

The Lund jet plane: a tool for precision and discovery at the LHC

Giovanni Stagnitto^{a,b,*}

^a*Università degli Studi di Milano-Bicocca & INFN,
Piazza della Scienza 3, I-20126 Milano, Italy*

^b*Department of Physics, University of Zürich,
Winterthurerstrasse 190, CH-8057 Zürich, Switzerland*

E-mail: giovanni.stagnitto@unimib.it

We provide a (very) short review of some of the recent developments concerning phenomenological applications of the Lund jet plane at the LHC.

*The Eleventh Annual Conference on Large Hadron Collider Physics (LHCP2023)
22-26 May 2023
Belgrade, Serbia*

*Speaker

The study of the substructure of hadronic jets at the LHC has seen a lot of developments in the last 15 years, to the point of becoming textbook material [1]. In this proceeding, I will focus on a single tool, adopted in a wide range of applications: the Lund plane [2], a way of depicting the pattern of QCD radiation, inside a jet or in a whole event. I will first define the Lund (jet) plane; then I will show some examples of analytic calculations, machine learning applications and heavy quark studies based on the Lund plane.

The Lund jet plane We can associate a kinematic structure to a given high-energy jet in the following way [3]. We first decluster the jet with the Cambridge/Aachen algorithm (based on a purely angular distance among particles). For each step of the declustering, involving pseudo-jets a and b with $p_{t,a} > p_{t,b}$, we record the variables:

$$\Delta \equiv \Delta_{ab} = \sqrt{(y_a - y_b)^2 + (\phi_a - \phi_b)^2}, \quad k_t = p_{t,b} \Delta_{ab}, \quad z = \frac{p_{t,b}}{p_{t,a} + p_{t,b}} \quad (1)$$

with $p_{t,i}$, y_i and ϕ_i the transverse momentum, the rapidity and the azimuthal angle of the pseudo-jet i , with $i = a, b$, respectively. We iterate the collection of variables (1) on both branches of the declustering tree. At the end, we can plot the set of points $(\ln 1/\Delta, \ln k_t)$ (in some variants also the choice $(\ln 1/\Delta, \ln 1/z)$ is considered) in a *primary* Lund plane (if related to an emission off the hardest branch) or in a *secondary*, *tertiary*, etc. Lund plane. The choice of variables $(\ln 1/\Delta, \ln k_t)$ or $(\ln 1/\Delta, \ln 1/z)$ is such that, in a first approximation, QCD emissions are uniformly distributed in the Lund plane(s). Moreover, there is a clear separation between QCD regimes, with the e.g. non-perturbative region confined to small k_t values, the initial-state radiation region confined to large values of Δ , the hard-collinear region to large value of z , etc.

Lund plane & analytics Once given with such a structure, the simplest observable defined on the primary Lund plane is the Lund jet plane density, counting the number of emissions falling in each “pixel” of the plane (the observable is infrared and collinear safe provided the pixel has area different from zero). First studies [3] have shown up to 20-30% difference in predictions from different Monte Carlo generators in some slices of the plane. Measurement by ATLAS [4], CMS [5] and ALICE [6] have further investigated the ability of the Lund jet plane to isolate physical effects, hence providing useful inputs to perturbative and non-perturbative model development and tuning.

Analytical calculations of the Lund plane density have been performed. In [7], the logarithmically dominant terms with structure $\alpha_s^{n+1} \ln^m \Delta \ln^{n-m} z$, $0 \leq m \leq n$, are resummed to all-orders, and then matched to the fixed-order NLO result. Their resummation require to deal with running coupling corrections (numerically dominant), hard-collinear logarithms, soft effects and clustering logarithms. Non-perturbative effects are estimated through Monte Carlo codes. The results show a good agreement with ATLAS data in several slices of the plane, with a clear separation of contributions, between non-perturbative, resummation and fixed-order region.

On the analytical side, other recent developments concern the study of Lund multiplicities, both at LEP and at the LHC. At LEP, Lund multiplicity is defined as the (average) number of Lund declusterings (in the full Lund tree) with $k_t \geq k_{t,\text{cut}}$. In [8] it has been computed up to next-to-next-to-double logarithmic accuracy (NNDL), with $L = \ln(Q/k_{t,\text{cut}})$, with a novel method, based on recycling DL results with insertions of NDL or NNDL genuine ingredients. At the LHC, Lund multiplicity is defined by counting the mean number of sub-jets per anti- k_t jet, with relative

$k_t \geq k_{t,\text{cut}}$. The resummation of this observable is presented in [9], up to NNDL, with the relevant log equal to $L = \ln(p_t R/k_{t,\text{cut}})$. The calculation exploits universal ingredients from the e^+e^- event-wide result, with the presence of jet radius impacting the large-angle components starting at NDL in a process-dependent way (e.g. Z +jets or dijets). Both analytical results, at LEP and at the LHC, have the potential to serve as benchmarks to test and develop MC event generators.

