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Determination of quark and lepton masses and mixings in the microscopic model

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Recently, formulas for the mixing matrices of quarks and leptons have been put forward in [1]. My contribution here describes the relevant foundational and technical aspects which have led to those results.
The work has been carried out in the framework of the microscopic model[2]. The most general ansatz for the interactions among tetrons leads to a Hamiltonian H involving Dzyaloshinskii-Moriya (DM), Heisenberg and torsional isospin forces. Diagonalization of the Hamiltonian provides for 24 eigenvalues which are identified as the quark and lepton masses. While the masses of the third and second family arise from DM and Heisenberg type of isospin interactions, light family masses are related to torsional interactions among tetrons. Neutrino masses turn out to be special in that they are given in terms of tiny isospin non-conserving DM, Heisenberg and torsional couplings.
The approach not only leads to masses, but also allows to calculate the quark and lepton eigenstates, an issue, which is important for the determination of the CKM and PMNS mixing matrices. The almost exact isospin conservation of the system dictates the form of the lepton states and makes them independent of all the couplings in H. Much in contrast, there is a strong dependence of the quark states on the coupling strengths, and a promising hierarchy between the quark family mixings shows up.

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In the microscopic model[2] quarks and leptons arise as eigenmode excitations of an internal tetrahedral fiber structure, which is made up from 4 constituents and extends into 3 extra dimensions. The constituents are called tetrons and transform under the fundamental spinor representation 8 of SO(6,1). More in detail, the ground state of the model looks like illustrated in Fig. 1. Each tetrahedron is made up from 4 tetrons, depicted as dots. The picture is a little misleading because physical space and the extra dimensions are assumed to be completely orthogonal. With respect to the decomposition of $SO(6,1) \rightarrow SO(3,1) \times SO(3)$ into the (3+1)-dimensional base space and the 3-dimensional internal space, a tetron Ψ possesses spin $\frac{1}{2}$ and isospin $\frac{1}{2}$. This means it can rotate both in physical space and in the extra dimensions, and corresponds to the fact that Ψ decomposes into an isospin doublet $\Psi = (U, D)$ of two ordinary SO(3,1) Dirac fields U and D.

$$8 \rightarrow (1,2,2) + (2,1,2) = ((1,2) + (2,1), 2) \tag{1}$$

For the Ψ field left and right handed 'isospin vectors' may be defined

$$\vec{Q}_L = \frac{1}{4}\Psi^\dagger(1-\gamma_5)\vec{\tau}\Psi \qquad\qquad \vec{Q}_R = \frac{1}{4}\Psi^\dagger(1+\gamma_5)\vec{\tau}\Psi \tag{2}$$

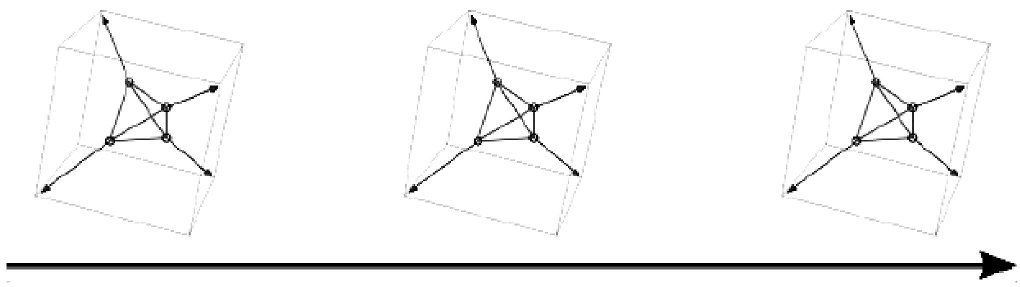

Figure 1: The global ground state after the electroweak symmetry breaking has occurred, considered at Planck scale distances. The big black arrow represents 3-dimensional physical space. Before the symmetry breaking the isospin vectors are directed randomly, thus exhibiting a local SU(2) symmetry, but once the temperature drops below the Fermi scale Λ_F, they become ordered into a repetitive tetrahedral structure, thereby spontaneously breaking the initial SU(2). Note that the SM Higgs vev corresponds to the length of the aligned isospin vectors.

The SM SSB being realized by an alignment of the tetron isospins, it is not surprising that the masses of quarks and leptons, and thus the SM Yukawa couplings are determined by the interactions among those isospins. The simplest interaction Hamiltonian between isospin vectors of 2 tetrons i and j is of the form $H = -J\,\vec{Q}_i\vec{Q}_j$. So it has the form of a Heisenberg interaction - but for isospins, not for spins. The coupling J may be called an 'isomagnetic exchange coupling'.

