The discovery of the Higgs boson at the LHC in 2012 by the ATLAS and CMS experiments marked a significant milestone, subsequently elevating the measurement of Higgs boson properties to a priority within the experimental particle physics community. As of now, measurements at the LHC have including the Yukawa couplings to the third family (t, b, and τ) and the Higgs boson mass. The European Organization for Nuclear Research (CERN) has proposed the Future Circular Collider (FCC) as a successor to the Large Hadron Collider, with an aim to achieve high-precision measurements of Higgs boson properties. This document explains the utilization of the "recoil mass" technique to assess several key parameters at the FCC, including the Higgs boson mass, ZH cross-section, Higgs boson self-coupling, various Higgs boson couplings, Higgs boson width, and the electron Yukawa coupling. This document aims to present the potential advancements in our understanding of Higgs boson properties with FCC.
1. FCC Program

The Future Circular Collider (FCC) is a project proposed by the European Organization for Nuclear Research (CERN). The current strategy for the FCC, settled in 2020, designates an electron–positron Higgs boson factory as the top-priority facility following the Large Hadron Collider (LHC), coupled with the examination of the technical and financial feasibility of such a Higgs boson factory, succeeded by a hadron collider situated in the same tunnel, which is about 90km in length. Over the 18 years of preparation, the five-year feasibility study for FCC is slated to commence in parallel with the LHC Run 3 in 2021. The subsequent European Strategy Update is scheduled around 2026 to deliberate on the project’s approval. If green-lighted, the civil engineering, along with accelerator and detector constructions, will start. The FCC integrated program (FCC-INT) is similar to the LEP-LHC program (the Large Electron-Positron collider followed by the LHC in the same tunnel).

The FCC is planned to first operate as an electron-positron collider for 15 years, working at different centre-of-mass energy. It starts running at the $Z$ pole for four years, aiming to collect $150 \text{ ab}^{-1}$ of data. Next, it will focus on $W^+W^-$ production for two years, followed by operating at the ZH threshold (240 GeV) for three years to gather $7.2 \text{ ab}^{-1}$ of data. This phase will help in accurately measuring Higgs boson properties. The final stage involves increasing energy to 365 GeV to study $t\bar{t}$ events. There’s also an option to operate at the Higgs boson mass (125 GeV) for direct measurement of the electron Yukawa coupling.

After this phase, the FCC will be upgraded to a proton-proton collider, called FCC-hh, aiming for a maximum energy of 100 TeV and $30 \text{ ab}^{-1}$ of integrated luminosity. This upgrade will allow for detailed exploration of TeV scale physics and the production of Higgs bosons at high transverse momentum through various channels.

2. "Recoil mass" method

The recoil mass technique is frequently employed in lepton collider physics studies due to the precise understanding it provides regarding the initial details of a given physics process at lepton colliders. The mass $m_{\text{rec}}$ recoiling against the lepton pair is deduced utilizing total energy-momentum conservation, as depicted in Equation 1 and illustrated in Figure 1a by calculating the difference of the four-vector of centre-of-mass energy and lepton pair system.

$$m_{\text{recoil}}^2 = (\sqrt{s} - E_{\ell^+\ell^-})^2 - p_{\ell^+\ell^-}^2 = s - 2E_{\ell^+\ell^-}\sqrt{s} + m_{\ell^+\ell^-}^2$$  \hspace{1cm} (1)

In this equation, $\sqrt{s}$ signifies the centre-of-mass energy, $E_{\ell^+\ell^-}$ denotes the energy of the di-lepton pair, and $m_{\ell^+\ell^-}$ refers to the invariant mass of the di-lepton pair.

As it uses the centre-of-mass energy, the recoil mass is sensitive to its exact knowledge, which might be impacted by the beam energy spread (BES) and initial state radiation (ISR). The main backgrounds come from the WW, ZZ, and $Z/\gamma$ processes, as illustrated in Figure 1b, which exhibits the $m_{\text{rec}}$ distribution of both signal and background events in the range 40 to 160 GeV. Two distinct peaks are apparent: the larger one, around 91 GeV, arises from the ZZ process; the other one, around 125 GeV, derives from the $e^+ + e^- \rightarrow ZH$ process.
3. Higgs boson measurements at FCC

Various feasibility studies are underway within the FCC community to understand the capability to measure the Higgs boson properties. In this document, a selection of significant results will be presented.

