PROCEEDINGS OF SCIENCE

Soft gluon resummation for the production of four top quarks at the LHC

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We consider the effects of multiple soft gluon emission in the absolute mass threshold limit for the production of four top quarks at the LHC. We obtain predictions for the cross section at next-to-leading logarithmic (NLL) accuracy, including additionally $O(\alpha_s)$ non-logarithmic contributions which do not vanish at threshold. The threshold corrections increase the NLO total cross section by 15% and reduce the theoretical uncertainty due to scale variation by up to a factor of 2.

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1. Introduction

The production of four top quarks ($t\bar{t}t\bar{t}$) at the Large Hadron Collider (LHC) is one of the rarest Standard Model (SM) processes currently accessible. It therefore provides an important test of the SM. In particular, the cross section receives potential contributions from beyond the Standard Model (BSM) processes, leading to a possible enhancement of the cross section, e.g. [1–9]. Despite its small cross section, the process has already been measured at the LHC by both the ATLAS [10–13] and the CMS [14–17] collaborations. In the most recent ATLAS analysis for the combined channels [12] the measured cross section shows a two standard deviation excess compared to the SM prediction, computed at next-to-leading order (NLO) in Quantum Chromodynamics (QCD) and including NLO electroweak (EW) corrections [18].

Regarding the status of theoretical predictions, NLO QCD corrections as well as a combination of NLO QCD and EW corrections have been obtained [18–21]. The calculation of the next-to-next-to-leading order (NNLO) cross section for such a process is currently out of reach. Nonetheless, the precision of theoretical calculations can be increased by taking into consideration the effects of multiple soft gluon emission near the absolute mass threshold limit. This results in the appearance of logarithmic terms of the form $\alpha_s^n [\log^m(1-\hat{\rho})/(1-\hat{\rho})]_+$ in the cross section at all orders in the coupling constant, with $m \leq 2n$, $\hat{\rho} = M^2/\hat{s} = (4m_t)^2/\hat{s}$ and $\sqrt{\hat{s}}$ the partonic center-of-mass energy. In this work, we extend the precision of theoretical predictions for the production of four top quarks beyond the known NLO by performing soft gluon resummation up to next-to-leading logarithmic (NLL) accuracy, including additionally non-logarithmic $\mathcal{O}(\alpha_s)$ contributions that do not vanish at threshold. In the following, we present results for expanded and resummed cross sections in the absolute mass threshold, i.e. $\hat{\rho} \to 1$, using the Mellin-space approach in direct QCD.

2. Resummation at absolute mass threshold

The partonic resummed cross section at NLL accuracy in Mellin space can be written as [22, 23]

$$\hat{\sigma}^{\mathrm{NLL}}_{ij\to t\bar{t}t\bar{t}}(N) = \mathrm{Tr}\left[\mathbf{H}_{ij\to t\bar{t}t\bar{t}}(N)\,\mathbf{S}_{ij\to t\bar{t}t\bar{t}}(N+1)\right]\,\Delta_i(N+1)\,\Delta_j(N+1)\,, \tag{1}$$

where the hard piece $\mathbf{H}_{ij\to t\bar{t}t\bar{t}}$ accounts for the hard dynamics and the soft function $\mathbf{S}_{ij\to t\bar{t}t\bar{t}}$ gathers contributions from soft wide-angle emission. Soft-collinear contributions from the incoming partons are collected in the incoming jet functions $\Delta_{i,j}$, which are universal and well-known, and can be found at NLL accuracy e.g. in Ref. [24].

The evolution of the soft matrix is driven by the soft anomalous dimension (SAD) matrix $\mathbf{\Gamma}$, which can be expanded perturbatively i.e. $\mathbf{\Gamma} = (\alpha_s/\pi)\,\mathbf{\Gamma}^{(1)} + \ldots$. In order to obtain predictions at NLL accuracy, only the one-loop SAD matrix $\mathbf{\Gamma}^{(1)}$ is required, which together with the hard and the soft functions, is a matrix in colour space. Therefore, one needs to know the colour structure of the process. The dimensions of the colour spaces for the $q\bar{q}$- and gg-initiated channels are 6 and 14, respectively. We work in a colour basis (denoted with the subscript R) where $\mathbf{\Gamma}^{(1)}$ is diagonal in the absolute mass threshold limit. The one-loop SAD matrices for $N_c = 3$ read

$$2\,\mathrm{Re}\left[\mathbf{\Gamma}^{(1)}_{R,q\bar{q}\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(0, 0, -3, -3, -3, -3\Big), \tag{2}$$

$$2\,\mathrm{Re}\left[\mathbf{\Gamma}^{(1)}_{R,gg\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(-8, -6, -6, -4, -3, -3, -3, -3, -3, -3, -3, -3, 0, 0\Big), \tag{3}$$

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where the elements of the matrix correspond to the negative values of the quadratic Casimir invariants of the respective irreducible representation of the final-state colour structure.

