PROCEEDINGS OF SCIENCE

Loop-corrected Fierz Identities in EFTs and Flavor Physics

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We give a general prescription for the transformation between four-fermion effective operator bases using one-loop corrections to tree-level transformations. The procedure has the major advantage of only relating physical operators between bases, eliminating the necessity for evanescent operator insertions in such transformations. Additionally, when performing basis changes using loop-corrected transformations, the dependence on renormalization scheme factorizes between the two bases, implying that such transformations simultaneously change renormalization scheme along with the operator basis. The utility of this method has been exemplified in flavor physics through several examples in BSM phenomenology.

The European Physical Society Conference on High Energy Physics (EPS-HEP2023)
21-25 August 2023
Hamburg, Germany

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PoS(EPS-HEP2023)354

1. Introduction

In $d = 4$ spacetime dimensions, there exist several relations between fermionic operator structures which allow for the reduction of a chosen physical operator basis. Schematically, redundant operators which admit such reductions, $\tilde{O}_i$, can be related to a set of physical operators, O_i, via

$$\tilde{O}_i \overset{d=4}{=} R_{ij} O_j \,. \tag{1}$$

However, many of these reductions, such as Fierz relations and those relying on the purely four-dimensional objects γ_5 and the Levi-Civita tensor, are only well-defined when $d = 4$. When computing higher-order corrections, divergent loop integrals are rendered finite using dimensional regularization, where the number of spacetime dimensions is continued away from four, thus explicitly breaking these relations. This breaking is compensated by the introduction of additional, unphysical operators which vanish in the limit $d \to 4$. These so-called evanescent operators are defined as the difference between the redundant operator and its projection onto the physical basis in four spacetime dimensions

$$E_i = \tilde{O}_i - \big(R_{ij} + \epsilon \Sigma_{ij}\big) O_j + O(\epsilon^2) \,, \tag{2}$$

where conventionally, $\epsilon = (4 - d)/2$ and Σ fixes the evanescent scheme. Although these operators are unphysical and drop from the basis when continuing back to $d = 4$, it is well known that they produce physical effects and cannot, in general, be neglected [1–3].

Of particular interest is the case of basis transformations: when transforming from one basis to another, evanescent insertions into divergent loop integrals can lead to finite contributions to the transformation of the physical basis. The standard way of accounting for such contributions is to include a finite counterterm to physical Wilson coefficients. For a transformation $\{Q_i, E_j\} \to \{Q'_i, E'_j\}$, parameterized by

$$Q' = R\big(Q + W\,E\big), \quad E' = M\big[\epsilon U\,Q + \big(1 + \epsilon U\,W\big)E\big] \,, \tag{3}$$

the finite one-loop counterterm is given by [4]

$$Z^{(1;0)}_{Q'Q'} = R\Big[W\,Z^{(1;0)}_{EQ} - \Big(Z^{(1;1)}_{QE} + W Z^{(1;1)}_{EE} - Z^{(1;1)}_{QQ} W\Big)U\Big]R^{-1} \,. \tag{4}$$

From Eqs. (3) and (4), it is clear that, not only do we need to relate the physical operators between the two bases, but also the evanescent operators. Additionally, one must compute the insertions of evanescent operators into one-loop diagrams.

An alternative method, originally proposed in Ref. [5], is to instead view this finite counterterm as a "shift" to the tree-level basis transformation. This shift is given only in terms of one-loop amplitudes of physical operator insertions in the respective bases. This has the clear advantage that one no longer needs to relate all evanescent operators between the two bases or explicitly insert any evanescent operators into one-loop diagrams.

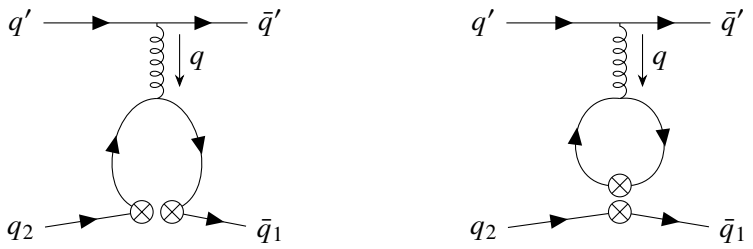

Figure 1: Two topologies of penguin diagrams with four-fermion operator insertions: open (left) and closed (right). Here, the fermion lines follow spinor indices.

