PROCEEDINGS OF SCIENCE

Influence of the nonextensivity on the transport properties of a magnetized hot QCD medium

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This work focuses on the influence of the nonextensivity on the transport properties related to charge and heat in a magnetized hot QCD medium. We have determined the electrical conductivity, Hall conductivity, thermal conductivity and Hall-type thermal conductivity using the nonextensive Tsallis framework within the kinetic theory approach. The deviation of the nonextensive parameter q from 1 in the Tsallis distribution function expresses the degree of the nonextensivity. The nonextensive Tsallis framework is beneficial to use, because the matter produced in heavy ion collisions is not exactly in the locally equilibrated state, rather it may slightly deviate from being in that state. It is observed that the abovementioned transport coefficients increase with the nonextensivity, thus the deviations of both charge and heat transport coefficients from their counterparts at $q = 1$ get enhanced. The presence of magnetic field further increases the deviations of the transport coefficients from their respective equilibrated values, whereas these deviations get decreased at finite chemical potential. Present work is also extended to explore the effect of the nonextensivity on the elliptic flow.

The Eleventh Annual Conference on Large Hadron Collider Physics (LHCP 2023)
22-26 May 2023
Belgrade, Serbia

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1. Introduction

The energetic heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) can produce quark-gluon plasma (QGP) at high temperature and/or high density. The noncentral events of such collisions could generate extremely strong magnetic fields with strengths varying from $eB = m_\pi^2$ at RHIC to $eB = 15\, m_\pi^2$ at LHC [1, 2]. These magnetic fields become weak as time progresses, however, their lifetimes get extended in an electrically conducting medium [3, 4]. The matter produced in aforesaid collisions may slightly deviate from being exactly in the equilibrated state and for an accurate description of such matter, the nonextensive Tsallis mechanism is followed, where the deviation of parameter q from unity measures the extent of the nonextensivity [5]. Using this mechanism, we have studied electrical ($\sigma_{\rm el}$), Hall ($\sigma_{\rm H}$), thermal (κ) and Hall-type thermal ($\kappa_{\rm H}$) conductivities, and elliptic flow (v_2) in the present work.

2. Transport coefficients

In the nonextensive Tsallis formalism, the fermion distribution function is written [6, 7] as $f = 1/\left[\left\{1+(q-1)\beta(u^\alpha p_\alpha \mp \mu_f)\right\}^{\frac{1}{q-1}}+1\right]$, where '−' ('+') sign is for f_q ($\bar{f}_q$), $p_\alpha \equiv \left(\omega_f, \mathbf{p}\right)$, u^α is the four-velocity of fluid and $T = \beta^{-1}$. External electric field can induce an electric current density, which is expressed in terms of the infinitesimal deviations (δf_q and $\delta\bar{f}_q$) of f_q and $\bar{f}_q$ as

$$J^i = \sum_f g_f \int \frac{d^3{\rm p}}{(2\pi)^3\omega_f} p^i \left[q_f \delta f_q + \bar{q}_f \delta\bar{f}_q\right]. \tag{1}$$

When a magnetic field is applied perpendicular to the electric field ($\mathbf{E} \perp \mathbf{B}$), j^i is written as

$$J^i = \sigma^{ij} E_j = \left(\sigma_{\rm el}\delta^{ij} + \sigma_{\rm H}\epsilon^{ij}\right) E_j. \tag{2}$$