Lund plane & machine learning The Lund plane features a different structure of emissions in the case of jets from the QCD background or originating from the hadronic decays of W , H , top, etc. This property can be exploited in several ways: by using the Lund plane density to build likelihood functions; by using the sequence of Lund declusterings as input to a LSTM or a DNN architecture; or by feeding a CNN architecture with the “image” of the Lund plane density.

In the original paper [3], an application to the tagging of W decay is presented. Further studies focused on the tagging of Higgs decays, both $H \rightarrow b\bar{b}$ and $H \rightarrow gg$, by using primary Lund plane images [10, 11]. In particular, it has been observed how a simple one-variable discriminant (the color ring [12]) performs well in the $H \rightarrow b\bar{b}$ case, but it fails in the $H \rightarrow gg$ case, whereas the Lund plane CNN maintains its discrimination power also in latter scenario.

The studies cited so far retain only information from the *primary* Lund plane. In [13], the structure of the *full* Lund tree i.e. with the inclusion of secondary, tertiary, etc. planes, is exploited as input to a graph neural network (GNN), dubbed LundNet. It reaches state-of-the-art performances (comparable to ParticleNet performances [14]) both on W tagging and top tagging scenarios. LundNet is proposed in two variants: LundNet-3, trained on $(\ln k_t, \ln \Delta, \ln z)$ tuples; LundNet-5, trained on $(\ln k_t, \ln \Delta, \ln z, \ln m, \ln \psi)$ tuples, with m the invariant mass of the pair and ψ the azimuthal angle around the subjet’s axis. LundNet-5 appears to be more performant, but LundNet-3 is found to be more resilient to non-perturbative effects (by *resilience*, we mean the degree of insensitivity to potential mismodelling aspects or to specific details of an event sample, see [15] for in depth discussion). This behaviour could be related to the fact that LundNet-5 is potentially extrapolating information on emissions below the transverse momentum cut $k_{t,\text{cut}}$.

The Lund jet plane can also be exploited as a powerful tool for quark vs. gluon discrimination (roughly speaking, the ability of discriminating between jets originating from a hard quark or gluon). In [16], the likelihood ratio between the probability of having a gluon jet over the probability of having a quark jet given the observed Lund primary or full tree is calculated analytically up to single logs, and compared to results based on machine learning models, such as a LSTM (trained on the primary tree) or a GNN (trained on the full tree). One observes a gain in performance when considering the full tree compared to the primary one; however, one also observes better performances in the case of ML models compared to the analytical one. In order to understand the source of the difference, a toy setup where events are generated in the strong angular-ordered limit is proposed; in this toy model, the analytic approach should corresponds to the *exact* likelihood ratio discriminant. The approach is similar in spirit to what done in [17]. In the toy setup, ML and analytical approaches provide similar performances. This behaviour is also confirmed by a second test, with the limit $\alpha_s \rightarrow 0$ at fixed $\alpha_s \ln(Q/k_{t,\text{cut}})$, in order to isolate only the single-logarithmic terms: the difference between the two approaches is seen to reduce in this asymptotic limit. Hence, it seems that the gain in performance for ML come from effects that are not fully under control (subleading effects beyond single logarithms, large-angle soft emissions, non-perturbative effects).

In addition, studies related to the usage of Lund plane images for b -jets tagging have appeared [18]. In particular, they have focused on the boosted region $p_t > 500$ GeV (where the b -tagging performance usually degrades). The Lund plane CNN is found to have performances similar to dedicated tagging algorithms, such as JetFitter and IP3D, which are low-level algorithms based on charged particle track reconstruction.

Lund plane & heavy quarks Finally, ideas based on the Lund plane declustering tree have found applications also in heavy quark physics. ALICE has recently reported about the observation of the dead-cone effect [19] for charm quarks, by using an iterative technique (introduced in [20]) based on a Cambridge/Aachen declustering sequence, by following the D^0 meson and by keeping track of the angle θ and the relative transverse momentum k_t between the splittings.

Ideas for dead-cone searches in heavy-ion environments have also appeared. In [21], it has been suggested as a new grooming strategy to select the most collinear splitting above a certain $k_{t,\text{cut}}$. Such a new groomer, dubbed Late- k_t , is suited to heavy-ion environment, as it reduces the impact of uncorrelated thermal background, typically manifesting as fake large angle splittings.

