## PROCEEDINGS OF SCIENCE



## Vector boson modeling for precision physics

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W- and Z- boson production are among of the most precisely analyzed processes at the LHC, enabling applications that range from precision determinations of couplings to parton distribution functions to particle masses. Direct measurements are meanwhile only limited by luminosity uncertainties of about 1%. On the other hand, the required theory predictions are pushing the boundaries of theoretical methods, with a level of sophistication reached that is setting the stage for the HL-LHC's demand for higher multiplicity processes at a similar level. In these proceedings we briefly summarize recent progress in the theoretical modeling of W- and Z-bosons.

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© Copyright owned by the author(s) under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License (CC BY-NC-ND 4.0). **Experimental measurements.** *W* and *Z* boson production are among the most important processes in LHC physics and constitute standard candles with a wide range of applications. Measurements at the level of 1% or better for both *Z* production [1–5] and *W* production by ATLAS, CMS and LHCb [5–9] [10–14] [15–19] demand the development of theoretical predictions to an unprecedented level. In the future this will be needed at a much broader scope for other processes to ensure a successful HL-LHC program. Only recently has the precision reached been limited by the measurement of the LHC luminosity to about 1%, improving upon earlier levels of 2-3% [20, 21]. Apart from direct kinematic measurements, the possibilities through this precision are manyfold, demonstrated for example by precision *W*-mass measurements [22–25], charge asymmetries [14, 26–29], parton distribution functions (PDFs) [9, 30–33], as well as the strong coupling  $\alpha_s$  [34–36].

**Theory predictions.** However, these highly complex collider analyses have a strong dependence on the theoretical predictions for background suppression and subtraction as well as the signal process. Without equally precise predictions the statistically precise measurements cannot be fully interpreted. We are currently in a situation where theory predictions are behind, limiting this ability, with many individual uncertainties contributing at the percent level. This requires increasing the precision of individual components like fixed-order expansions in QCD, QED and electroweak couplings, higher-order resummation, parton showers, non-perturbative effects in PDFs and TMDs, possibly including higher power terms in (collinear) factorization, understanding phenomenological modeling and tuning, and even decreasing numerical precision and computational resource requirements. With uncertainties contributing at the per mill to percent level in all of these components, only the combination allows for comprehensive predictions aiming to match experimental precision.

First N<sup>3</sup>LO QCD ( $\alpha_s^3$ ) predictions for W and Z boson production were calculated at a fully inclusive level for total cross-sections [37, 38] and rapidity distributions [39]. These calculations revealed unexpectedly large corrections of about -2.5% due to cancellations between partonic initial-state channels, but did not take into account effects from N<sup>3</sup>LO PDFs. The current state-of-the-art in N<sup>3</sup>LO QCD is at a fiducial and fully differential level [40–46], typically including the effect of transverse-momentum ( $q_T$ ) resummation at a similar level in  $\alpha_s$ . Generally the residual QCD truncation uncertainties at the level of  $\alpha_s^3$  are estimated to be at the level of 1 - 2% inclusively, and at small transverse momenta  $q_T \leq m_V$  due to the higher-order  $q_T$  resummation. Note that both fixed-order and resummed calculations require N<sup>3</sup>LO PDFs for a consistent  $\alpha_s^3$  precision. The formal logarithmic accuracy of N<sup>4</sup>LL ( $\alpha_s^3$ ) in particular relies on the four-loop DGLAP evolution.

Higher-order transverse-momentum resummation up the level of N<sup>3</sup>LL' matched to  $\alpha_s^2$  fixed order predictions has also been studied in refs. [47–49]. Recent studies of threshold resummation in rapidity distributions were presented in refs. [50–53]. Transverse-momentum resummation is also considered with a focus on TMD fits in the literature, see e.g. refs. [54–57]. Attention will have to be paid in disentangling perturbative and non-perturbative contributions [58].

Currently, all of these fully differential calculations at  $\alpha_s^3$  rely on the idea of  $q_T$  slicing subtractions [59]. They are made possible through calculations of the corresponding three-loop beam-functions [60–62], complete three-loop hard function [63–67] and the existence of a NNLO calculation of V+jet production [68–72].