We use $\mathbf{E} = E\hat{x}$ and $\mathbf{B} = B\hat{z}$. To determine δf_q, we use the relativistic Boltzmann transport equation in the relaxation time approximation within the nonextensive Tsallis mechanism,

$$p^\mu \frac{\partial f_q'}{\partial x^\mu} + \mathcal{F}^\mu \frac{\partial f_q'}{\partial p^\mu} = -\frac{p_\nu u^\nu}{\tau_f}\delta f_q\ , \tag{3}$$

where $f_q' = f_q + \delta f_q$, $\mathcal{F}^\mu = (p^0\mathbf{v}\cdot\mathbf{F}, p^0\mathbf{F})$ and $\mathbf{F} = q_f(\mathbf{E} + \mathbf{v}\times\mathbf{B})$. The relaxation time for quark (antiquark), τ_f ($\tau_{\bar{f}}$) is given [8] by $\tau_{f(\bar{f})} = 1/\left[5.1T\alpha_s^2 \log\left(1/\alpha_s\right)\left\{1+0.12(2N_f+1)\right\}\right]$. For a spatially homogeneous distribution function with steady-state condition, eq. (3) becomes

$$\tau_f q_f E v_x \frac{\partial f_q'}{\partial p_0} + \tau_f q_f B v_y \frac{\partial f_q'}{\partial p_x} - \tau_f q_f B v_x \frac{\partial f_q'}{\partial p_y} = f_q - f_q' - \tau_f q_f E \frac{\partial f_q}{\partial p_x}\ . \tag{4}$$

Equation (4) can be solved by using an ansatz: $f_q' = f_q - \tau_f q_f \mathbf{E}\cdot\frac{\partial f_q}{\partial \mathbf{p}} - \mathbf{\Gamma}\cdot\frac{\partial f_q}{\partial \mathbf{p}}$, where the quantity $\mathbf{\Gamma}$ needs to be calculated. Using ansatz and eq. (4), we get δf_q and $\delta\bar{f}_q$. Substituting them in eq. (1) and then comparing with eq. (2), we calculate $\sigma_{\rm el}$ and $\sigma_{\rm H}$ respectively [9] as

$$\sigma_{\rm el} = \sum_f \frac{\beta g_f q_f^2}{3\pi^2}\int d{\rm p}\frac{{\rm p}^4}{\omega_f^2}\left[\frac{\tau_f\left[1+(q-1)\beta\omega\right]^{\frac{q}{1-q}}}{1+\omega_c^2\tau_f^2} + \frac{\tau_{\bar{f}}\left[1+(q-1)\beta\bar{\omega}\right]^{\frac{q}{1-q}}}{1+\omega_c^2\tau_{\bar{f}}^2}\right], \tag{5}$$

$$\sigma_{\rm H} = \sum_f \frac{\beta g_f q_f^2}{3\pi^2}\int d{\rm p}\frac{{\rm p}^4\omega_c}{\omega_f^2}\left[\frac{\tau_f^2\left[1+(q-1)\beta\omega\right]^{\frac{q}{1-q}}}{1+\omega_c^2\tau_f^2} + \frac{\tau_{\bar{f}}^2\left[1+(q-1)\beta\bar{\omega}\right]^{\frac{q}{1-q}}}{1+\omega_c^2\tau_{\bar{f}}^2}\right], \tag{6}$$

where the cyclotron frequency $\omega_c = \frac{q_f B}{\omega_f}$, $\omega = \omega_f - \mu_f$ and $\bar{\omega} = \omega_f + \mu_f$.

The heat flow in a medium is associated with the energy diffusion and the enthalpy diffusion as $Q_\mu = \Delta_{\mu\alpha} T^{\alpha\beta} u_\beta - h\Delta_{\mu\alpha} N^\alpha$, where $\Delta_{\mu\alpha} = g_{\mu\alpha} - u_\mu u_\alpha$, the enthalpy per particle $h = (\varepsilon + P)/n$, the particle number density $n = N^\alpha u_\alpha$, the energy density $\varepsilon = u_\alpha T^{\alpha\beta} u_\beta$ and the pressure $P = -\Delta_{\alpha\beta} T^{\alpha\beta}/3$. The spatial component of the heat flow is written as

$$Q^i = \sum_f g_f \int \frac{d^3\mathrm{p}}{(2\pi)^3 \omega_f}\, p^i \left[(\omega_f - h_f)\delta f_q + (\omega_f - \bar{h}_f)\delta \bar{f}_q\right]. \tag{7}$$

