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Solitons as ground states in supergravity theories

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We discuss recent soliton solutions in a class of $D = 4$ supergravity theories. For suitable values of the parameters, the new configurations can be embedded in the gauged maximal $\mathcal{N} = 8$ theory and uplifted in the higher-dimensional $D = 11$ theory. We also consider supersymmetric solutions and their stability, in light of the prescriptions of the positive energy theorem.

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1. Introduction

Extended supergravity theories are of great importance in high-energy physics, as they can arise as the field-theoretical limit of more fundamental superstring theories. It is therefore interesting to study how the structure of supergravity models is enriched by including non-perturbative degrees of freedom, this latter investigation also being motivated by the "duality conjecture", whereby classical solutions play an important role in the strong-coupling limit.

Solitons have emerged as important tools for studying non-perturbative aspects of quantum field theories. They are stationary, finite energy, non-singular solutions of the field equations. Solitons are non-perturbative since they are solutions to non-linear equations which cannot be found by perturbation of the linearized field equations. Their existence suggests that the full theory may turn out to be a much richer framework than appears in perturbation theory. Gravitational solitons in AdS were first considered in [1] and later developed as models of scalar fields coupled to gravity [2], allowing for non-trivial boundary conditions for the scalars. The latter conditions correspond to deformations of the boundary CFT by multitrace operators [3, 4].

Soliton solutions have been extensively explored in the context of positive energy theorems for AdS spacetimes [1], also in the presence of multitrace boundary conditions [5, 6]. In these asymptotically AdS models, CFT states correspond to the condensation of charged fields in the bulk gravity theory, where charged solitons were argued to be the ground states of the theory with fixed charges [7, 8].

A considerable interest has been generated by the existence of supersymmetric solitons. These solutions play an important role in the study of the non-perturbative sector of string theory and in understanding string dualities. In such configurations supersymmetry can be only partially broken, and each of the unbroken supersymmetries is associated with a Killing spinor satisfying a suitable set of linear differential constraints. The corresponding integrability conditions are then formulated as non-linear BPS equations for the solitonic background. The analysis of the BPS conditions has proven to be an efficient way to investigate the non-perturbative dynamics of the full theory. In this regard, the analysis of supersymmetric configurations provides an efficient tool for solving the full equations of motion, being the resulting equations typically of first order as compared to the second order equations of motion.

In the following we consider soliton solutions in a $D = 4$, $\mathcal{N} = 2$ gauged supergravity theory, in the presence of Fayet-Iliopoulos terms [9]. The latter solutions are obtained by a double Wick rotation applied to the electrically charged hairy black hole solutions of [10]. We also consider the existence of BPS conditions preserving part of the supersymmetry. Surprisingly, it is possible to show that, for suitable choices of the parameters, non-susy solutions exist characterized by lower energy than susy configurations, for the same boundary conditions and asymptotic charges.

2. The model

We consider a dilaton truncation of the STU model [11–14] in a $\mathrm{SO}(8)$ gauged maximal $\mathcal{N} = 8$ supergravity. If we set all the dilatons to the same value, imposing at the same time suitable conditions on the vector fields, we obtain the so-called T^3 model, featuring an action of the form

$$\mathcal{S}=\frac{1}{\kappa}\int d^4x\,\sqrt{-g}\left(\frac{R}{2}-\frac{1}{2}\,(\partial\phi)^2+\frac{3}{L^2}\,\cosh\left(\sqrt{\frac{2}{3}}\,\phi\right)-\frac{1}{4}\,e^{3\sqrt{\frac{2}{3}}\,\phi}\left(F^1\right)^2-\frac{1}{4}e^{-\sqrt{\frac{2}{3}}\,\phi}\left(F^2\right)^2\right). \quad (1)$$

Soliton solution. Starting from a black hole solution, a soliton can be found acting by a double analityc continuation of the former black hole configuration [15–18]. Let us then consider the class of black hole solutions of [10], where a class of $\mathcal{N}=2$ supergravities is found to interpolate between all possible single-dilaton truncations of the ω-deformed [19–21] gauged maximal theory. Restricting to their $\nu=-2$ case, we obtain a model embedded in a T^3 theory with action (1), that is, a consistent dilatonic truncation of the gauged maximal $\mathcal{N}=8$ supergravity. If we now apply the double Wick rotation

$$t\to i\,\varphi\,,\qquad \varphi\to i\,t\,, \quad (2)$$

to the electrically charged black holes of [10], we obtain a new solution that reads [9]

$$\begin{aligned}
&e^0=\sqrt{\Upsilon(x)}\,dt\,,\qquad e^1=\sqrt{\frac{\Upsilon(x)}{f(x)}}\,\eta\,dx\,,\qquad e^2=\sqrt{\Upsilon(x)\,f(x)}\,d\varphi\,,\qquad e^3=\sqrt{\Upsilon(x)}\,dz\,,\\
&\phi=\sqrt{\frac{3}{2}}\,\ln(x)\,,\qquad A^1=Q_1\left(x^{-2}-x_0^{-2}\right)\,d\varphi\,,\qquad A^2=Q_2\left(x^2-x_0^2\right)\,d\varphi\,,
\end{aligned} \quad (3)$$