3.1 Higgs boson mass measurement

The Higgs boson mass ($m_H$) can be directly extracted from the recoil mass ($m_{\text{recoil}}$) distribution. A customised probability distribution function (p.d.f.) termed 2CBG (two Crystal Ball functions sharing the same Gaussian core but with exponential tails on different sides; a Gaussian distribution is applied to cope with the heavy right tail) was employed to model the signal shape. The backgrounds are modeled using a third-order polynomial. The Higgs boson mass ($m_H$) is injected as a Parameter Of Interest (POI) in the fit. We conclude that with $7.2 \, \text{ab}^{-1}$ integrated luminosity, the statistic-only uncertainty on Higgs boson mass is $3.1$ MeV. After including systematic uncertainties, it is downgraded to $4.0$ MeV. The beam energy spread (BES), centre-of-mass energy ($\sqrt{s}$), and lepton scales are considered in the systematic uncertainties, with the centre-of-mass energy varied at $2$ MeV emerging as the leading systematic uncertainty.

Extended studies are also conducted to understand detector effects. Transitioning from a crystal calorimeter to dual readout, and degrading the electron resolution, the uncertainty increases to $4.1$ MeV. Elevating the magnetic field from $2$ T to $3$ T for better tracking reduces the uncertainty to $2.6$ MeV. Replacing the IDEA drift chamber with a CLD silicon tracker decreases the uncertainty to $4.7$ MeV. Increasing the BES from $1\%$ (nominal) to $6\%$ raises the uncertainty to $4.7$ MeV. Disabling the BES decreases the uncertainty to $3.0$ MeV. Assuming perfect momentum resolution, the uncertainty further diminishes to $3.3$ MeV.

3.2 ZH cross-section

The evaluation of the ZH cross-section is conducted in a model-independent manner, which implies a selection efficiency independent of the Higgs boson decay modes. This model-independent measurement is a distinguishing feature of lepton colliders, because of the known initial state.

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**Figure 1:** Figure a: Feynman diagram illustrating the Higgsstrahlung process and the recoil mass ($m_{\text{recoil}}$) calculation. From [3]. Figure b: The inclusive $m_{\text{recoil}}$ distribution for events where a $Z$ boson decays into a $\mu^+\mu^-$ pair. The Z and Higgs boson mass peaks are clearly visible in this distribution. Reproduced according to [1].
Subsequently, the ZH cross-section can be utilized to measure the Higgs boson couplings \( (H \rightarrow X \bar{X}) \) in a model-independent way employing the formula below:

\[
\sigma_{ZH} \times Br(H \rightarrow X \bar{X}) \propto \frac{g_{HZZ}^2 \times g_{HXX}^2}{\Gamma_H} \quad \text{and} \quad \sigma_{H \nu\nu} \times Br(H \rightarrow \bar{X}X) \propto \frac{g_{HWW}^2 \times g_{HXX}^2}{\Gamma_H}
\]  

(2)

In this expression, the left equation represents the cross-section for the ZH production mode, while the right equation represents the Vector Boson Fusion production mode. To maintain the model-independence, the ZH cross-section measurement employs a Boosted Decision Tree (BDT) approach to separate the signal from the background, thereby avoid varying selection efficiency on Higgs boson decay modes. A fit on the BDT response is applied to deduce the uncertainty on the cross-section. We conclude that with an integrated luminosity of \( 7.2 \text{ ab}^{-1} \), the statistical-only uncertainty of the ZH cross-section stands at 0.68%. Upon including the same systematics as in the Higgs boson mass measurement, the uncertainty marginally escalates to 0.69%. The impact of the systematic uncertainty on this measurement is negligible.

### 3.3 Higgs boson couplings

At the FCC, Higgs boson couplings \( (g_{HXX}) \) can be directly measured across various Higgs boson decay final states of the ZH process \( (H \rightarrow HH \text{ and } Z \rightarrow YY) \), as described in Equation (2).