**Figure 1:**  $W^+$  cross-sections at various perturbative orders in  $\alpha_s$ , with and without  $q_T$  resummation, in comparison with the 5.02 TeV ATLAS measurement [5]. Error bars show uncertainties from scale variation and from the MSHT20 PDF sets [86, 87] corresponding to the perturbative order. The  $\alpha_s^3$  results have an additional numerical and slicing cutoff uncertainty of 0.5% that was added linearly to the scale uncertainties for display. This figure is taken from ref. [41]

Recently there has been a shift from relying on fixed-order calculations for total fiducial cross-sections to resummed calculations. This is because convergence issues in the perturbative series due to fiducial cuts have been identified [73–75] that are resolved in resummation-improved perturbation theory without requiring modification of analysis cuts [74]. The difference between symmetric and product lepton cuts has been studied in ref. [43].

Apart from QCD effects, other Standard Model effects play a role at the level of 1% precision. Among these, mixed QCD $\otimes$ EW corrections were reported in refs. [76–78] for Z production and in refs. [79–83] for W production, and with an application to W-mass determinations in ref. [84]. QED-QCD transverse-momentum resummation has been considered in ref. [85]. Of particular importance are effects from PDFs, which currently dominate the uncertainty budget, see figs. 1, 2. They will require careful examination to resolve systematic issues and an extension towards N<sup>3</sup>LO [86].

With *W* and *Z*-boson predictions entering crucially in *W*-mass analyses, there is a strong interest in how uncertainties propagate in *W*-mass analyses. A comprehensive review of how theoretical contributions and uncertainties impact the *W*-boson mass measurement was presented in ref. [88] (2016), while the impact of PDF [89] and higher-order [90] uncertainties have also been separately assessed more recently. An estimate for the impact of mixed QCD⊗EW corrections has since also been performed [84].

Due to the significance of this process and the complication of higher-order corrections, as well as the flexibility in approaches beyond fixed-order, it is important to compare different approaches, cross-check results, and allow for public and sustainable predictions [91, 92]. These aspects, especially public reproducibility, are increasingly important with very precise collider measurements that might indicate Standard Model tensions, see e.g. ref. [25].

Public codes for the calculation of W and Z production include RadISH+MATRIX at NNLO QCD





**Figure 2:** Relative PDF uncertainties of the  $W^+$  transverse momentum distribution. Note that MSHT20an310 includes uncertainties from missing higher orders, which are not included in the other sets. This figure is taken from ref. [41]

including  $q_T$  resummation and NLO EW [93–95], DYTurbo matching  $q_T$ -resummed predictions to NNLO QCD [45, 96, 97], CuTe-MCFM at N<sup>3</sup>LO QCD including  $q_T$  resummation and NLO EW [40, 41, 98, 99], MiNNLO+PS+POWHEG NNLO QCD matched to parton shower [100], artemide [101] and Nanga Parbat [56] with resummed-only predictions focusing on transverse-momentum parton distribution functions, FEWZ [102, 103] at NNLO QCD using local subtractions, and Horace [104, 105] including NLO EW corrections with matching to QED parton shower. While fixed order NNLOjet predictions [69, 71] have been matched to RadISH resummation [106, 107] at the level of  $\alpha_s^3$ , e.g. ref. [44], the code is not publicly available. Note that codes like DYTurbo can use NNLO V+jet results from MCFM [40, 41, 68, 70, 72] to also achieve full N<sup>3</sup>LO accuracy.

**Challenges and outlook.** The experimental precision in *W* and *Z*-boson production demonstrates the LHC's capabilities for precision measurements, the success of modern data-analysis techniques, and showcases a level of precision that will be reached for a wider range of processes at the HL-LHC. Matching the precision in theoretical predictions is a future challenge, requiring not just higher perturbative orders in individual components and more precise non-perturbative inputs like PDFs, but the combination of a multitude of effects that contribute at the percent level. Overcoming these challenges will require an unprecedented community effort in novel developments and open collaboration. Ultimately this will ensure that we maximize the return on investment of the LHC in our goal of describing the fundamental laws on nature.