In reality, the Hamiltonian H is more complicated, for several reasons:

• The appearance of antitetron degrees of freedom. This can be accounted for by using interactions both of $\vec{Q}_L$ and $\vec{Q}_R$ defined in (2)

$$H_H = -J_{LL}\,\vec{Q}_{Li}\vec{Q}_{Lj} - J_{LR}\,\vec{Q}_{Li}\vec{Q}_{Rj} - J_{RR}\,\vec{Q}_{Ri}\vec{Q}_{Rj} \tag{3}$$

for tetron fields located at tetrahedral sites $i, j = 1, 2, 3, 4$. As seen below, the 3 couplings J_{LL}, J_{LR} and J_{RR} can be roughly associated to the quark and lepton masses of the second family.

• In addition to the Heisenberg Hamiltonian (3) Dzyaloshinskii–Moriya interactions are to be considered. They will be shown to give the dominant mass contributions to the heavy family.

• Heisenberg and DM terms do not contribute at all to the masses m_e, m_u and m_d of the first family. Therefore, small torsional interactions have to be introduced. They are characterized by the exerting torques $dQ_{L,R}/dt$ being proportional to the isospins $Q_{L,R}$ themselves.

• The masses of the neutrinos are yet another story. While the interactions discussed so far are isospin conserving and leave the neutrinos massless, neutrino masses can arise only from isospin violation[2].

My presentation of the mass calculations begins with the Dzyaloshinskii-Moriya (DM) coupling, firstly because it is the dominant isospin interaction and secondly it gives masses only to the third family, i.e. to top, bottom and τ, while leaving all other quarks and leptons massless. Among all the fermion masses the top quark mass is by far the largest and is of the order of the Fermi scale. As turns out, this is no accident, but has to do with the largeness of the relevant DM coupling. The complete DM Hamiltonian reads

$$H_D = -K_{LL}\,(\vec{Q}_{Li} \times \vec{Q}_{Lj})^2 - K_{LR}\,(\vec{Q}_{Li} \times \vec{Q}_{Rj})^2 - K_{RR}\,(\vec{Q}_{Ri} \times \vec{Q}_{Rj})^2 \tag{4}$$

with DM couplings K_{LL}, K_{LR} and K_{RR}.

It is convenient to already include at this point the Heisenberg terms (3). They give masses both to the second and third family (but not to the first one) and their couplings J are typically smaller than 1 GeV, while the DM couplings K are larger. Altogether, Heisenberg and DM terms provide the most general isotropic and isospin conserving interactions within the internal space. Apart from that there will only be tiny torsional interactions responsible for the mass of the first family.

The masses of the excitations δQ can be calculated by diagonalizing torque equations of the generic form $dQ/dt = i[H, Q]$, and using the angular momentum commutation relations for the isospin vectors

$$[Q^a_{Ri}, Q^b_{Rj}] = i\delta_{ij}\epsilon^{abc}Q^c_{Ri} \qquad [Q^a_{Li}, Q^b_{Lj}] = i\delta_{ij}\epsilon^{abc}Q^c_{Li} \qquad [Q^a_{Ri}, Q^b_{Lj}] = 0 \tag{5}$$

where $i, j = 1, 2, 3, 4$ count the 4 tetrahedral edges and $a, b, c = 1, 2, 3$ the 3 internal directions(=extra dimensions).

The 24 first order differential equations for the δQ are rather lengthy. In linear approximation they read

$$\begin{aligned} \frac{d\vec{\delta}_{Li}}{dt} &= 2K_{LL}\{\vec{Q}_0 \times \vec{\Delta}_{LLi} + i[-\vec{\Delta}_{LLi} + (\vec{\Delta}_{LLi}.\vec{Q}_0)\,\vec{Q}_0]\} + 2K_{LR}\{\vec{Q}_0 \times \vec{\Delta}_{LRi} + i[-\vec{\Delta}_{LRi} \\ &\quad +(\vec{\Delta}_{LRi}.\vec{Q}_0)\,\vec{Q}_0]\} + J_{LL}(\vec{Q}_0 \times \vec{\Delta}_{LLi}) + J_{LR}(\vec{Q}_0 \times \vec{\Delta}_{LLi}) \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{d\vec{\delta}_{Ri}}{dt} &= 2K_{RR}\{\vec{Q}_0 \times \vec{\Delta}_{RRi} + i[-\vec{\Delta}_{RRi} + (\vec{\Delta}_{RRi}.\vec{Q}_0)\,\vec{Q}_0]\} + 2K_{LR}\{\vec{Q}_0 \times \vec{\Delta}_{RLi} + i[-\vec{\Delta}_{RLi} \\ &\quad +(\vec{\Delta}_{RLi}.\vec{Q}_0)\,\vec{Q}_0]\} + J_{RR}(\vec{Q}_0 \times \vec{\Delta}_{RRi}) + J_{LR}(\vec{Q}_0 \times \vec{\Delta}_{RLi}) \end{aligned} \tag{7}$$