#### Higgs boson to visible

The Higgs boson decays to bottom quarks \( (H \rightarrow b\bar{b}) \), charm quarks \( (H \rightarrow c\bar{c}) \), strange quarks \( (H \rightarrow s\bar{s}) \), and gluon pairs \( (H \rightarrow gg) \) are examined using the ZH production mode, at a centre-of-mass energy of \( \sqrt{s} = 240 \text{ GeV} \). For the Z leptonic decay channels \( (Z \rightarrow \ell^+ \ell^-) \), although the environment is clean, the signal acceptance is relatively smaller. A 1-D fit is employed on the recoil mass \( (m_{\text{rec}}) \) distribution. In the case of Z decays to neutrinos \( (Z \rightarrow \nu\bar{\nu}) \), there exists a favorable balance between signal acceptance and purity. A 2-D fit is applied on the missing mass \( (m_{\text{miss}}) \) and visible mass \( (m_{\text{vis}}) \) distributions. The Z hadronic decay channel \( (Z \rightarrow q\bar{q}) \) exhibits the largest signal acceptance, and the corresponding studies are currently in progress.

The initial results were derived with an integrated luminosity of 5 \text{ ab}^{-1}, and subsequently scaled to 7.2 \text{ ab}^{-1} for further analysis.

#### Table 1: Uncertainty of the Higgs boson couplings with Z decays to leptons \( (Z(\ell^+ \ell^-)H) \) or neutrinos \( (Z(\nu\bar{\nu})H) \). The initial results were derived with an integrated luminosity of 5 \text{ ab}^{-1}, and subsequently scaled to 7.2 \text{ ab}^{-1} for further analysis

<table>
<thead>
<tr>
<th>( \delta\mu/\mu ) [%]</th>
<th>( b\bar{b} )</th>
<th>( c\bar{c} )</th>
<th>( gg )</th>
<th>( s\bar{s} )</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z(\ell^+ \ell^-)H )</td>
<td>0.68</td>
<td>4.11</td>
<td>2.28</td>
<td>314.84</td>
<td>1.83</td>
</tr>
<tr>
<td>( Z(\nu\bar{\nu})H )</td>
<td>0.31</td>
<td>2.08</td>
<td>1.04</td>
<td>133.33</td>
<td>1.22</td>
</tr>
</tbody>
</table>

#### Higgs boson to invisible

Dark matter may interact with ordinary matter solely through the Higgs boson (Higgs Portal models). Hence, it is imperative to investigate the potential of this channel in discovery new physics. The Z boson decays into various particles were utilized for this exploration,
including decays to electrons ($Z \rightarrow e^+e^-$), muons ($Z \rightarrow \mu^+\mu^-$), bottom quarks ($Z \rightarrow b\bar{b}$), and light quarks ($Z \rightarrow q\bar{q}$), while the Higgs boson decays into a $Z$ boson pair and subsequently into neutrinos ($H \rightarrow ZZ^* \rightarrow \nu\bar{\nu}\nu\bar{\nu}$). A fit was conducted on the missing mass distribution to further analyze this interaction. We deduce that with an integrated luminosity of $5 \text{ ab}^{-1}$, the Standard Model branching ratio $Br(H \rightarrow \text{invisible})$ can be measured with an uncertainty of about 45%, predominantly driven by the hadronic channels. If the Standard Model signal is treated as background, for a $5\sigma$ discovery, the branching ratio for Higgs boson decaying to Dark Matter, $Br(H \rightarrow D.M.)$, should be at least $0.23\%$.

### 3.4 Higgs boson self-coupling

The direct measurement of Higgs boson self-coupling is not feasible at FCC-ee. However, the Higgs boson self-coupling can be measured through the Next-to-Leading Order (NLO) variations on the total $ZH$ cross-section, because of the sufficient statistics. The relationship is expressed as:

$$\Sigma_{NLO} = Z_H \Sigma_{LO}(1 + \kappa_4 C_1)$$

Here, $\Sigma_{NLO}$ denotes the NLO total $ZH$ cross-section, $Z_H$ is a normalization factor, $\Sigma_{LO}$ signifies the LO total $ZH$ cross-section, and $\kappa_4$ represents the ratio of the measured Higgs boson self-coupling to the Standard Model prediction value ($\kappa_4 \equiv \lambda_3/\lambda_3^{SM}$). The decay of the $Z$ boson to electrons ($Z \rightarrow e^+e^-$), muons ($Z \rightarrow \mu^+\mu^-$), and quark channels are explored. For this analysis, assumptions include a 0.1% luminosity uncertainty, a 1% selection efficiency uncertainty, a 2.8 MeV uncertainty on the centre-of-mass energy, $m_H = 125.38 \pm 0.14$ GeV, with the Higgs boson decay branching ratio $Br(H \rightarrow b\bar{b})$ fixed to Standard Model values. We conclude that the uncertainty of the Higgs boson self-coupling will be narrowed down to 25% (initially obtained with an integrated luminosity at $5 \text{ ab}^{-1}$, and then re-scaled to $7.2 \text{ ab}^{-1}$). The sensitivity is primarily driven by the $Z(q\bar{q})H$ channel. Incorporating the $ZH$ production at $\sqrt{s} = 365$ GeV resolves the degenerated minima of the $\kappa_4$ scan.