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## References

[1] ATLAS collaboration, Measurement of the transverse momentum and  $\phi_{\eta}^*$  distributions of Drell–Yan lepton pairs in proton–proton collisions at  $\sqrt{s} = 8$  TeV with the ATLAS detector,

Eur. Phys. J. C 76 (2016) 291 [1512.02192].

- [2] CMS collaboration, Measurement of the transverse momentum spectra of weak vector bosons produced in proton-proton collisions at  $\sqrt{s} = 8$  TeV, JHEP **02** (2017) 096 [1606.05864].
- [3] CMS collaboration, Measurements of differential Z boson production cross sections in proton-proton collisions at  $\sqrt{s} = 13$  TeV, JHEP 12 (2019) 061 [1909.04133].
- [4] ATLAS collaboration, *Measurement of the transverse momentum distribution of Drell–Yan lepton pairs in proton–proton collisions at*  $\sqrt{s} = 13$  *TeV with the ATLAS detector, Eur. Phys. J. C* **80** (2020) 616 [1912.02844].
- [5] ATLAS collaboration, Precise measurements of W and Z transverse momentum spectra with the ATLAS detector at  $\sqrt{s} = 5.02$  TeV and 13 TeV, tech. rep., CERN, Geneva, 2023.
- [6] ATLAS collaboration, *Measurement of*  $W^{\pm}$ -boson and Z-boson production cross-sections in pp collisions at  $\sqrt{s} = 2.76$  TeV with the ATLAS detector, Eur. Phys. J. C **79** (2019) 901 [1907.03567].
- [7] ATLAS collaboration, *Measurements of W and Z boson production in pp collisions at*  $\sqrt{s} = 5.02$  TeV with the ATLAS detector, Eur. Phys. J. C **79** (2019) 128 [1810.08424].
- [8] ATLAS collaboration, Measurement of differential cross sections and  $W^+/W^-$  cross-section ratios for W boson production in association with jets at  $\sqrt{s} = 8$  TeV with the ATLAS detector, JHEP **05** (2018) 077 [1711.03296].
- [9] ATLAS collaboration, Precision measurement and interpretation of inclusive  $W^+$ ,  $W^-$  and  $Z/\gamma^*$  production cross sections with the ATLAS detector, Eur. Phys. J. C **77** (2017) 367 [1612.03016].
- [10] CMS collaboration, Measurements of differential production cross sections for a Z boson in association with jets in pp collisions at  $\sqrt{s} = 8$  TeV, JHEP **04** (2017) 022 [1611.03844].
- [11] CMS collaboration, Measurement of inclusive W and Z boson production cross sections in pp collisions at  $\sqrt{s} = 8$  TeV, Phys. Rev. Lett. **112** (2014) 191802 [1402.0923].
- [12] CMS collaboration, Measurement of the Inclusive W and Z Production Cross Sections in pp Collisions at  $\sqrt{s} = 7$  TeV, JHEP 10 (2011) 132 [1107.4789].
- [13] CMS collaboration, *Measurements of Inclusive W and Z Cross Sections in pp Collisions at*  $\sqrt{s} = 7$  TeV, JHEP **01** (2011) 080 [1012.2466].
- [14] CMS collaboration, Measurements of the W boson rapidity, helicity, double-differential cross sections, and charge asymmetry in pp collisions at  $\sqrt{s} = 13$  TeV, Phys. Rev. D 102 (2020) 092012 [2008.04174].
- [15] LHCB collaboration, *Measurement of forward*  $W \rightarrow ev$  production in pp collisions at  $\sqrt{s} = 8$  TeV, JHEP **10** (2016) 030 [1608.01484].