where we have also redefined the charges as $Q_{1,2}\to i\,Q_{1,2}$, while $\Upsilon(x)$, $f(x)$ are expressed as

$$\Upsilon(x)=\frac{4\,L^2\,x}{\left(x^2-1\right)^2\,\eta^2}\,,\qquad f(x)=1+\frac{\eta^2\,\left(x^2-1\right)^3\,\left(3\,Q_1^2-x^2\,Q_2^2\right)}{6\,L^2\,x^2}\,. \quad (4)$$

The conformal boundary is reached for $x=1$, the spacetime being splitted in two inequivalent regions $x\in(0,1)$ and $x\in(1,\infty)$, characterizing the sign of ϕ. The point x_0 is such that $f(x_0)=0$, and determine the location where a φ-circle contracts in the interior of the geometry (x_0 identifies a sort of axis of symmetry of the solution and is not the location of an horizon). The above (3) then identifies a regular and horizon-free spacetime configuration, a charged gravitational soliton.

An expansion of the metric around x_0 leads to the definition of a parameter Δ such that $\varphi\in[0,\,\Delta]$, with

$$\Delta^{-1}=\left|\frac{1}{4\pi\,\eta}\,\frac{df}{dx}\right|_{x=x_0}=\left|\frac{\eta\,\left(x_0^2-1\right)^2}{4\pi\,L^2\,x_0^3}\,\left(Q_1^2\left(1+2\,x_0^2\right)-Q_2^2\,x_0^4\right)\right|\,. \quad (5)$$

3. Discussion

In the following, we briefly discuss supersymmetric solutions and their stability under phase transitions.

3.1 BPS solutions

A direct computation of the Killing spinor equations shows that supersymmetric configurations satisfy the following relation between the charges [9]:

$$Q_1 = -\frac{1}{\sqrt{3}}\,Q_2\,. \tag{6}$$

In particular, four chiral spinors are found and, since we are working in a $\mathcal{N}=2$ theory, the susy configuration is 1/4 BPS [22]. If we consider the embedding of our model in the maximal theory, the supersymmetric configuration turns out to be 1/8 BPS with respect to the $\mathcal{N}=8$ framework.

3.2 Fixed charges boundary conditions

We now want to study the stability of our solution for special boundary conditions. In particular, we consider a fixed charge background (sometimes referred to as canonical ensemble). To this end, we keep fixed the ratios Q_1/η, Q_2/η and the period Δ, and define the rescaled charges as

$$q_{1,2} \equiv \frac{\Delta^2}{4\pi^2 L}\,\frac{Q_{1,2}}{\eta}\,. \tag{7}$$

We also write the parameter η in terms of the above quantities as [9]

$$\eta = \frac{3\,\Delta}{2\pi}\,\frac{\left|\left(q_2^2-3\,q_1^2\right)-2\left(1-x_0^2\right)\left(q_2^2-q_1^2\right)+\left(1-x_0^2\right)^2 q_2^2\right|}{x_0\left(1-x_0^2\right)\left(3\,q_1^2-q_2^2+q_2^2\left(1-x_0^2\right)\right)}\,. \tag{8}$$

Supersymmetric solutions and stabilty. In this canonical (fixed charges) ensemble, to study the phase structure and stability, we must observe the system free energy. The latter is expressed as

$$\frac{S_{\mathrm{E}}}{V} = -\frac{\mu}{2\,L^2\,\kappa}\,, \tag{9}$$

in terms of the μ mass parameter [9]

$$\mu = \mp\frac{4\,L^2}{3\,\eta}\left(3\,Q_1^2-Q_2^2\right) = \mp\left(\frac{2\sqrt{2}\,\pi\,L}{\Delta}\right)^4\frac{\left(3\,q_1^2-q_2^2\right)\,\eta}{3}\,. \tag{10}$$

The above (6) can be rewritten in terms of the rescaled charges as

$$q_1 = -\frac{1}{\sqrt{3}}\,q_2\,, \tag{11}$$

while the expressions of the metric function $f(x_0)$ and the η parameter simplify in

$$f(x_0) = 1-\frac{q_1^2}{2\,x_0^6}\left(1+3\,x_0^2\right)^4 = 0\,, \qquad \eta = \frac{\Delta}{2\pi}\,\frac{1+3\,x_0^2}{x_0\left(1-x_0^2\right)}\,. \tag{12}$$

In order to study the stability of the above hairy supersymmetric configuration, we consider the pure Einstein-Maxwell solution of [23], characterized by vanishing scalars and a single vector field with charge Q. The latter solution can be found as a limit of our hairy solution under the following identifications:

$$\phi = 0\,, \qquad A^1 = \frac{1}{\sqrt{3}} A^2 = \frac{1}{2\sqrt{2}} A\,, \qquad q_1 = \frac{\Delta^2}{4\pi^2 L}\,\frac{Q}{\sqrt{8}\,L^2}\,. \tag{13}$$

The boundary conditions of the hairy soliton and those of the Einstein-Maxwell configuration [23] match for

$$q_2^2 - 3\,q_1^2 = 0\,, \tag{14}$$

also fulfilling the susy condition (11).

