

## New approaches to determine $B^\pm$ and $B^0$ production fractions

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This proceeding summarises a flavour physics session talk presented at the Conference on High Energy Physics. The talk explored novel methods for precisely determining the decay rates of  $\Upsilon(4S)$  into charged and neutral  $B$  meson pairs. We present the studies from the paper in [1]. These include an overview of previous measurements and introduces a new updated average. The presentation further outlined new determinations for  $R^{\pm 0}$  at Belle II and the LHC. Theoretical assumptions of previous measurements and the experimental challenges affecting the precision of  $R^{\pm 0}$  were also discussed.

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## 1. Introduction and present status

The talk will summarise the work from [1]. It began by highlighting the importance of precise measurements of the decay rates of  $\Upsilon(4S)$  into  $B^+B^-$  and  $B^0\bar{B}^0$ . Accurate values for these decay rates are essential for numerous applications, including precision determinations of CKM matrix elements and flavor symmetry relations. The ratio of these decay rates  $R^{\pm 0}$  is directly related to any absolute branching fraction measurement at electron-positron colliders. This highlights the crucial role of a precise determination of  $R^{\pm 0}$ , since its uncertainty directly affects the precision of measurements.

To calculate a new average value for  $R^{\pm 0}$ , we revisit the analysis conducted in Ref. [2], but this time considering the possibility of  $\Upsilon(4S)$  decaying into states without  $B$  mesons, so by allowing  $f_B$  to be non-zero. The goal is to precisely determine the decay rates of  $\Upsilon(4S)$  into charged and neutral  $B$  meson pairs while accounting for theoretical assumptions and experimental challenges. This determination involves three common measurement categories, which are outlined below.

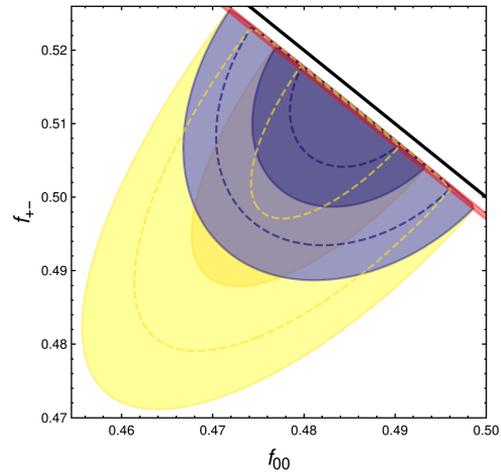
### I. Cancellation of final-state dependence.

This technique leverages the properties of double-tagged events in  $\Upsilon(4S)$  decays, where the production fractions of  $B^+B^-$  and  $B^0\bar{B}^0$  enter linearly, while the decay rate enters quadratically. By examining the ratio of the number of single-tag events squared to the number of double-tag events, it is possible to eliminate the influence of the decay rate. This allows a clean measurement of isospin violation in production.

**II. Known ratio of decay rates.** This method is based on the proportional relationship between experimentally determined ratios of charged and neutral  $B$  meson decays and  $R^{\pm 0}$ , assuming knowledge of the ratio of decay rates. While this ratio can be obtained from external measurements, it currently relies on isospin symmetry for the required level of precision, with specific cases, such as inclusive semileptonic  $B$  meson decays, benefiting from suppressed isospin breaking.

**III. (Pseudo-)Isospin symmetry.** By assuming small isospin breaking in (pseudo-)isospin-related quantities in a decay, it is possible to extract the isospin violation in production. This method relies on the premise that the isospin breaking in production is significantly larger than that in decay. This is the case, e.g., for  $B \rightarrow J/\psi K$  decays, where the annihilation amplitude contributing only to the charged mode is often argued to be negligible.

