay rates for these processes to first order in the mixing and CP-violating parameters of the neutral D meson system read

$$\begin{aligned}\Gamma(B \to [f]_D h) \ \propto\ & 1 + r^2_{D[f]} r^2_{B[Dh]} + 2\kappa_{D[f]}\kappa_{B[Dh]} r_{D[f]} r_{B[Dh]} \cos\big(\Delta_f + \delta_{B[Dh]} - \gamma\big) \\ & - \alpha y_{12}\Big[\kappa_{D[f]} r_{D[f]} \cos\Delta_f (1 + r^2_{B[Dh]}) + \kappa_{B[Dh]} r_{B[Dh]} \cos\big(\delta_{B[Dh]} - \gamma\big)(1 + r^2_{D[f]})\Big] \\ & + \alpha x_{12}\Big[\kappa_{D[f]} r_{D[f]} \sin\Delta_f (1 - r^2_{B[Dh]}) + \kappa_{B[Dh]} r_{B[Dh]} \sin\big(\delta_{B[Dh]} - \gamma\big)(1 - r^2_{D[f]})\Big],\end{aligned} \tag{9}$$

where we introduced the ratio of magnitudes of the beauty decay amplitudes $r_{B[Dh]}$, the corresponding strong phase $\delta_{B[Dh]}$, the time selection efficiency α and the coherence factors $\kappa_{D[f]}$, $\kappa_{B[Dh]}$ that are needed for multi-body decays of the D and B mesons, respectively.
Since the decay rate of the CP conjugated process is obtained simply replacing γ with $-\gamma$ in Eq. (9), the difference between $\Gamma(B \to [f]_D h)$ and $\Gamma(\overline{B} \to [\overline{f}]_D \overline{h})$ gives access to $\sin\gamma$. This is commonly referred to as the integrated CP asymmetry, which is an example of the so-called Gronau-London-Wyler (GLW) and Atwood-Dunietz-Soni (ADS) observables, that group together many other combinations of the rates of the cascade decays sensitive to γ. They have been measured extensively in recent years (see [3] and all the references therein for a complete set of measurements by the LHCb Collaboration). Similarly to what happens in the charm sector, phase space-dependent

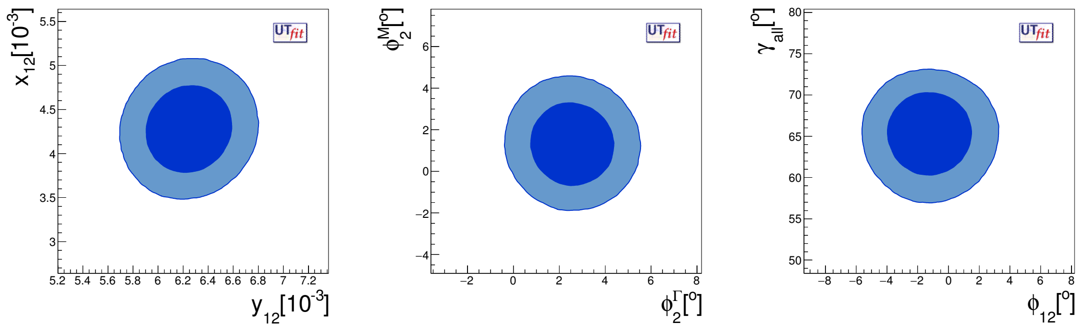

Figure 2: Pdfs of the charm mixing and *CP*-violating parameters and the CKM angle γ, obtained combining all the measurements. Darker (lighter) contours correspond to 68.3% (95.4%) probability.