The Einstein-Maxwell configuration features a normalized free energy [23]

$$\frac{F_{\text{EM}}}{\Delta\,\Delta_z} = \frac{2\pi^3 L^2}{\Delta^3\,\kappa}\,X^2\,(5 - 4\,X)\,, \tag{15}$$

with $q_1^2 = 2^{-7}\,X^3\,(4 - 3\,X)$. The free energy of the supersymmetric hairy solution reads [9]

$$\frac{F_\phi}{|G_0|} = \frac{27}{\sqrt{2}}\,|q_1|\,, \tag{16}$$

having used the free energy of the AdS soliton [1]

$$G_0 = -\frac{32}{27}\,\frac{\pi^3 L^2}{\Delta^3\,\kappa}\,\Delta\,\Delta_z\,, \tag{17}$$

as a suitable normalization factor. The two free energies are compared in Figure 1: as one can see, there exists a branch of susy hairy solutions featuring an higher free energy than the corresponding non-susy Einstein-Maxwell, for the same boundary conditions.

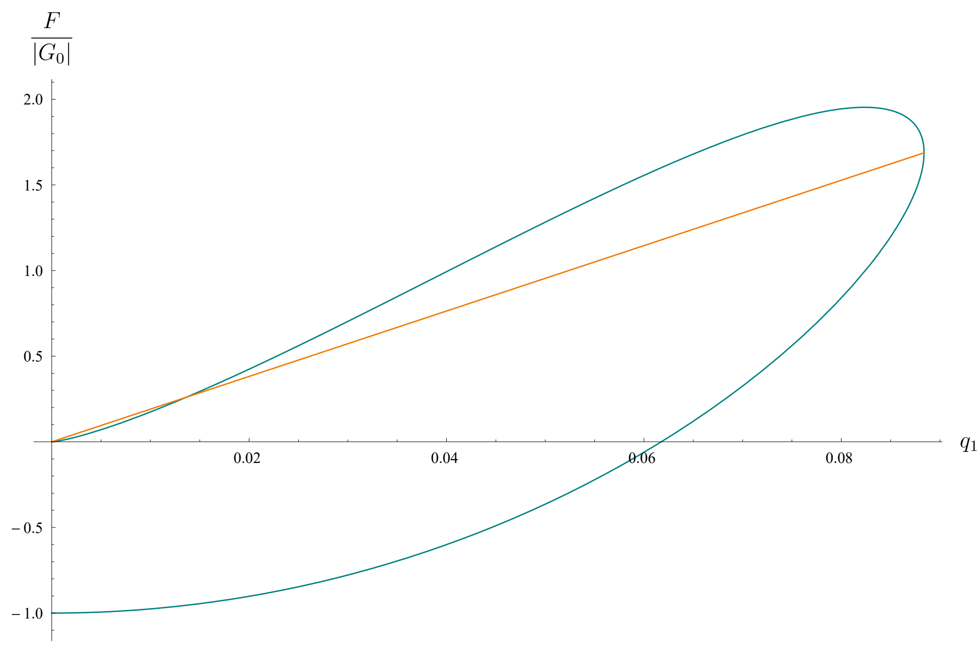

Figure 1: Rescaled free energy $\frac{F}{|G_0|}$ as a function of q_1 with $q_2 = -\sqrt{3}\,q_1$. The blue line represents the pure Einstein-Maxwell configuration, while the yellow line is the hairy supersymmetric soliton. We can see that there exist susy solutions characterized by higher free energy than non-susy configurations, for the same boundary conditions and fixed asymptotic charges [9].

This unconventional result does not conflict with the positive energy theorem [24–26]. The latter implies that the energy of a susy configuration is lower than the energy of any other configuration having the same boundary conditions. However, a necessary condition for the positive energy theorem to apply is the existence, for the non-susy configuration, of an asymptotic Killing spinor coinciding, up to $O(1/r^2)$ terms at infinity, with the Killing spinor of the susy one. In the case under consideration, the susy hairy solution has antiperiodic boundary conditions at infinity [9]: then, the consequences of the positive energy theorem only apply for non-susy solutions with an asymptotic Killing spinor featuring the same properties. This last situation only occurs for values of the charges at infinity for which the free energy of the susy solution is lower than the non-supersymmetric one, as one naively expects [9]. Then, there is no conflict with the prescriptions of the positive energy theorem and the presented scenario is of interest, as an example of framework in which an instability under quantum phase transitions can occur [27].

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