In the Navier-Stokes equation at finite magnetic field, if the gradients of temperature and pressure are perpendicular to the magnetic field, then we get

$$Q^i = -\left(\kappa\delta^{ij} + \kappa_{\mathrm{H}}\epsilon^{ij}\right)\left[\partial_j T - \frac{T}{\varepsilon + P}\partial_j P\right]. \tag{8}$$

Using the ansatz in eq. (3) and then dropping the electric field part, we have

$$\frac{\tau_f p^\mu}{p_0}\frac{\partial f_q}{\partial x^\mu} + \beta\left[1 + (q-1)\beta\omega\right]^{\frac{q}{1-q}} \mathbf{\Gamma}\cdot\mathbf{v} - q_f B \tau_f \left(v_x \frac{\partial f_q'}{\partial p_y} - v_y \frac{\partial f_q'}{\partial p_x}\right) = 0. \tag{9}$$

From eq. (9) and ansatz, we get δf_q and $\delta \bar{f}_q$. Substituting them in eq. (7) and then comparing with eq. (8), we calculate κ and κ_{H} respectively [9] as

$$\kappa = \sum_f \frac{\beta^2 g_f}{6\pi^2}\int d\mathrm{p}\frac{\mathrm{p}^4}{\omega_f^2}\left[\frac{\tau_f\left[1 + (q-1)\beta\omega\right]^{\frac{q}{1-q}}\left(\omega_f - h_f\right)^2}{1 + \omega_c^2\tau_f^2} + \frac{\tau_{\bar{f}}\left[1 + (q-1)\beta\bar{\omega}\right]^{\frac{q}{1-q}}\left(\omega_f - \bar{h}_f\right)^2}{1 + \omega_c^2\tau_{\bar{f}}^2}\right], \tag{10}$$

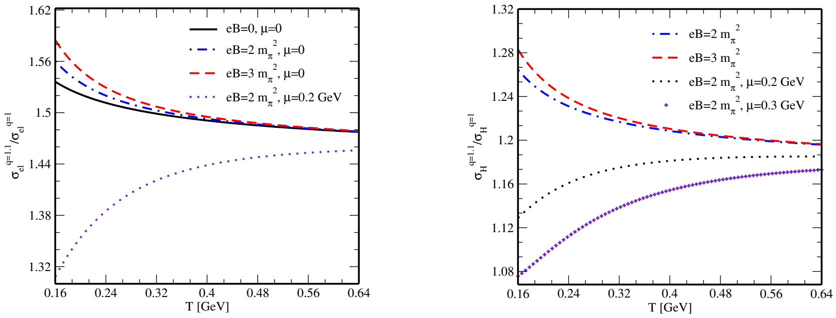
$$\kappa_{\mathrm{H}} = \sum_f \frac{\beta^2 g_f}{6\pi^2}\int d\mathrm{p}\frac{\mathrm{p}^4\omega_c}{\omega_f^2}\left[\frac{\tau_f^2\left[1 + (q-1)\beta\omega\right]^{\frac{q}{1-q}}\left(\omega_f - h_f\right)^2}{1 + \omega_c^2\tau_f^2} + \frac{\tau_{\bar{f}}^2\left[1 + (q-1)\beta\bar{\omega}\right]^{\frac{q}{1-q}}\left(\omega_f - \bar{h}_f\right)^2}{1 + \omega_c^2\tau_{\bar{f}}^2}\right]. \tag{11}$$

In this study, we use the thermal mass (squared) of quark [10], $m_{fT}^2 = \frac{g^2T^2}{6}\left(1 + \frac{\mu_f^2}{\pi^2T^2}\right)$. We also assume that the chemical potentials for all flavors (u, d and s) are same, *i.e.* $\mu_f = \mu$.