### 3.5 Higgs boson Width

Following obtaining of the $ZH$ cross-section, the Higgs boson width ($\Gamma_H$) can be measured utilizing individual Higgs boson decay modes ($H \rightarrow XX$). Equation (2) can be reformulated as follows:

$$\Gamma_H \propto \frac{\sigma(e^+e^- \rightarrow ZH)^2}{\sigma(e^+e^- \rightarrow ZH, H \rightarrow ZZ)}$$

and

$$\Gamma_H \propto \frac{\sigma(e^+e^- \rightarrow \nu\bar{\nu}H, H \rightarrow b\bar{b})\sigma(e^+e^- \rightarrow ZH)^2}{\sigma(e^+e^- \rightarrow ZH, H \rightarrow b\bar{b})\sigma(e^+e^- \rightarrow ZH, H \rightarrow W^+W^-)}$$

Equation (4) clarifies the method of measuring the Higgs boson width at 240 GeV, while Equation (5) explained the method at 365 GeV. At 240 GeV, the Higgs boson width can be measured using the $H \rightarrow ZZ$ channel, whereas both 240 GeV and 365 GeV facilitate measurements through the $H \rightarrow b\bar{b}$ channel. The present study is centered on $\sqrt{s} = 240$ GeV. At this energy point, the Higgs boson decays to one $Z$ boson and one off-shell $Z$ boson ($H \rightarrow ZZ^*$), counting in a total of three $Z$ bosons. Scenarios where one $Z$ decays to a lepton pair, another $Z$ boson decays to jets, and the final $Z$ boson...
decays to neutrinos are examined \((Z(\ell^+\ell^-)Z(jj)Z(\bar{\nu}\nu))\). The uncertainty on the Higgs boson width is gauged utilizing the cross-section \(\sigma(e^+e^- \rightarrow ZH, H \rightarrow ZZ)\). A fit is administered on the BDT scores. We infer that an uncertainty of 3.8% can be attained using this channel (results were initially procured using 5 ab\(^{-1}\) and then scaled to 7.2 ab\(^{-1}\)). By including all other channels, we anticipate achieving a 1% uncertainty on the Higgs boson width.

3.6 The electron Yukawa coupling

The examination of the electron Yukawa coupling has been explored in [2]. Upon attaining MeV precision on the Higgs boson mass, the electron Yukawa coupling could be measured through the resonant s-channel process \(e^+e^- \rightarrow H\), by deploying the FCC-ee at \(\sqrt{s} = m_H\). To date, measurements of Yukawa couplings have been conducted for the top \((t)\), bottom \((b)\) quarks, and the \(\tau\) lepton. As the high luminosity LHC (HL-LHC), efforts to probe the Higgs boson Yukawa couplings to second family fermions will have been initiated. However, the endeavor to probe the Yukawa coupling to the first family fermion poses a challenge, given the Higgs boson branching ratio’s proportionality to the square of the fermion mass. Several challenges confront this measurement: First, accurately determining the Higgs boson mass through \(ZH\) recoil studies is essential. Second, precision to the MeV level is required for understanding ISR and BES. Third, a thorough understanding of various backgrounds is critical. Despite these hurdles, the project is highly motivated by its potential to advance fundamental physics, enabling experimental investigation of the Higgs mechanism in the first fermion family and study of particles nearly identical in mass to the Higgs boson.

3.7 FCC-hh

The rare decay channels will remain statistically limited at FCC-ee, but can benefit from the high-luminosity of FCC-hh. Most of the Higgs boson couplings will achieve statistical precision at 1% level. The systematics largely cancel by measuring ratios \(BR(\gamma\gamma/4l), BR(\mu\mu/4l), BR(\gamma\gamma/4l)\), and \(BR(\gamma\gamma/\mu\mu)\).

4. Conclusion

Various Feasibility Studies matured and on-going. Together, FCC-ee and FCC-hh provide the highest possible precision among all future facilities in the Higgs sector.

References