- [16] LHCB collaboration, Measurement of forward W and Z boson production in association with jets in proton-proton collisions at  $\sqrt{s} = 8$  TeV, JHEP **05** (2016) 131 [1605.00951].
- [17] LHCB collaboration, *Measurement of forward W and Z boson production in pp collisions at*  $\sqrt{s} = 8 \text{ TeV}, \text{ JHEP 01} (2016) 155 [1511.08039].$
- [18] LHCB collaboration, *Measurement of the forward W boson cross-section in pp collisions at*  $\sqrt{s} = 7 \text{ TeV}$ , *JHEP* **12** (2014) 079 [1408.4354].
- [19] LHCB collaboration, *Inclusive W and Z production in the forward region at*  $\sqrt{s} = 7$  *TeV*, *JHEP* **06** (2012) 058 [1204.1620].
- [20] CMS collaboration, Precision luminosity measurement in proton-proton collisions at  $\sqrt{s} = 13$ TeV in 2015 and 2016 at CMS, Eur. Phys. J. C 81 (2021) 800 [2104.01927].
- [21] ATLAS collaboration, Luminosity determination in pp collisions at  $\sqrt{s} = 13$  TeV using the ATLAS detector at the LHC, 2212.09379.
- [22] ATLAS collaboration, Improved W boson Mass Measurement using 7 TeV Proton-Proton Collisions with the ATLAS Detector, tech. rep., CERN, Geneva, 2023.
- [23] ATLAS collaboration, Measurement of the W-boson mass in pp collisions at  $\sqrt{s} = 7$  TeV with the ATLAS detector, Eur. Phys. J. C 78 (2018) 110 [1701.07240].
- [24] LHCB collaboration, Measurement of the W boson mass, JHEP 01 (2022) 036 [2109.01113].
- [25] CDF collaboration, High-precision measurement of the W boson mass with the CDF II detector, Science 376 (2022) 170.
- [26] ATLAS collaboration, Measurement of the cross-section and charge asymmetry of W bosons produced in proton–proton collisions at  $\sqrt{s} = 8$  TeV with the ATLAS detector, Eur. Phys. J. C **79** (2019) 760 [1904.05631].
- [27] CMS collaboration, Measurement of the differential cross section and charge asymmetry for inclusive pp  $\rightarrow$  W<sup>±</sup> + X production at  $\sqrt{s}$  = 8 TeV, Eur. Phys. J. C **76** (2016) 469 [1603.01803].
- [28] CMS collaboration, Measurement of the Electron Charge Asymmetry in Inclusive W Production in pp Collisions at  $\sqrt{s} = 7$  TeV, Phys. Rev. Lett. **109** (2012) 111806 [1206.2598].
- [29] CMS collaboration, Measurement of the lepton charge asymmetry in inclusive W production in pp collisions at  $\sqrt{s} = 7$  TeV, JHEP 04 (2011) 050 [1103.3470].
- [30] ATLAS collaboration, Determination of the parton distribution functions of the proton from ATLAS measurements of differential W and Z boson production in association with jets, JHEP 07 (2021) 223 [2101.05095].

