Determination of CP-violating Higgs couplings with transversely-polarized beams at the ILC

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We study possible CP-violation effects of the Higgs to Z-boson coupling at a future e^+e^- collider, e.g. the International Linear Collider (ILC). We find that the azimuthal angular distribution of the muon pair, produced by $e^+e^- \rightarrow HZ \rightarrow H\mu^+\mu^-$, can be sensitive to such a CP-violation effect when we apply initial transversely polarized beams. Based on this angular distribution, we construct a CP sensitive asymmetry and obtain this asymmetry by Whizard simulation. By comparing the SM prediction with 2σ range of this asymmetry, we estimate the limit of the CP-odd coupling in HZZ interaction.

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1. Introduction

As we know from the Cosmic Microwave Background observation [1], the universe has much larger baryon number density than the SM prediction. In order to explain this discrepancy, the additional source of CP-violation has to be introduced according to Sakharov conditions for Baryogenesis [2]. In particular, one of the possible CP-violation source can be introduced in the Higgs sector, which is generated by the Two-Higgs-Doublet Model (2HDM) with spontaneous CP violation [3]. In this model, the CP-even 125 GeV Higgs boson gets the CP-odd admixture and the Higgs to fermions couplings are CP-violating. So far, such a Baryogenesis and CP-violation effect has been exploited by [4] via Htt interaction at LHC , and via $H\tau\tau$ at Higgs factories [5].

Furthermore, the tree-level CP-violating Higgs to fermions couplings can contribute a CP-violating Higgs to gauge bosons couplings at one-loop level. So far the LHC experiments, ATLAS [6] and CMS [7, 8] perform the measurement for the HVV coupling via $H \to 4\ell$ decay, and yield a limit of CP-odd HVV coupling, which can be interpreted as $(\tilde{c}_{AZZ})_{\text{CMS}} \sim [-1.65, 0.63]$ and $(\tilde{c}_{AZZ})_{\text{ATLAS}} \sim [-1.2, 1.75]$. On the other hand, one can probe directly the CP-violating HVV couplings at e^+e^- colliders (CEPC [9] and ILC), since a CP-violating HVV coupling can lead to deviations in the Higgs strahlung process at e^+e^- colliders. In addition, the initial beams polarization can be applied at the ILC [10], where particularly the transverse polarization can be used to construct new CP sensitive observables [10, 11].

In this work, we will focus on the Higgs strahlung process at the ILC with a center of mass energy of 250 GeV and apply transversely polarized electron-positron beams. In such a case, we will study the azimuthal angular distribution of the muon pair, produced by the Z decay, and construct the naive T-odd observable to probe the CP-odd HZZ coupling. Eventually, we try to determine the limit of the CP-odd HZZ coupling by comparing with the SM CP-conserving 2σ limit.

2. Theoretical framework

2.1 CP-violation in HZZ coupling and EFT interpretation

Concerning the Higgs to Z boson interaction, we have the following effective lagrangian [12]:

$$\mathcal{L}_{\text{EFF}} = c_{\text{SM}} Z_\mu Z^\mu H + \frac{c_{HZZ}}{v} Z_{\mu\nu} Z^{\mu\nu} H + \frac{\tilde{c}_{AZZ}}{v} Z_{\mu\nu} \tilde{Z}^{\mu\nu} H \tag{1}$$

$$= \kappa_{\text{SM}} \cos\xi_{CP} Z_\mu Z^\mu H + \frac{\kappa_{HZZ}}{v} \cos\xi_{CP} Z_{\mu\nu} Z^{\mu\nu} H + \frac{\tilde{\kappa}_{AZZ}}{v} \sin\xi_{CP} Z_{\mu\nu} \tilde{Z}^{\mu\nu} H, \tag{2}$$

where the CP-violation angle is parameterized by ξ_{CP} and $\tilde{c}_{AZZ}$ denotes the loop-induced CP-odd coupling. In this case, the total cross-section with HZZ interaction is the linear combinations of the following three terms

$$\sigma_{\text{tot}} = |c_{\text{SM}}|^2 \sigma_{\text{SM}} + |c_{HZZ}|^2 \sigma_{HZZ} + |\tilde{c}_{AZZ}|^2 \sigma_{AZZ}. \tag{3}$$

In this study, we ignore the CP-even operator c_{HZZ} [13]. Furthermore, we can define a cross-section fraction [14] to parameterize the CP-violation:

$$f_{CPV} = \frac{|\tilde{c}_{AZZ}|^2 \sigma_{AZZ}}{|c_{\text{SM}}|^2 \sigma_{\text{SM}} + |\tilde{c}_{AZZ}|^2 \sigma_{AZZ}}, \tag{4}$$

where the SM contribution is fixed by setting $|c_{\text{SM}}| = 1$.