The soft function is expressed in terms of the evolution matrices $\mathbf{U}_{ij\to t\bar{t}t\bar{t}}$ and the boundary condition $\tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}$, such that $\mathbf{S}_{ij\to t\bar{t}t\bar{t}} = \bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}\mathbf{U}_{ij\to t\bar{t}t\bar{t}}$, with

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}} = \mathrm{P}\exp\left[\int_{\mu_R}^{M/\bar{N}} \frac{\mathrm{d}q}{q}\,\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]. \tag{4}$$

In the R basis, the soft function at NLL reads

$$\mathbf{S}_{R,ij\to t\bar{t}t\bar{t}} = \tilde{\mathbf{S}}_{R,ij\to t\bar{t}t\bar{t}}\exp\left[\mathrm{Re}\left[\mathbf{\Gamma}^{(1)}_{R,ij\to t\bar{t}t\bar{t}}\right]\log(1-2\lambda)/(\pi b_0)\right], \tag{5}$$

with $\lambda = \alpha_s b_0 \log\bar{N}$ and $\bar{N} \equiv N e^{\gamma_E}$.

The hard and soft functions can be expanded perturbatively in powers of α_s: $\mathbf{H}_R = \mathbf{H}_R^{(0)} + \frac{\alpha_s}{\pi}\mathbf{H}_R^{(1)} + \ldots$ and $\tilde{\mathbf{S}}_R = \tilde{\mathbf{S}}_R^{(0)} + \frac{\alpha_s}{\pi}\tilde{\mathbf{S}}_R^{(1)} + \ldots$. The function $\mathbf{H}^{(1)}$ takes into account one-loop virtual corrections and non-logarithmic constant terms from collinear enhancements, whereas the first-order soft function accounts for eikonal corrections to $\tilde{\mathbf{S}}^{(0)}$. To perform NLL resummation, only the lowest order contributions are required. The first-order contributions enter formally at next-to-next-to-leading logarithmic (NNLL) accuracy, but one can increase the precision of the predictions beyond NLL by including non-logarithmic $O(\alpha_s)$ contributions, i.e. $\mathbf{H}^{(1)}$ and $\tilde{\mathbf{S}}^{(1)}$, such that

$$\mathrm{Tr}\left[\mathbf{H}_R\,\tilde{\mathbf{S}}_R\right] = \mathrm{Tr}\left[\mathbf{H}_R^{(0)}\,\tilde{\mathbf{S}}_R^{(0)} + \frac{\alpha_s}{\pi}\mathbf{H}_R^{(1)}\,\tilde{\mathbf{S}}_R^{(0)} + \frac{\alpha_s}{\pi}\mathbf{H}_R^{(0)}\,\tilde{\mathbf{S}}_R^{(1)}\right], \tag{6}$$

resulting in an accuracy referred to as NLL′. The resummation-improved predictions for the total cross section at NLO+NLL$^{(\prime)}$ accuracy are obtained through matching to the full NLO result

$$\begin{aligned}\sigma^{\mathrm{NLO+NLL}^{(\prime)}}_{t\bar{t}t\bar{t}}(\rho) = \sigma^{\mathrm{NLO}}_{t\bar{t}t\bar{t}}(\rho) + \sum_{i,j}\int_C \frac{\mathrm{d}N}{2\pi i}\,\rho^{-N}\, f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2) \\ \times\left[\hat{\sigma}^{\mathrm{NLL}^{(\prime)}}_{ij\to t\bar{t}t\bar{t}}(N) - \hat{\sigma}^{\mathrm{NLL}^{(\prime)}}_{ij\to t\bar{t}t\bar{t}}(N)|_{\mathrm{NLO}}\right],\end{aligned} \tag{7}$$

with $\rho = M^2/s$ the hadronic threshold variable. The last term in the second line of eq. (7) corresponds to the expansion of the resummed cross section truncated at NLO, and the inverse Mellin transform relies on the Minimal Prescription [25].