- [31] CMS collaboration, Measurement of the Muon Charge Asymmetry in Inclusive  $pp \rightarrow W + X$ Production at  $\sqrt{s} = 7$  TeV and an Improved Determination of Light Parton Distribution Functions, Phys. Rev. D 90 (2014) 032004 [1312.6283].
- [32] R. Boughezal, A. Guffanti, F. Petriello and M. Ubiali, *The impact of the LHC Z-boson transverse momentum data on PDF determinations*, *JHEP* 07 (2017) 130 [1705.00343].
- [33] ATLAS collaboration, *Impact of ATLAS V* + *jets data on PDF fits*, *Nucl. Part. Phys. Proc.* **312-317** (2021) 6 [2011.08481].
- [34] ATLAS collaboration, A precise determination of the strong-coupling constant from the recoil of Z bosons with the ATLAS experiment at  $\sqrt{s} = 8$  TeV, 2309.12986.
- [35] NNPDF collaboration, *Precision determination of the strong coupling constant within a global PDF analysis, Eur. Phys. J. C* **78** (2018) 408 [1802.03398].
- [36] S. Camarda, G. Ferrera and M. Schott, *Determination of the strong-coupling constant from the Z-boson transverse-momentum distribution*, 2203.05394.
- [37] C. Duhr and B. Mistlberger, *Lepton-pair production at hadron colliders at N*<sup>3</sup>LO in QCD, *JHEP* **03** (2022) 116 [2111.10379].
- [38] C. Duhr, F. Dulat and B. Mistlberger, *Charged current Drell-Yan production at N*<sup>3</sup>LO, *JHEP* **11** (2020) 143 [2007.13313].
- [39] X. Chen, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang and H. X. Zhu, *Dilepton Rapidity Distribution in Drell-Yan Production to Third Order in QCD*, *Phys. Rev. Lett.* **128** (2022) 052001 [2107.09085].
- [40] T. Neumann and J. Campbell, Fiducial Drell-Yan production at the LHC improved by transverse-momentum resummation at N4LLp+N3LO, Phys. Rev. D 107 (2023) L011506 [2207.07056].
- [41] J. Campbell and T. Neumann, *Third order QCD predictions for fiducial W-boson production*, *JHEP* **11** (2023) 127 [2308.15382].
- [42] X. Chen, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang and H. X. Zhu, *Transverse mass distribution and charge asymmetry in W boson production to third order in QCD*, *Phys. Lett. B* 840 (2023) 137876 [2205.11426].
- [43] X. Chen, T. Gehrmann, E. W. N. Glover, A. Huss, P. Monni, E. Re et al., *Third order fiducial predictions for Drell-Yan at the LHC*, 2203.01565.
- [44] W. Bizon, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, P. F. Monni et al., *The transverse momentum spectrum of weak gauge bosons at N<sup>3</sup>LL + NNLO, Eur. Phys. J. C* 79 (2019) 868 [1905.05171].
- [45] S. Camarda, L. Cieri and G. Ferrera, *Drell-Yan lepton-pair production:*  $q_T$  *resummation at*  $N^3LL$  accuracy and fiducial cross sections at  $N^3LO$ , 2103.04974.

- [46] E. Re, L. Rottoli and P. Torrielli, Fiducial Higgs and Drell-Yan distributions at N<sup>3</sup>LL'+NNLO with RadISH, 2104.07509.
- [47] J. Isaacson, Y. Fu and C. P. Yuan, *Improving ResBos for the precision needs of the LHC*, 2311.09916.
- [48] W.-L. Ju and M. Schönherr, The  $q_T$  and  $\Delta \phi$  spectra in W and Z production at the LHC at  $N^3LL'+N^2LO$ , 2106.11260.
- [49] S. Camarda, L. Cieri and G. Ferrera, Drell–Yan lepton-pair production: qT resummation at N4LL accuracy, Phys. Lett. B 845 (2023) 138125 [2303.12781].
- [50] M. Bonvini and G. Marinelli, On the approaches to threshold resummation of rapidity distributions for the Drell-Yan process, 2306.03568.
- [51] G. Das,  $Z, W^{\pm}$  rapidity distributions at NNLL and beyond, 2303.16578.
- [52] P. Banerjee, G. Das, P. K. Dhani and V. Ravindran, *Threshold resummation of the rapidity distribution for Drell-Yan production at NNLO+NNLL*, *Phys. Rev. D* 98 (2018) 054018 [1805.01186].
- [53] G. Lustermans, J. K. L. Michel and F. J. Tackmann, *Generalized Threshold Factorization with Full Collinear Dynamics*, 1908.00985.
- [54] V. Moos, I. Scimemi, A. Vladimirov and P. Zurita, Extraction of unpolarized transverse momentum distributions from fit of Drell-Yan data at N<sup>4</sup>LL, 2305.07473.
- [55] I. Scimemi and A. Vladimirov, Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan scattering at small transverse momentum, JHEP 06 (2020) 137 [1912.06532].
- [56] A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, F. Delcarro, F. Piacenza et al., *Transverse-momentum-dependent parton distributions up to N<sup>3</sup>LL from Drell-Yan data, JHEP* 07 (2020) 117 [1912.07550].
- [57] D. Gutierrez-Reyes, S. Leal-Gomez and I. Scimemi, W-boson production in TMD factorization, Eur. Phys. J. C 81 (2021) 418 [2011.05351].
- [58] M. A. Ebert, J. K. L. Michel, I. W. Stewart and Z. Sun, *Disentangling Long and Short Distances in Momentum-Space TMDs*, 2201.07237.
- [59] S. Catani and M. Grazzini, An NNLO subtraction formalism in hadron collisions and its application to Higgs boson production at the LHC, Phys. Rev. Lett. 98 (2007) 222002 [hep-ph/0703012].
- [60] M.-x. Luo, T.-Z. Yang, H. X. Zhu and Y. J. Zhu, Unpolarized quark and gluon TMD PDFs and FFs at N<sup>3</sup>LO, JHEP 06 (2021) 115 [2012.03256].
- [61] M. A. Ebert, B. Mistlberger and G. Vita, *Transverse momentum dependent PDFs at N<sup>3</sup>LO*, *JHEP* **09** (2020) 146 [2006.05329].