2.2 The scattering amplitude with polarized electron-positron beams

Concerning the polarization of the initial electron and positron beams, one can define a projection operator, which is given by:

$$\frac{1}{2}(1 - \boldsymbol{P} \cdot \boldsymbol{\sigma}) = \frac{1}{2}(\delta_{\lambda\lambda'} - P^a \sigma^a_{\lambda\lambda'}) = \frac{1}{2}\begin{pmatrix} 1 - P_3 & P_1 - iP_2 \\ P_1 + iP_2 & 1 + P_3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 - f\cos\theta_P & f\sin\theta_P e^{-i\phi_P} \\ f\sin\theta_P e^{i\phi_P} & 1 + f\cos\theta_P \end{pmatrix}, \quad (5)$$

where f is the polarization fraction. The Higgs strahlung $e^+ e^- \to ZH$ is the dominant Higgs production process at $e^+ e^-$ collider. By taking the polarization of the initial beams into account, the spin density matrix of the Higgs strahlung $\rho_{\lambda_r \lambda_u}$ can be derived, see following diagram,
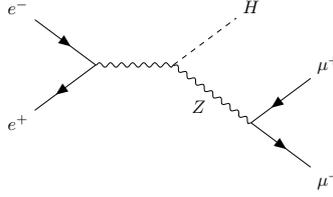
where the λ_r, λ_u are the spin indices of the initial electron and positron. Particularly, we obtain the spin density matrix by applying the Bouchiat-Michel formula [13, 15]:

$$u(p, \lambda'_r)\bar{u}(p, \lambda_r) = \frac{1}{2}\left(\delta_{\lambda_r \lambda'_r} + \gamma_5 \slashed{s}^a \sigma^a_{\lambda_r \lambda'_r}\right)(\slashed{p} + m), \quad (6)$$

$$v(p, \lambda'_u)\bar{v}(p, \lambda_u) = \frac{1}{2}\left(\delta_{\lambda_u \lambda'_u} + \gamma_5 \slashed{s}^a \sigma^a_{\lambda_u \lambda'_u}\right)(\slashed{p} - m), \quad (7)$$

where σ^a denotes the Pauli matrices, and the four-vector s^a_μ, $a = 1, 2, 3$ are the three spin vectors, which are both orthogonal to each other and the corresponding four momentum.

Furthermore, we introduced the CP-odd operator for the HZZ interaction as Eq. (2), and obtain the following form of the total amplitude squared:

$$
\begin{aligned}
|\mathcal{M}|^2 = {} & (1 - P^3_- P^3_+)(\cos^2 \xi_{CP}\, \mathcal{A}_1^{\text{CP-inv}} + \sin 2\xi_{CP}\, \mathcal{A}^{\text{CP-mix}} + \sin^2 \xi_{CP}\, \mathcal{A}_2^{\text{CP-inv}}) \\
& + (P^3_- - P^3_+)(\cos^2 \xi_{CP}\, \mathcal{B}_1^{\text{CP-inv}} + \sin 2\xi_{CP}\, \mathcal{B}^{\text{CP-mix}} + \sin^2 \xi_{CP}\, \mathcal{B}_2^{\text{CP-inv}}) \\
& + \sum_{mn}^{1,2} P^m_- P^n_+ \left(\cos^2 \xi_{CP}\, C_1^{mn\,\text{CP-inv}} + \sin 2\xi_{CP}\, C_{mn}^{\text{CP-mix}} + \sin^2 \xi_{CP}\, C_2^{mn\,\text{CP-inv}}\right).
\end{aligned}
\quad (8)
$$

Note that, the CP-invariant parts with $\cos^2 \xi_{CP}$ and $\sin^2 \xi_{CP}$ are both CP-conserving, while the CP-mixing terms with $\sin 2\xi_{CP}$ violate the CP symmetry. According to the analytical calculation, via the transversely polarized terms the contributions of the CP-violating terms $C_{mn}^{\text{CP-mix}}$ can be extracted by another triple-product, which is given by

$$C_{\text{CP-mix}}^{mn} \propto \epsilon_{\mu\nu\rho\sigma}[(p_{e^-} + p_{e^+})^\mu p_H^\nu p_{\mu^-}^\rho s_{e^-}^\sigma] \propto (\vec{p}_H \times \vec{p}_{\mu^-}) \cdot \vec{s}_{e^-} \propto (\vec{p}_{\mu^+} \times \vec{p}_{\mu^-}) \cdot \vec{s}_{e^-}. \quad (9)$$