2. One-Loop Fierz Relations

The general strategy for computing shifted basis changes was originally applied to Fierz relations, which relate tensor products of Dirac structures, O_i, to a linear combination of Fierz-conjugated structures, $\tilde{O}_i$ in $d = 4$ via the matrix $\mathcal{F}$, i.e.

$$\tilde{O}_i \overset{d=4}{=} \mathcal{F}_{ij} O_j \,. \tag{5}$$

When performing a next-to-leading order (NLO) computation, one-loop corrections to (5) must be accounted for when transforming physical quantities such as Wilson coefficients (WCs) or anomalous dimension matrices (ADMs). This is accomplished by introducing the one-loop shifted transformation

$$\tilde{O}_i = \left(\mathcal{F} + \Delta\right)_{ij} O_j \,. \tag{6}$$

The one-loop shift is computed via

$$\Delta_{ij} \left\langle O_j \right\rangle^{(0)} = \left\langle \tilde{O}_i \right\rangle^{(1)} - \mathcal{F}_{ij} \left\langle O_j \right\rangle^{(1)} \,, \tag{7}$$

where $\langle Q \rangle^{(n)}$ denotes the n-loop amplitude with Q-operator insertion.

Here, we emphasize that O_i and $\tilde{O}_i$ are *strictly physical* operators in the two respective bases; no evanescent insertions are necessary to compute the shifts at one-loop. It is, however, important to note that in the computation of Eq. (7), the Fierz-evanescent operators which are rotated into the physical basis under the transformation in Eq. (3) feature trivial scheme dependences, i.e.

$$E_i^{\text{Fierz}} = \tilde{O}_i - \mathcal{F}_{ij} O_j \,. \tag{8}$$

This is equivalent to setting $U = 0$ in Eq. (3), and will be discussed further in Sec. 3.

The shifts to Fierz relations arising from four-fermion operators in the low-energy effective theory (LEFT) were originally computed in Ref. [5] in the BMU scheme [6, 7]. In Ref. [5], mass effects were neglected and therefore contributions from dipole operators were not accounted for. These results were extended to include dipole effects in Ref. [8]. However, closed-penguin topologies in dipole diagrams with insertions of four-fermion operators, see Fig. 1, result in traces featuring odd numbers of γ_5 which cannot be evaluated in naive dimensional regularization (NDR). Hence, the scheme used for evalutating d-dimensional Dirac algebra was NDR augmented by the 't Hooft-Veltman (HV) scheme to evaluate such traces. Similar results in the standard model effective theory (SMEFT) have also been computed in Ref. [9].

3. Scheme Factorization

Since the one-loop shifts depend on the finite piece of one-loop amplitudes, they are naturally scheme-dependent quantities. In Refs. [5, 8], calculations were performed in a fixed scheme, irrespective of the choice of physical basis, namely the use of HV-augmented NDR with evanescent quantities derived from the so-called Greek identities [10].

It is often the case that one wishes to change physical basis in order to avoid computing in a more complicated renormalization scheme, e.g. one which features closed fermionic loops with insertions of γ_5 that would require the HV or Larin scheme for evaluating traces. It is then necessary to not only change basis, but also change renormalization scheme. In Eq. (4), this is handled by relating all evanescent operators between the two bases and computing all one-loop diagrams, including those with evanescent insertions.

The story is much simpler when using shifted basis transformations; instead of including all evanescent operator insertions, one only needs to compute the physical operator insertions in each basis, in the respective scheme of that basis. If one wishes to transform from some physical basis $\{O_i\}$ with scheme S to another basis $\{\tilde{O}_i\}$ with scheme $\tilde{S}$ such that, in $d = 4$, $\tilde{O}_i = R_{ij}O_j$, then the shift is given by

$$\Delta_{ij} \langle O_j \rangle^{(0)} = \langle \tilde{O}_i \rangle^{(1)}_{\tilde{S}} - R_{ij} \langle O_j \rangle^{(1)}_{S} \,, \tag{9}$$

where $\langle Q \rangle_{\mathcal{S}}$ denotes the matrix element of operator Q computed in scheme $\mathcal{S}$.

From Eq. (9), it is clear that the one-loop shift depends on both schemes, S and $\tilde{S}$, as is necessary to properly change schemes for scheme-dependent NLO quantities. This dual scheme-dependence can be split into three pieces: the scheme-independent, S-scheme-dependent, and $\tilde{S}$-scheme-dependent pieces

$$\Delta = \Delta^{\text{S.I.}} + \Delta^{S} + \Delta^{\tilde{S}} \,, \tag{10}$$

respectively.

Suppose one has a set of scheme-dependent WCs in the original basis whose scheme dependence can similarly be represented as

$$C_i = C_i^{\text{S.I.}} + C_i^{S} \,. \tag{11}$$

Then, when transforming using the shifted relation Eq. (6) with $\mathcal{F} \to R$, one finds the transformation of the WCs

$$\tilde{C}_i = C_j R^{-1}_{ji} - C_j R^{-1}_{jk} \Delta_{k\ell} R^{-1}_{\ell i} \,, \tag{12}$$

to one-loop order. Since $\tilde{C}_i$ can only depend on the scheme $\tilde{S}$, the scheme-dependence of Δ must factorize such that

$$\begin{aligned} \tilde{C}_i^{\text{S.I.}} &= C_j^{\text{S.I.}} R^{-1}_{ji} - C_j^{\text{S.I.}} R^{-1}_{jk} \Delta^{\text{S.I.}}_{k\ell} R^{-1}_{\ell i} \,, \\ 0 &= C_j^{S} R^{-1}_{ji} - C_j^{\text{S.I.}} R^{-1}_{jk} \Delta^{S}_{k\ell} R^{-1}_{\ell i} \,, \\ \tilde{C}_i^{\tilde{S}} &= -C_j^{\text{S.I.}} R^{-1}_{jk} \Delta^{\tilde{S}}_{k\ell} R^{-1}_{\ell i} \,. \end{aligned} \tag{13}$$