3. Results and discussions

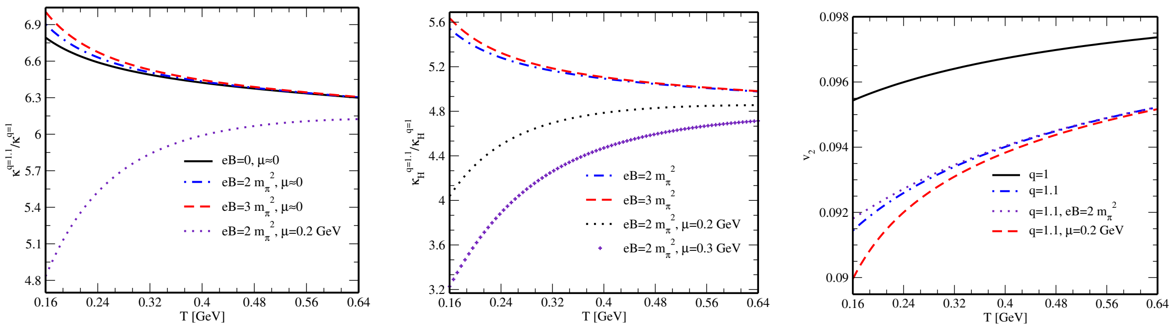
Figures 1a and 1b respectively depict how the electrical and Hall conductivities vary with temperature for $q > 1$. It is observed that, their magnitudes get enhanced when $q = 1.1$, which explains that the nonextensivity facilitates the charge transport in the medium. In addition, the deviations of the nonextensive σ_{el} as well as σ_{H} from their corresponding equilibrated values increase with the increase of magnetic field, whereas the introduction of chemical potential brings σ_{el} and σ_{H} a bit closer to their equilibrated values.

In figures 2a and 2b, the thermal and Hall-type thermal conductivities are respectively plotted as functions of temperature for $q > 1$. Observation shows that, both κ and κ_{H} are larger for $q = 1.1$ as compared to $q = 1$. Thus the nonextensivity enhances the heat transport in the medium. Further, as the magnetic field increases, the deviations of the nonextensive κ and κ_{H} from their respective



a b

Figure 1: Variations of (a) $\sigma_{\rm el}$ and (b) $\sigma_{\rm H}$ with T for different values of q parameter.



a b c

Figure 2: Variations of (a) κ, (b) $\kappa_{\rm H}$ and (c) v_2 with T for different values of q parameter.

thermally equilibrated values increase, unlike the chemical potential case which decreases these deviations.

The elliptic flow is defined [11–13] as $v_2 = \frac{v_2^h}{1+(\Omega/\Omega_h)}$, where the Knudsen number $\Omega = \frac{\lambda}{l}$, the mean free path $\lambda = \frac{3\kappa}{vC_V}$, v_2^h is the elliptic flow in hydrodynamic limit, Ω_h is the Knudsen number obtained by observing the transition between the hydrodynamic regime and the free streaming particle regime, C_V is the specific heat at constant volume, l and v are the characteristic length and velocity of flow, respectively. As per the transport calculation in ref. [13], $\Omega_h \approx 0.7$ and $v_2^h \approx 0.1$. In this work, we have used $v \simeq 1$ and $l = 4$ fm. Figure 2c displays a decrease of elliptic flow as q increases. This deviation is small in the presence of the weak magnetic field, whereas it is large at finite chemical potential. Thus, for $q > 1$, the mean free path gets closer to the characteristic length of the medium and it takes the medium a bit away from the local equilibrium state.

4. Conclusions

In this work, we studied the effects of the nonextensivity on different transport coefficients related to charge and heat of the QCD medium, which were calculated by using the kinetic theory approach. It was found that the nonextensivity enhances the transport coefficients of the medium. The elliptic flow also gets conspicuously affected by the nonextensive behavior of the medium.

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