As average over each of these categories and taking systematic effects into account, we find the



**Figure 1:** Impact of the treatment of  $f_B$  on the determination of the  $B\bar{B}$  production fractions. The black line corresponds to setting  $f_B = 0$ , i.e.,  $f_0 + f_\pm = 1$ . The red line corresponds to setting  $f_B$  to a lower bound obtained from [3]. The yellow shaded areas use our results in Table ?? and treat the value of [3] as a lower limit. Meanwhile the blue shaded constraints use the measurement from [4] as lower limit. The lighter and darker regions show  $\Delta\chi^2 \leq 5.99$  and  $2.28$ , respectively, while the dashed lines correspond to  $\Delta\chi^2 = 1, 4$  (illustrating one-dimensional limits).

following average:

$$R_{\text{I+II+III}}^{\pm 0} = 1.057 \pm 0.023. \quad (1)$$

The detailed determination  $R^{\pm 0}$  can be found in [1]. This analysis provides a robust alternative to the HFLAV average  $R^{\pm 0}$  value, incorporating additional uncertainties from the assumptions made. It also validates the assumption that isospin violation in the considered decay modes is not anomalously large, especially in  $B \rightarrow J/\psi K$  decays.

The interplay between  $R^{\pm 0}$ ,  $f_{00}$ , and  $f_\pm$  in determining the precision of absolute branching fractions should be highlighted here. As we consider  $f_B$  in our calculations, the value of  $R^{\pm 0}$  is no longer in a linear relation to  $f_{00}$  and  $f_\pm$ . Figure 1 highlights the importance of precisely determining  $f_{00}$ ,  $f_\pm$  and  $f_B$ , as different treatments of  $f_B$  show the impact on  $f_{00}$  and  $f_\pm$  for a given  $R^{\pm 0}$ .

## 2. Determining $R^{\pm 0}$ using $\Upsilon(5S)$ decays

The main source of isospin violation in  $\Upsilon(4S)$  decays stems from the small available phase space of only 20 MeV. In contrast to that the  $\Upsilon(5S)$  resonance has a mass difference of around 326 MeV, which offers a larger phase space and no such large enhancement of isospin violation. An  $e^+e^-$  collider operating near the  $\Upsilon(5S)$  resonance produces a multitude of different final states. Experimentally, the branching fraction  $\Gamma(\Upsilon(5S) \rightarrow BBX)$  has been determined to be  $(76.2_{-4.0}^{+2.7})\%$ , with only roughly 5.5% corresponding to direct  $B\bar{B}$  production, alongside  $BB^*$  (13.7%) and  $B^*B^*$  (38.1%) production. Additionally, multi-body final states, such as  $B^{(*)}B^{(*)}\pi$  and  $BB\pi\pi$ , contribute to the spectrum. For (quasi-)two-body final states, we can assume

$$R_{5S}^{\pm 0} = \frac{\Gamma(\Upsilon(5S) \rightarrow B^{(*)+}B^{(*)-})}{\Gamma(\Upsilon(5S) \rightarrow B^{(*)0}\bar{B}^{(*)0})} \simeq 1. \quad (2)$$

Using this assumption, one can determine  $R^{\pm 0}$  with a novel approach. By studying the double ratio of decays at the  $\Upsilon(4S)$  and  $\Upsilon(5S)$  resonances,

$$r(f, f') = \left[ \frac{N(B^+ \rightarrow f)}{N(B^0 \rightarrow f')} \right]_{\Upsilon(4S)} \bigg/ \left[ \frac{N(B^+ \rightarrow f)}{N(B^0 \rightarrow f')} \right]_{\Upsilon(5S)}, \quad (3)$$

directly probing the production rate ratio  $R^{\pm 0}$  becomes feasible. Here  $N$  denotes the acceptance- and efficiency-corrected yields in  $\Upsilon(4S)$  and  $\Upsilon(5S)$  decays, in the latter case including  $B$  mesons from all (quasi-)two-body decays  $\Upsilon(5S) \rightarrow \bar{B}^{(*)}B^{(*)}$ . In this ratio the  $\mathcal{B}(B^+ \rightarrow f)$  and  $\mathcal{B}(B^0 \rightarrow f')$  branching fractions cancel. This is crucial, since no information on the size of isospin breaking in the decay rates is needed. The states  $f$  and  $f'$  do not even need to be (pseudo-)isospin related, so any pair of states can be chosen to minimize the experimental uncertainties. Thus, the double ratio in Eq. (3) directly probes the ratio of production rates,  $R^{\pm 0}$ , assuming Eq. (2) holds.