3. Numerical Results

We next present numerical results for the resummation-improved total cross sections for the production of four top quarks for LHC collisions at $\sqrt{s}$ = 13 TeV, and we make use of m_t = 172.5 GeV and LUXqed_plus_PDF4LHC15_nnlo_100 PDF set [26, 27]. Fixed-order calculations are obtained from aMC@NLO [28, 29]. One-loop virtual contributions are also numerically extracted from aMC@NLO code. We consider NLO cross sections including only QCD corrections, as well as a combination of QCD and EW corrections, with EW corrections up to $O(\alpha^2)$ [18].

We first present on the left and center plots of Figure 1 the scale dependence of the total cross section at several accuracies. We set $\mu_R = \mu_F$ and vary them simultaneously around the central scale $\mu_0 = 2m_t$. On the left plot, we compare the NLO cross section where the quark-gluon channel has been subtracted - denoted NLO (no qg) - with the expansion of the resummed cross section up to NLO - denoted NLL$'|_{\rm NLO}$. It can be seen that the expansion up to NLO is a good approximation of the NLO (no qg), indicating that the dominant part of the NLO corrections is captured by soft gluon emission. On the center plot, one can see that the NLL resummed cross section calculated at the central scale is higher by 4%, and the scale dependence is mildly reduced, compared to the NLO cross section. With the inclusion of $O(\alpha_s)$ contributions, the NLO (QCD+EW)+NLL$'$ cross section is 15% higher than the full NLO (QCD+EW) cross section. The scale dependence of the cross sections with NLL$'$ corrections is significantly reduced.

On the right plot of Figure 1 we consider the 7-point scale variation, and show the central value of the cross section for the fixed-order and resummation-improved results. In red we present the theoretical error associated with the scale uncertainty. It can be seen that the associated uncertainty is highly reduced, even up to a factor of two, when including NLL$'$ resummation effects compared to fixed-order calculations. As an estimate of the PDF error we study the PDF error of the NLO (QCD+EW) result, which amounts to ±6.9%, by scanning over all the members of the PDF set.

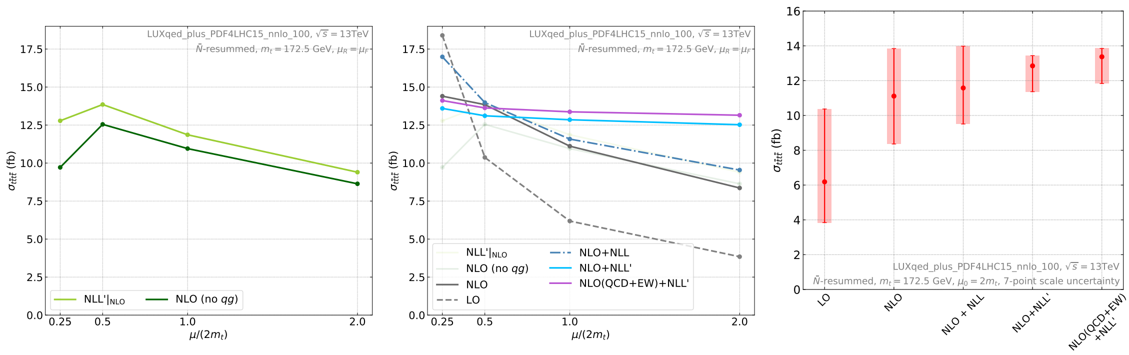

Figure 1: Scale dependence of the NLO (no qg) and NLL$'|_{\rm NLO}$ (left), and LO, NLO, NLO+NLL, NLO+NLL$'$ and NLO(QCD+EW)+NLL$'$ (center) cross sections at $\sqrt{s}$ = 13 TeV. LO and NLO include only QCD effects, while NLO(QCD+EW) includes as well electroweak corrections. Right: total cross sections for the production of four top quarks at $\sqrt{s}$ = 13 TeV for fixed-order calculations and resummation-improved results.

Our state-of-the-art prediction for the $t\bar{t}t\bar{t}$ production at $\sqrt{s}$ = 13 TeV, reads

$$\sigma_{t\bar{t}t\bar{t}}^{\rm NLO(QCD+EW)+NLL'} = 13.37(2)\,^{+3.6\%}_{-11.4\%}\,({\rm scale})\,^{+6.9\%}_{-6.9\%}\,({\rm pdf})\ {\rm fb}.$$

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