- [62] M.-x. Luo, T.-Z. Yang, H. X. Zhu and Y. J. Zhu, Quark Transverse Parton Distribution at the Next-to-Next-to-Leading Order, Phys. Rev. Lett. 124 (2020) 092001 [1912.05778].
- [63] T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli and C. Studerus, *Calculation of the quark and gluon form factors to three loops in QCD*, *JHEP* 06 (2010) 094 [1004.3653].
- [64] P. A. Baikov, K. G. Chetyrkin, A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Quark and gluon form factors to three loops, Phys. Rev. Lett. 102 (2009) 212002 [0902.3519].
- [65] R. N. Lee, A. V. Smirnov and V. A. Smirnov, Analytic Results for Massless Three-Loop Form Factors, JHEP 04 (2010) 020 [1001.2887].
- [66] T. Gehrmann and A. Primo, *The three-loop singlet contribution to the massless axial-vector quark form factor*, *Phys. Lett. B* **816** (2021) 136223 [2102.12880].
- [67] L. Chen, M. Czakon and M. Niggetiedt, *The complete singlet contribution to the massless quark form factor at three loops in QCD*, 2109.01917.
- [68] R. Boughezal, C. Focke, X. Liu and F. Petriello, W-boson production in association with a jet at next-to-next-to-leading order in perturbative QCD, Phys. Rev. Lett. 115 (2015) 062002 [1504.02131].
- [69] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss and T. A. Morgan, Precise QCD predictions for the production of a Z boson in association with a hadronic jet, Phys. Rev. Lett. 117 (2016) 022001 [1507.02850].
- [70] R. Boughezal, J. M. Campbell, R. K. Ellis, C. Focke, W. T. Giele, X. Liu et al., *Z-boson production in association with a jet at next-to-next-to-leading order in perturbative QCD*, *Phys. Rev. Lett.* **116** (2016) 152001 [1512.01291].
- [71] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss and D. M. Walker, Next-to-Next-to-Leading-Order QCD Corrections to the Transverse Momentum Distribution of Weak Gauge Bosons, Phys. Rev. Lett. 120 (2018) 122001 [1712.07543].
- [72] R. Boughezal, X. Liu and F. Petriello, W-boson plus jet differential distributions at NNLO in QCD, Phys. Rev. D 94 (2016) 113009 [1602.06965].
- [73] M. A. Ebert, J. K. L. Michel, I. W. Stewart and F. J. Tackmann, Drell-Yan q<sub>T</sub> resummation of fiducial power corrections at N<sup>3</sup>LL, JHEP 04 (2021) 102 [2006.11382].
- [74] G. P. Salam and E. Slade, Cuts for two-body decays at colliders, JHEP 11 (2021) 220 [2106.08329].
- [75] G. Billis, B. Dehnadi, M. A. Ebert, J. K. L. Michel and F. J. Tackmann, *Higgs pT Spectrum and Total Cross Section with Fiducial Cuts at Third Resummed and Fixed Order in QCD*, *Phys. Rev. Lett.* **127** (2021) 072001 [2102.08039].