We provide estimates of the expected sensitivities to  $r(f, f')$  using this novel method in Table 1. The available Belle data, as well as the anticipated Belle II data with a partial (10%) and full data set are considered. Our study encompasses various modes representing different parton-level transitions, experimental signatures, and uncertainties. The uncertainties are estimated based on existing data, with an assumed factor of two improvement in systematic uncertainties for Belle II. We scale statistical uncertainties with integrated luminosity ratios and project the  $\Upsilon(5S)$  analyses'

	Belle	Belle II partial	Belle II full
$\mathcal{L}_{\Upsilon(5S)} / \mathcal{L}_{\Upsilon(4S)}$ [ $\text{ab}^{-1}/\text{ab}^{-1}$ ]	0.12 / 0.71	0.5 / 5	5 / 50
$N_{B^{(*)}B^{(*)}}^{\Upsilon(5S)} / N_{BB}^{\Upsilon(4S)}$	$2.74 \times 10^7 / 7.72 \times 10^8$	$1.13 \times 10^8 / 5.55 \times 10^9$	$1.13 \times 10^9 / 5.55 \times 10^{10}$
$f, f'$	$\Delta r(f, f')/r(f, f')$		
$J/\psi K^+, J/\psi K^0$	7.1%	3.5%	1.1%
$\bar{D}^0 \pi^+, D^- \pi^+$	2.4%	1.2%	0.4%
$\bar{D}^{*0} \ell^+ \nu, D^{*-} \ell^+ \nu$	4.5%	2.2%	0.7%
$\bar{D}^0 \pi^+, D^{*-} \ell^+ \nu$	1.8%	0.9%	0.3%

**Table 1:** Estimated sensitivity to  $r(f, f')$  in Eq. (3) depending on the modes used for the states  $f$  and  $f'$ . They are based on the available Belle data and anticipated partial and full Belle II data.

precision based on  $\Upsilon(4S)$  measurements, assuming equivalent systematic uncertainties, but correspondingly larger statistical uncertainties. We also assume that common systematic uncertainties cancel between the  $\Upsilon(4S)$  and  $\Upsilon(5S)$  measurements.

For decays like  $B \rightarrow J/\psi K$ , our analysis indicates that a precision of 7.1% on  $R^{\pm 0}$  can be achieved with the current Belle data. The limiting factor in this case is the statistical uncertainty of the  $\Upsilon(5S)$  measurement. In the case of  $B \rightarrow D\pi^\pm$  decays, a determination could already reach a precision comparable to the current world average. Semileptonic  $B \rightarrow D^* \ell \bar{\nu}$  decays also offer a clean path but are constrained by the precision of  $B^+ \rightarrow \bar{D}^{*0} \ell^+ \nu$  at  $\Upsilon(5S)$ . An additional enhancement in precision is attainable by focusing on  $B \rightarrow DX \ell \bar{\nu}$  decays. These findings suggest the potential to reach uncertainties of 1% or even sub-1% with a Belle II data set of  $5 \text{ ab}^{-1}$  of  $\Upsilon(5S)$  data. Importantly, our analysis emphasises that  $f$  and  $f'$  can be independently chosen to minimise experimental uncertainties, thereby enhancing the accuracy of the  $r(f, f')$  determination.

In conclusion, our study underscores the promise of the double-ratio method employing either  $B \rightarrow D\pi^\pm$  decays or mixed  $D\pi$  and semileptonic decays, making it a viable approach with existing Belle data.