The triple-product is the azimuthal-angle difference between the $\mu^+\mu^-$ plane and the spin of the electron. Therefore, defining the orientation of the azimuthal plane by fixing the direction of electron transverse polarization, the $C_{mn}^{\text{CP-mix}}$ depends directly on the azimuthal angle of the final state μ^-.
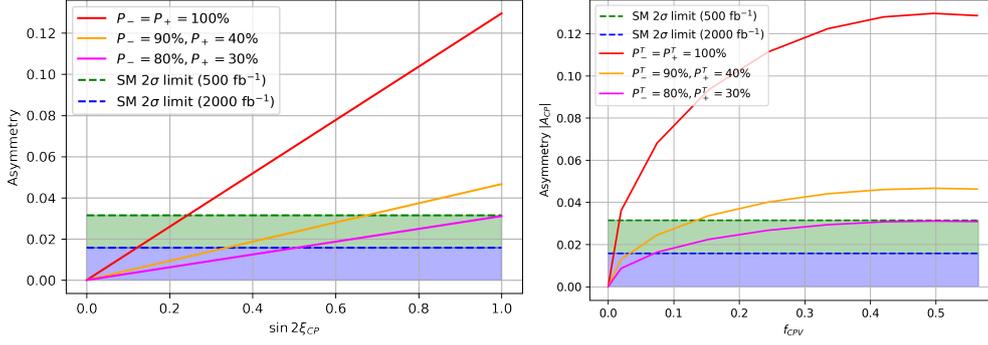
Figure 1: The analytical results of the asymmetries from Eq. (10) with the statistical uncertainties of the asymmetries. Left panel: varying $|\sin 2\xi_{CP}|$ and fixed total cross section $\sigma_{\text{tot}} = \sigma_{\text{SM}}$; right panel: varying CP-odd coupling and fixing $c_{\text{SM}} = 1$. The red solid line corresponds to the completely polarized beams $(P_-^T, P_+^T) = (100\%, 100\%)$, while the orange line and magenta line demonstrate the asymmetries with $(P_-^T, P_+^T) = (90\%, 40\%)$ polarized beams and $(P_-^T, P_+^T) = (80\%, 30\%)$ polarized beams respectively. The blue and green region are the region below the 2σ limit of SM CP-conserving case for 500 fb^{-1} and 2000 fb^{-1} respectively.

2.3 The asymmetry

Regarding the azimuthal angular distribution of the muon plane, one can define an observable $O_{CP} = \eta_H \sin 2\phi_{\mu^-}$. which sensitive to the CP-mixing effect. Hence, this asymmetry can be given by:

$$\mathcal{A}_{CP} = \frac{1}{\sigma_{\text{tot}}} \int \text{sgn}(O_{CP}) d\sigma = \frac{N(O_{CP} < 0) - N(O_{CP} > 0)}{N(O_{CP} < 0) + N(O_{CP} > 0)}, \quad (10)$$

where N denotes the corresponding number of events. Since the SM is CP conserving for the neutral current, the SM background for this asymmetry is negligible. However, the number of events fluctuates statistically leading to the uncertainty of this asymmetry, which is based on a binomial distribution and given by $\Delta\mathcal{A}_{CP} = \sqrt{\frac{1 - \mathcal{A}_{CP}^2}{N_{\text{tot}}}}$.

3. Analytical results

By taking the uncertainties of the asymmetry into account, one can potentially distinguish the CP-mixing cases from the SM case with a given integrated luminosity and derive the unique CP-violation effect.

Thus, we vary the CP-mixing angles from the CP-conserving case $|\sin 2\xi_{CP}| = 0$ to the maximal CP-mixing case $|\sin 2\xi_{CP}| = 1$, and present the asymmetries in Fig. 1, where we still fix the total cross section $\sigma_{\text{tot}} = \sigma_{\text{SM}}$ as used before.