- [76] M. Heller, A. von Manteuffel, R. M. Schabinger and H. Spiesberger, *Mixed EW-QCD* two-loop amplitudes for  $q\bar{q} \rightarrow \ell^+ \ell^-$  and  $\gamma_5$  scheme independence of multi-loop corrections, *JHEP* **05** (2021) 213 [2012.05918].
- [77] R. Bonciani, L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano et al., *Mixed Strong-Electroweak Corrections to the Drell-Yan Process*, *Phys. Rev. Lett.* **128** (2022) 012002 [2106.11953].
- [78] F. Buccioni, F. Caola, H. A. Chawdhry, F. Devoto, M. Heller, A. von Manteuffel et al., *Mixed QCD-electroweak corrections to dilepton production at the LHC in the high invariant mass region*, *JHEP* 06 (2022) 022 [2203.11237].
- [79] L. Buonocore, M. Grazzini, S. Kallweit, C. Savoini and F. Tramontano, *Mixed QCD-EW* corrections to  $p p \rightarrow \ell \nu_{\ell} + X$  at the LHC, *Phys. Rev. D* **103** (2021) 114012 [2102.12539].
- [80] A. Behring, F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov et al., *Mixed QCD-electroweak corrections to W-boson production in hadron collisions*, *Phys. Rev. D* 103 (2021) 013008 [2009.10386].
- [81] S. Dittmaier, T. Schmidt and J. Schwarz, *Mixed NNLO QCD*×*electroweak corrections of*  $O(N_f \alpha_s \alpha)$  to single-W/Z production at the LHC, JHEP **12** (2020) 201 [2009.02229].
- [82] S. Dittmaier, A. Huss and C. Schwinn, *Mixed QCD-electroweak*  $O(\alpha_s \alpha)$  corrections to *Drell-Yan processes in the resonance region: pole approximation and non-factorizable corrections*, *Nucl. Phys. B* **885** (2014) 318 [1403.3216].
- [83] S. Dittmaier, A. Huss and C. Schwinn, *Dominant mixed QCD-electroweak O*( $\alpha_s \alpha$ ) *corrections to Drell–Yan processes in the resonance region*, *Nucl. Phys. B* **904** (2016) 216 [1511.08016].
- [84] A. Behring, F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov et al., *Estimating the impact of mixed QCD-electroweak corrections on the W-mass determination at the LHC*, *Phys. Rev. D* 103 (2021) 113002 [2103.02671].
- [85] A. Autieri, L. Cieri, G. Ferrera and G. F. R. Sborlini, *Combining QED and QCD transverse-momentum resummation for W and Z boson production at hadron colliders*, *JHEP* 07 (2023) 104 [2302.05403].
- [86] J. McGowan, T. Cridge, L. A. Harland-Lang and R. S. Thorne, Approximate N<sup>3</sup>LO Parton Distribution Functions with Theoretical Uncertainties: MSHT20aN<sup>3</sup>LO PDFs, 2207.04739.
- [87] S. Bailey, T. Cridge, L. A. Harland-Lang, A. D. Martin and R. S. Thorne, *Parton distributions from LHC, HERA, Tevatron and fixed target data: MSHT20 PDFs, Eur. Phys. J. C* 81 (2021) 341 [2012.04684].
- [88] C. M. Carloni Calame, M. Chiesa, H. Martinez, G. Montagna, O. Nicrosini, F. Piccinini et al., Precision Measurement of the W-Boson Mass: Theoretical Contributions and Uncertainties, Phys. Rev. D 96 (2017) 093005 [1612.02841].