### 3. Decay-channel-independent determination of $R^{\pm 0}$

An alternative to using specific decay channels in  $R^{\pm 0}$  determinations involves using the full spectrum of  $B$  meson decays. This method also does not rely on any isospin-violation assumptions. It relies on the fact that  $B$  mesons leave a distinct signature that can be easily triggered on with high efficiency. Many triggers used in  $e^+e^-$   $B$ -factory experiments rely on properties that are nearly identical for  $B^0 \bar{B}^0$  and  $B^+ B^-$  events. Nevertheless, to distinguish between  $B^0 \bar{B}^0$  and  $B^+ B^-$  events, an additional event property can be used: the total number of detector-stable charged daughter particles. While challenging to reconstruct directly, it can be approximated using the number of charged-particle tracks. These numbers exhibit distinct patterns for  $B^+$  and  $B^0$  meson decays, where the number of charged daughters is odd for  $B^+$  and even for  $B^0$ . This distinction becomes less

pronounced when looking at the number of charged daughters from a pair of  $B$  mesons produced in  $\Upsilon(4S)$  decay. The distributions of these quantities are, however, sensitive to the modelling of  $B$  meson decays. To mitigate this sensitivity, one can measure this distribution or its proxy in data using  $B$  meson decays that can identify their charge, such as  $B^0 \rightarrow D^- \pi^+$  decays, which have a small branching fraction but can be reconstructed with excellent experimental precision.

In practice, by assigning the final-state particles of one  $B$  meson decay to the signal, the rest of the collision event can be analysed, enabling the measurement of the multiplicity distribution of the number of charged-particle tracks with high precision. Similar measurements can be conducted with  $B^\pm$  decays and other exclusive channels, providing the necessary components for predicting the  $B$  meson pair distributions solely from data.

To assess the separation power, we construct an Asimov fit, assuming the calibration of charged and neutral  $B$  meson multiplicities can be achieved using  $B^0 \rightarrow D^- \pi^+$  and  $B^+ \rightarrow D^0 \pi^+$  decays. We scale the statistical uncertainty of an existing reference, factoring in the expected increase in integrated luminosity. Since the goal is to measure the distribution of reconstructed tracks rather than branching fractions, many leading systematic uncertainties do not compromise the sensitivity. With templates, correlated uncertainties, and predictions, we determine the achievable relative uncertainties in various luminosity scenarios.

For example, with Belle data, the estimated  $\Delta(R^{\pm 0})/R^{\pm 0}$  uncertainty is around 2.2%, which can be significantly reduced to 0.9% with Belle II's partial data set and even further to 0.3% with the full Belle II data set. These estimates consider the calibration uncertainty in the number of charged-particle tracks from  $B^0 \rightarrow D^- \pi^+$  decays and assume a similar calibration precision can be obtained for  $B^\pm \rightarrow D^0 \pi^\pm$ . Without the calibration uncertainty, the statistical component alone would be sub-percent even with the currently available data.

Practically, various reconstruction effects may lead to differences between the number of charged particles and tracks. These effects, including finite detector acceptance and the presence of misidentified or duplicate tracks, can shift and broaden the  $B^0 \bar{B}^0$  and  $B^+ B^-$  distributions, necessitating a comprehensive assessment of the method's feasibility within experimental setups.

#### 4. Production fraction ratios at hadron colliders

At hadron colliders, the production fractions of charged ( $f_u$ ) and neutral ( $f_d$ )  $B$  mesons serve a role analogous to the decay rates of  $\Upsilon(4S)$  mesons at  $e^+e^-$   $B$ -factories. However, establishing the equality of these production fractions is more challenging, because the initial and final states at hadron colliders are more complex: At the Tevatron for instance, the initial  $p\bar{p}$  state is a superposition of an isosinglet and an isotriplet, whereas at the LHC, the  $pp$  initial state is purely isotriplet. Additionally, distinguishing between  $B$  mesons and their isospin states is intricate due to the presence of additional particles in the final state. Nonetheless, the primary mechanisms for  $b\bar{b}$  production at the LHC, including gluon splitting and  $t$ -channel flavor creation, are isospin invariant. However, the fragmentation of these  $b$  quarks into  $B$  mesons introduces complications, especially when considering  $B_s$  mesons, where large corrections to  $SU(3)$  flavor symmetry are observed in the  $f_s/f_d$  ratio. Hence, determining the  $f_u/f_d$  ratio is an experimental question. The assumption of  $f_u/f_d = 1$  should be tested experimentally, considering potential kinematic dependencies as seen in  $f_s$  and  $f_{\Lambda_b}$ .