In Ref. [11], it was proven that the one-loop shifts, Eq. (9), satisfy exactly such a factorization of renormalization scheme, allowing for the simultaneous change of basis and renormalization scheme without the need to insert evanescent operators into one-loop diagrams. This method was then used

to reproduce the two-loop ADM corresponding to the mixing of BSM LEFT operators into the electron dipole moment in the Larin scheme [12] using a much simpler computation in NDR.

As mentioned in the previous section, Eq. (9) only holds in the case that evanescent operators which rotate into the physical basis do not feature an $O(\epsilon)$ scheme-dependence (though, other evanescent operators can have an arbitrary scheme dependence). In the case where such $O(\epsilon)$ terms exist, as in Ref. [4], additional care must be taken. In Ref. [5], the proper basis transformation was found by computing the additional "U-terms" of Eq. (4) which correspond to the evanescent-physical rotation. A generalization to Eq. (9) which accounts for these cases will be presented in a future update of [11].

4. Conclusions

A change of basis is often useful in effective field theory computations when different parts of the calculation are better suited for different bases, e.g. when the basis which is most convenient for matching leads to complications in the calculation of ADMs which do not arise in another basis. However, when working at NLO and higher, additional care must be taken to account for loop effects which alter the naive tree-level basis changes.

In these proceedings, we have presented a method for directly including loop effects into basis changes via shifts without the need to explicitly calculate the one-loop insertions of evanescent operators. Additionally, we have discussed how this method can be used to change renormalization schemes simultaneously with a change of basis. Explicit results for the shifts between Fierz-conjugated LEFT operators in the BMU scheme are given in Refs. [5, 8] and several known literature results are reproduced using this method in Refs. [5, 8, 11].

Acknowledgements

We would like to thank the organizers of EPS-HEP2023 for the opportunity to present this research. We acknowledge financial support from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement 833280 (FLAY), and from the Swiss National Science Foundation (SNF) under contract 200020-204428.

References

[1] M. J. Dugan and B. Grinstein, *On the vanishing of evanescent operators*, *Physics Letters B* **256** (1991) 239–244.

[2] A. J. Buras and P. H. Weisz, *Qcd nonleading corrections to weak decays in dimensional regularization and 't hooft-veltman schemes*, *Nuclear Physics B* **333** (1990) 66–99.

[3] S. Herrlich and U. Nierste, *Evanescent operators, scheme dependences and double insertions*, *Nucl. Phys. B* **455** (1995) 39–58, [hep-ph/9412375].

[4] M. Gorbahn and U. Haisch, *Effective Hamiltonian for non-leptonic $|\Delta F| = 1$ decays at NNLO in QCD*, *Nucl. Phys. B* **713** (2005) 291–332, [hep-ph/0411071].

[5] J. Aebischer and M. Pesut, *One-loop Fierz transformations*, *JHEP* **10** (2022) 090, [2208.10513].

[6] E. E. Jenkins, A. V. Manohar and P. Stoffer, *Low-Energy Effective Field Theory below the Electroweak Scale: Operators and Matching*, *JHEP* **03** (2018) 016, [1709.04486].

[7] A. J. Buras, M. Misiak and J. Urban, *Two loop QCD anomalous dimensions of flavor changing four quark operators within and beyond the standard model*, *Nucl. Phys. B* **586** (2000) 397–426, [hep-ph/0005183].

[8] J. Aebischer, M. Pesut and Z. Polonsky, *Dipole operators in Fierz identities*, *Phys. Lett. B* **842** (2023) 137968, [2211.01379].

[9] J. Fuentes-Martín, M. König, J. Pagès, A. E. Thomsen and F. Wilsch, *Evanescent Operators in One-Loop Matching Computations*, 2211.09144.

[10] N. Tracas and N. Vlachos, *Two-loop calculations in qcd and the $\delta i = 1/2$ rule in non-leptonic weak decays*, *Physics Letters B* **115** (1982) 419–421.

[11] J. Aebischer, M. Pesut and Z. Polonsky, *Renormalization scheme factorization of one-loop Fierz identities*, 2306.16449.

[12] J. Brod, Z. Polonsky and E. Stamou, *A Precise Electron EDM Constraint on CP-odd Heavy-Quark Yukawas*, 2306.12478.