Conclusions The Lund (jet) plane is a unique tool for collider phenomenology. The clear separation of perturbative and non-perturbative regimes is a key property that could be exploited in several way e.g. to extract the strong coupling constant. Its sensitivity to disparate scales, from few GeV up to several TeV, offers an ideal tool for resummation and parton showers studies. The observables based on the Lund plane are amenable to calculability up to high orders, thus allowing for precise comparisons with data and benchmark calculations. Finally, Lund trees and images can be adopted as theory-friendly input to machine learning algorithms, hence having the potential to reach good performance and resilience at the same time.

References

- [1] S. Marzani, G. Soyez and M. Spannowsky, *Looking inside jets: an introduction to jet substructure and boosted-object phenomenology*, vol. 958, Springer (2019), [10.1007/978-3-030-15709-8](https://doi.org/10.1007/978-3-030-15709-8), [[1901.10342](https://arxiv.org/abs/1901.10342)].
- [2] B. Andersson, G. Gustafson, L. Lonnblad and U. Pettersson, *Coherence Effects in Deep Inelastic Scattering*, *Z. Phys. C* **43** (1989) 625.
- [3] F.A. Dreyer, G.P. Salam and G. Soyez, *The Lund Jet Plane*, *JHEP* **12** (2018) 064 [[1807.04758](https://arxiv.org/abs/1807.04758)].
- [4] ATLAS collaboration, *Measurement of the Lund Jet Plane Using Charged Particles in 13 TeV Proton-Proton Collisions with the ATLAS Detector*, *Phys. Rev. Lett.* **124** (2020) 222002 [[2004.03540](https://arxiv.org/abs/2004.03540)].
- [5] CMS collaboration, *Measurement of the primary Lund jet plane density in proton-proton collisions at $\sqrt{s} = 13$ TeV*, [2312.16343](https://arxiv.org/abs/2312.16343).
- [6] ALICE collaboration, *Measurement of the primary Lund jet plane density in pp collisions at $\sqrt{s} = 13$ TeV with ALICE*, *PoS EPS-HEP2021* (2022) 364 [[2111.00020](https://arxiv.org/abs/2111.00020)].

- [7] A. Lifson, G.P. Salam and G. Soyez, *Calculating the primary Lund Jet Plane density*, *JHEP* **10** (2020) 170 [2007.06578].
- [8] R. Medves, A. Soto-Ontoso and G. Soyez, *Lund and Cambridge multiplicities for precision physics*, *JHEP* **10** (2022) 156 [2205.02861].
- [9] R. Medves, A. Soto-Ontoso and G. Soyez, *Lund multiplicity in QCD jets*, *JHEP* **04** (2023) 104 [2212.05076].
- [10] C.K. Khosa and S. Marzani, *Higgs boson tagging with the Lund jet plane*, *Phys. Rev. D* **104** (2021) 055043 [2105.03989].
- [11] L. Cavallini, A. Coccaro, C.K. Khosa, G. Manco, S. Marzani, F. Parodi et al., *Tagging the Higgs boson decay to bottom quarks with colour-sensitive observables and the Lund jet plane*, *Eur. Phys. J. C* **82** (2022) 493 [2112.09650].
- [12] A. Buckley, G. Callea, A.J. Larkoski and S. Marzani, *An Optimal Observable for Color Singlet Identification*, *SciPost Phys.* **9** (2020) 026 [2006.10480].
- [13] F.A. Dreyer and H. Qu, *Jet tagging in the Lund plane with graph networks*, *JHEP* **03** (2021) 052 [2012.08526].
- [14] H. Qu and L. Gouskos, *ParticleNet: Jet Tagging via Particle Clouds*, *Phys. Rev. D* **101** (2020) 056019 [1902.08570].
- [15] *Les Houches 2017: Physics at TeV Colliders Standard Model Working Group Report*, 3, 2018.
- [16] F.A. Dreyer, G. Soyez and A. Takacs, *Quarks and gluons in the Lund plane*, *JHEP* **08** (2022) 177 [2112.09140].
- [17] G. Kasieczka, S. Marzani, G. Soyez and G. Stagnitto, *Towards Machine Learning Analytics for Jet Substructure*, *JHEP* **09** (2020) 195 [2007.04319].
- [18] O. Fedkevych, C.K. Khosa, S. Marzani and F. Sforza, *Identification of b jets using QCD-inspired observables*, *Phys. Rev. D* **107** (2023) 034032 [2202.05082].
- [19] ALICE collaboration, *Direct observation of the dead-cone effect in quantum chromodynamics*, *Nature* **605** (2022) 440 [2106.05713].
- [20] L. Cunqueiro and M. Płoskoń, *Searching for the dead cone effects with iterative declustering of heavy-flavor jets*, *Phys. Rev. D* **99** (2019) 074027 [1812.00102].
- [21] L. Cunqueiro, D. Napoletano and A. Soto-Ontoso, *Dead-cone searches in heavy-ion collisions using the jet tree*, *Phys. Rev. D* **107** (2023) 094008 [2211.11789].