The experimental measurement of this quantity is complex, primarily due to the challenge of disentangling the production fractions from the decay rates. Category I measurements, as discussed for  $e^+e^-$   $B$ -factories, are not feasible because of the uncorrelated hadronization of  $b$  and  $\bar{b}$  quarks at hadron colliders. This leaves us with category II and III measurements.

A CMS Collaboration measurement in category II involves external inputs on the ratio of decay rates, leading to  $f_d/f_u = 1.015 \pm 0.051$ . The precision of this measurement is currently limited by the uncertainties in the analysis, as well as to a lesser extent, the uncertainty in  $R^{\pm 0}$ .

An alternative approach for determining  $f_d/f_u$  in category III involves exploiting the approximately equal rates of semi-inclusive decays, such as  $\Gamma(B^0 \rightarrow \bar{D}^{(*)} X \mu \nu) \approx \Gamma(B^+ \rightarrow \bar{D}^{(*)} X \mu \nu)$ . The relative simplicity of this approach arises from the equality of inclusive rates and the small fraction of decays that do not result in a  $D^{(*)}$  meson in the final state. While this method remains to be explored in detail experimentally, it offers the potential to measure  $B$  meson production fractions directly, aiming for an accuracy on the order of 1%.

In addition to these approaches, the copious samples of  $t\bar{t}$  events at the LHC, collected by ATLAS and CMS, could be utilised to test isospin invariance in the production and decay of  $B$  mesons. This is because the  $t\bar{t}$  system represents an isospin singlet, unlike the  $pp$  initial state. Testing the equality of  $f_u$  and  $f_d$  for  $B$  mesons produced in this process can provide insights into color reconnection effects. These possibilities, while speculative, open up a new avenue to experimentally probe isospin invariance in  $B$  meson decays and test the equality of  $B$  and  $\bar{B}$  production fractions.

The feasibility of these experimental approaches, their detailed studies, and their potential implications require dedicated experimental analyses to be conducted.

## 5. Discussion and Conclusions

This talk summarised the results in [1]. We investigated the determinations of the  $\Upsilon(4S) \rightarrow B^+B^-$  and  $B^0\bar{B}^0$  decay rates and proposed new methods to improve them. Presently the limited precision of these decay rates constitutes a lower limit of  $\sim 2\%$  on the uncertainties in absolute branching fraction measurements, and thereby in applications, such as precision determinations of CKM matrix elements or flavor symmetry relations. We revisited the theoretical assumptions in  $R^{\pm 0}$  measurements, and updated its world average, emphasising underestimated uncertainties in prior evaluations, in particular due to isospin violation and non-zero  $f_B$  value. Due to the inclusion of additional measurements, we obtained nevertheless an improved precision for  $R^{\pm 0}$  and the individual production fractions. We proposed two new methods to determine  $R^{\pm 0}$  precisely, using  $\Upsilon(5S)$  data anticipated at Belle II over the next decade, or using the different average number of charged-particle tracks between charged and neutral  $B$  meson decays. We also proposed possible measurements of  $f_d/f_u$  at the (HL-)LHC.

The issues raised and methods developed in this article will remain important at future colliders. In the meantime, progress could already be made by revisiting the measurement of Ref. [5] with the full data set, performing a double-tag analysis at Belle or Belle II, and by using our methods with the existing data.

## References

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