- [89] J. Gao, D. Liu and K. Xie, Understanding PDF uncertainty in W boson mass measurements\*, Chin. Phys. C 46 (2022) 123110 [2205.03942].
- [90] J. Isaacson, Y. Fu and C. P. Yuan, ResBos2 and the CDF W Mass Measurement, 2205.02788.
- [91] F. Febres Cordero, A. von Manteuffel and T. Neumann, *Computational Challenges for Multi-loop Collider Phenomenology: A Snowmass 2021 White Paper, Comput. Softw. Big Sci.* 6 (2022) 14 [2204.04200].
- [92] F. Caola, W. Chen, C. Duhr, X. Liu, B. Mistlberger, F. Petriello et al., *The Path forward to N<sup>3</sup>LO*, in 2022 Snowmass Summer Study, 3, 2022, 2203.06730.
- [93] S. Kallweit, E. Re, L. Rottoli and M. Wiesemann, Accurate single- and double-differential resummation of colour-singlet processes with MATRIX+RADISH: W<sup>+</sup>W<sup>-</sup> production at the LHC, JHEP 12 (2020) 147 [2004.07720].
- [94] M. Grazzini, S. Kallweit, J. M. Lindert, S. Pozzorini and M. Wiesemann, NNLO QCD + NLO EW with Matrix+OpenLoops: precise predictions for vector-boson pair production, JHEP 02 (2020) 087 [1912.00068].
- [95] L. Buonocore, S. Kallweit, L. Rottoli and M. Wiesemann, *Linear power corrections for two-body kinematics in the qT subtraction formalism*, *Phys. Lett. B* 829 (2022) 137118 [2111.13661].
- [96] S. Camarda et al., DYTurbo: Fast predictions for Drell-Yan processes, Eur. Phys. J. C 80 (2020) 251 [1910.07049].
- [97] S. Camarda, L. Cieri and G. Ferrera, Fiducial perturbative power corrections within the q<sub>T</sub> subtraction formalism, Eur. Phys. J. C 82 (2022) 575 [2111.14509].
- [98] T. Becher and T. Neumann, Fiducial  $q_T$  resummation of color-singlet processes at  $N^3LL+NNLO$ , JHEP **03** (2021) 199 [2009.11437].
- [99] J. M. Campbell, D. Wackeroth and J. Zhou, *Study of weak corrections to Drell-Yan, top-quark pair, and dijet production at high energies with MCFM, Phys. Rev. D* **94** (2016) 093009 [1608.03356].
- [100] P. F. Monni, P. Nason, E. Re, M. Wiesemann and G. Zanderighi, *MiNNLO<sub>PS</sub>: a new method to match NNLO QCD to parton showers*, *JHEP* 05 (2020) 143 [1908.06987].
- [101] I. Scimemi and A. Vladimirov, Analysis of vector boson production within TMD factorization, Eur. Phys. J. C 78 (2018) 89 [1706.01473].
- [102] R. Gavin, Y. Li, F. Petriello and S. Quackenbush, FEWZ 2.0: A code for hadronic Z production at next-to-next-to-leading order, Comput. Phys. Commun. 182 (2011) 2388 [1011.3540].
- [103] R. Gavin, Y. Li, F. Petriello and S. Quackenbush, W Physics at the LHC with FEWZ 2.1, Comput. Phys. Commun. 184 (2013) 208 [1201.5896].

- [104] C. M. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini, *Precision electroweak calculation of the production of a high transverse-momentum lepton pair at hadron colliders*, *JHEP* **10** (2007) 109 [0710.1722].
- [105] C. M. Carloni Calame, G. Montagna, O. Nicrosini and A. Vicini, Precision electroweak calculation of the charged current Drell-Yan process, JHEP 12 (2006) 016 [hep-ph/0609170].
- [106] W. Bizon, P. F. Monni, E. Re, L. Rottoli and P. Torrielli, Momentum-space resummation for transverse observables and the Higgs p<sub>⊥</sub> at N<sup>3</sup>LL+NNLO, JHEP 02 (2018) 108
  [1705.09127].
- [107] P. F. Monni, E. Re and P. Torrielli, *Higgs Transverse-Momentum Resummation in Direct Space*, *Phys. Rev. Lett.* **116** (2016) 242001 [1604.02191].