As we can see in the figure, the CP-conserving case with $|\sin 2\xi_{CP}| = 0$ shows the vanishing asymmetry $\mathcal{A}_{CP}$, while this CP-sensitive asymmetry would be enhanced by increasing $|\sin 2\xi_{CP}|$. One can determine the CP-violation effect at 95% confidence level, when the asymmetry $\mathcal{A}_{CP}$ exceed the 2σ region of SM results. However, in Fig. 1, the configuration $(P_-^T, P_+^T) = (80\%, 30\%)$ and 500 fb^{-1} cannot generate a large enough asymmetry $\mathcal{A}_{CP}$, which is always within the SM 2σ range. Furthermore, we can use the polarization configuration of transversely polarized beams with

$(P_-^T, P_+^T) = (90\%, 40\%)$, which is the best polarization fraction for the electron and positron beams expected by the current set-up [10]. In this case, the limit of $|\sin 2\xi_{CP}|$, where the asymmetry $\mathcal{A}_{CP}$ can be distinguished from the SM CP-conserving case, can be improved by the increment of the polarization fraction.

On the other hand, we can just fix the SM tree-level contribution by $c_{\text{SM}} = 1$, and vary the $\tilde{c}_{AZZ}$ individually, and present the results of asymmetry $\mathcal{A}_{CP}$ in right panel of Fig. 1. This figure demonstrates that the asymmetry $\mathcal{A}_{CP}$ would reach a maximum for $f_{CPV} \approx 0.5$, where the CP-odd and CP-even interaction have equal contributions to the total cross-section. Note that, the maximum values of $\mathcal{A}_{CP}$ can be decreased by smaller transverse polarization fraction. The configuration with $(P_-^T, P_+^T) = (80\%, 30\%)$ polarized beams and the luminosity of 500 fb^{-1} is still insufficient to determine the CP structure of the HZZ interaction. However, the luminosity of 2000 fb^{-1} improves this sensitivity significantly, and the fraction $f_{CPV} \sim 0.1$ can be determined even by $(P_-^T, P_+^T) = (80\%, 30\%)$.

## 4. The limits for CP-violation at ILC

In order to estimate the limit for CP-violating effects, one has to perform the Monte Carlo simulation and to determine the parameter space where the CP-odd observable is distinguishable from the SM CP-conserving prediction by 95% C.L. In Fig 2, we present the results of the asymmetry $\mathcal{A}_{C\mathcal{P}}$ with varying CP-mixing angle and the fluctuating total cross-section, which are generated by Whizard. As we see in Fig. 2, the asymmetry is consistent with the analytical calculation in Fig. 1, and the asymmetry shows the statistical fluctuation in Fig. 2 for certain $|\sin 2\xi_{CP}|$ and for the integral luminosity 2000 fb^{-1} and 5000 fb^{-1}. Since the 500 fb^{-1} is insufficient to determine the CP-violation effect based on the previous discussion, we do not present the results with 500 fb^{-1} for the Monte Carlo simulation. However, the blue points with maximal CP-violation effect $|\sin 2\xi_{CP}| = 1$ can be still below SM 2σ limit. This means one cannot distinguish the CP-violation effect with the integrate luminosity of 2000 fb^{-1} and $(P_{e^-}^T = 80\%, P_{e^+}^T = 30\%)$ initial polarization, only taking the statistical uncertainties into account. In the right panel of Fig. 2, we present the results applying $(P_{e^-}^T = 90\%, P_{e^+}^T = 40\%)$ initial polarization, where the luminosity of 2000 fb^{-1} can be used to determine the limit of $\sin 2\xi_{CP}$.

Furthermore, we present the results with fixed the SM tree-level contribution but increasing the CP-odd coupling in Fig 3. We can observe that the variation of the asymmetry $\mathcal{A}_{CP}$ is basically the same as the analytical results in Fig. 1 but with additional statistical fluctuation. As the same as the discussion for previous scenario, we could read out the limit of CP-odd coupling $\tilde{c}_{AZZ}$ from the vertical dashed lines in both panels.

In order to summarize all the determination results, we present all limits for the CP-violation effect for the scenarios we discussed in Tab. 1.