In these equations $\vec{\delta}_{Li} = \vec{Q}_{Li} - \langle\vec{Q}_{Li}\rangle$ and $\vec{\delta}_{Ri} = \vec{Q}_{Ri} - \langle\vec{Q}_{Ri}\rangle$, $a = 1, 2, 3, 4$, denote the isospin vibrations and the Δ's are certain linear combinations of them which are important to maintain isospin conservation[2]:

$$\vec{\Delta}_{LLi} = -3\,\vec{\delta}_{Li} + \sum_{j\neq i} \vec{\delta}_{Lj} \qquad \vec{\Delta}_{LRi} = -3\,\vec{\delta}_{Li} + \sum_{j\neq i} \vec{\delta}_{Rj} \qquad \vec{\Delta}_{RLi} = -3\,\vec{\delta}_{Ri} + \sum_{j\neq i} \vec{\delta}_{Lj} \tag{8}$$

Eqs. (6) and (7) correspond to a 24×24 eigenvalue problem which - after the SSB - leads to 6 singlet and 6 triplet states, the latter ones each consisting of 3 degenerate eigenstates (corresponding to three quark colors). After diagonalization one obtains the following results: the first family excitations are still massless at this point, but will get masses from the torsional interactions to be discussed below. The DM exchange coupling K_{LL} is consistently of the order of the transition energy Λ_F resp. the top quark mass, and the DM and Heisenberg couplings can be accommodated to reproduce the third and second family masses. Namely, assuming the DM couplings K to dominate over the Heisenberg couplings J, one can prove the following approximate relations

$$m_t = 4K_{LL} \qquad m_\tau = \frac{3}{2}K_{LR} \qquad m_b = 4K_{RR} \qquad m_c = J_{LL} \qquad m_\mu = \frac{3}{2}J_{LR} \qquad m_s = J_{RR} \tag{9}$$

One concludes that in this approximation, the masses of quarks and leptons arise from different isospin interaction terms in (3) and (4), each mass associated essentially to one of the interactions.

It was seen above how the heaviness of the third family is related to large DM couplings. Afterwards masses of the quarks and leptons of the second family were obtained from Heisenberg exchange. It then remains to show how the small masses of the first family can be obtained from isospin conserving torsional interactions. Actually, torsional interactions give contributions to the masses of all families. However, since they are assumed to be small, the 2 heavy families remain dominated by DM and Heisenberg couplings, as given in (9). The structure of torsional interactions is quite simple. Using the notation introduced in (8)

$$\frac{d\vec{\delta}_{Li}}{dt} = iC_{LL}\vec{\Delta}_{LLi} + iC_{LR}\vec{\Delta}_{LRi} \qquad\qquad \frac{d\vec{\delta}_{Ri}}{dt} = iC_{LR}\vec{\Delta}_{RLi} + iC_{RR}\vec{\Delta}_{RRi} \tag{10}$$

with torsional couplings C_{LL}, C_{LR} and C_{RR}. Since (10) gives the only mass contributions to the first family, the C-couplings can be chosen to accommodate the mass of the up quark, down quark and electron, respectively. Namely, one arrives at the mass formulas

$$m_e = 6C_{LR} \quad m_u = -2C_{LL} + 3C_{LR} + 2C_{RR} - W_C \quad m_d = -2C_{LL} + 3C_{LR} + 2C_{RR} + W_C \tag{11}$$

where $W_C := \sqrt{4(C_{LL}+C_{RR})^2 + C_{LR}^2}$. Then, using the phenomenological values $m_e = 0.51$ MeV, $m_u = 1.9$ MeV and $m_d = 4.7$ MeV one obtains

$$C_{LR} = 0.085\ MeV \qquad C_{LL} = 1.13\ MeV \qquad C_{RR} = 0.49 MeV \tag{12}$$

References

[1] B. Lampe, *Analytic and Parameter-Free Formula for the Neutrino Mixing Matrix*, arXiv:2308.08498 (hep-ph).

[2] B. Lampe, Int. J. Mod. Phys. A30 (2015) 1550025.