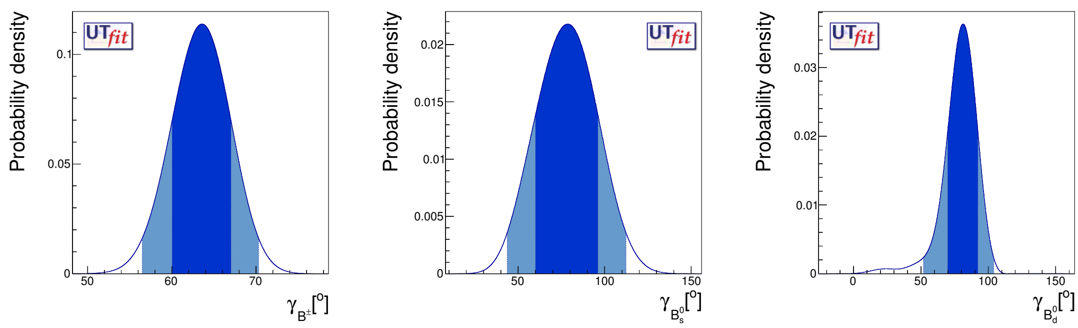

Figure 3: From left to right: pdfs of the CKM angle γ, obtained using only charged B, neutral B_s and neutral B_d measurements. Darker (lighter) contours correspond to 68.3% (95.4%) probability.

analyses are performed also in the beauty sector for the cascade decays $B \to [K^0_S\pi^+\pi^-(K^+K^-)]_D h$ and $B \to [K^+K^-(\pi^+\pi^-)\pi^+\pi^-]_D h$. The Dalitz plot of these processes has been studied using a model for the D meson decay amplitudes [14, 15] or by binning the phase space and solving a system of linear equations in a model-independent way [16]. The first method is more precise statistically with respect to the latter but introduces an additional systematic uncertainty due to the model. In both cases, it is possible to extract the so-called Giri-Grossmann-Soffer-Zupan (GGSZ) observables that depend on γ as

$$x_\pm^{Dh} = r_{B[Dh]}\cos\big(\delta_{B[Dh]} \pm \gamma\big),\ y_\pm^{Dh} = r_{B[Dh]}\sin\big(\delta_{B[Dh]} \pm \gamma\big). \tag{10}$$

Besides the cascade decays, we considered also measurements of the time-dependent rates of the neutral B meson decays to charmed mesons, such as $D^-\pi^+$ and $D_s^-K^+(\pi^+\pi^-)$ [17–19]. These decays depend on the *CP*-violating phase between $B_{d,s}$ mixing ($\phi_{d,s}$) and decay (γ), as depicted in Fig. 1. In the SM, one has $\phi_{d,s} = \mp 2\beta_{(s)}$, with $\beta = \arg[-\lambda^d_{tc}\lambda^{b*}_{tc}]$ and $\beta_s = \arg[-\lambda^{s*}_{tc}\lambda^b_{tc}]$. Beyond the SM one can still use the experimental value of $\phi_{d,s}$ to take into account $B_{d,s}$ mixing effects in the extraction of γ.

5. Results

We combine the observables described before in a Bayesian framework. We choose the likelihood as a product of Gaussian distributions for each set of correlated measurements, while the priors are assumed to follow uniform distributions defined over the physical ranges of the parameters. We perform four combinations using different subsets of beauty observables obtained using all the modes together, only charged B modes, only neutral B_s and B_d modes. The results for the most physically relevant parameters, when using all the measurements are

$$x_{12} = (4.28 \pm 0.32)‰, \; y_{12} = (6.24 \pm 0.23)‰, \; \phi_2^M = (1.3 \pm 1.3)^\circ, \; \phi_2^\Gamma = (2.6 \pm 1.2)^\circ, \\ \gamma_{\text{all}} = (65.4 \pm 3.3)^\circ, \tag{11}$$

and their probability density functions (pdfs) are shown in Fig. 2. Correlations between γ and the charm mixing parameters is below percent, while correlations between the charm mixing parameters are below 10%. The γ estimates extracted from measurements of B mesons with different charges are reported in Fig. 3 and are found to be

$$\gamma_{B^\pm} = (63.5 \pm 3.5)^\circ, \; \gamma_{B_s^0} = (78 \pm 18)^\circ, \; \gamma_{B_d^0} = (81 \pm 11)^\circ. \tag{12}$$

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