5. Conclusion

In this work, we investigate the CP-violation effects in the process $e^+ e^- \rightarrow HZ \rightarrow H\mu^+\mu^-$ with initial transversely polarized beams. Based on the azimuthal angular distribution of the final state muons, we constructed a CP sensitive observable and exploited its dependence on the CP-violation
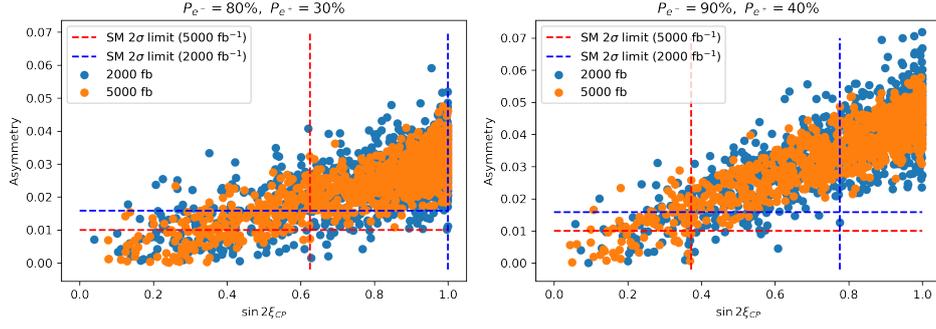
Figure 2: The Monte Carlo simulation results of the asymmetries generated by the `Whizard`, where the total cross-section is fixed to the SM tree-level value. The left panel corresponds to the polarization fraction $(P_-^T, P_+^T) = (80\%, 30\%)$, and the right panel corresponds to $(P_-^T, P_+^T) = (90\%, 40\%)$. For both figures, the results with luminosity 2000 fb^{-1} are demonstrated by the blue points and 5000 fb^{-1} for the orange points. In addition, the blue and orange horizontal dashed lines indicate the SM CP-conserving 2σ limit.
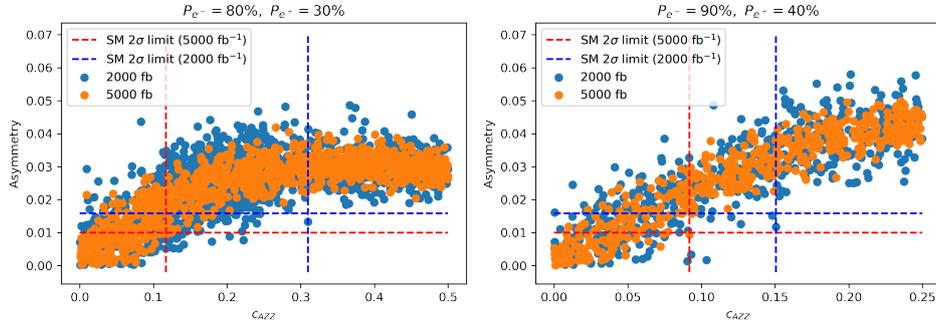
Figure 3: The Monte Carlo simulation results of the asymmetries generated by the `Whizard`, where the SM cross-section contribution is fixed. The left panel corresponds to the polarization fraction $(P_-^T, P_+^T) = (80\%, 30\%)$, and the right panel corresponds to $(P_-^T, P_+^T) = (90\%, 40\%)$. For both figures, the results with luminosity 2000 fb^{-1} are demonstrated by the blue points and 5000 fb^{-1} for the orange points. In addition, the blue and orange dashed horizontal lines indicate the SM CP-conserving 2σ limit for the asymmetry.

parameters. Furthermore, we could determine limit of the CP-violation effect by comparing with the 2σ limit of the SM CP-conserving prediction. Finally, we obtained the limit of the CP-odd coupling $|\tilde{c}_{AZZ}| \lesssim 0.09$ with 5000 fb^{-1} and $(P_{e^-}^T, P_{e^+}^T) = (90\%, 40\%)$ polarization, which is potentially better than the unpolarized result $|\tilde{c}_{AZZ}| \lesssim 0.15$ with 5600 fb^{-1} in [9]. Overall, these CP-odd coupling limit can be strongly improved at e^+e^- collider with initial beams polarization compared to current LHC measurement.

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$(P_{e^-}^T, P_{e^+}^T)$	Luminosity [fb^{-1}]	$\sin 2\xi_{CP}$ limit	c_{AZZ} limit
(80%, 30%)	2000	-	[-0.31,0.31]
(80%, 30%)	5000	[-0.62,0.62]	[-0.12,0.12]
(90%, 40%)	2000	[-0.79,0.79]	[-0.15,0.15]
(90%, 40%)	5000	[-0.39,0.39]	[-0.09,0.09]

Table 1: The summary table for the polarization fraction $(P_{e^-}^T, P_{e^+}^T) = (80\%, 30\%)$ and $(90\%, 40\%)$, and the integrated luminosity of 2000 fb^{-1} and 5000 fb^{-1}. The limit of $\sin 2\xi_{CP}$ can be determined by assuming the total cross-section is fixed, and the limit of individual $\tilde{c}_{AZZ}$ is determined by fixing to the SM tree-level cross-section.

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