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Electromagnetic Interaction of Colliding Q-Gaussian Beams

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The electromagnetic interaction of colliding beams causes beams to be perturbed as they approach each other. As luminosity depends on colliding beam parameters, luminosity is influenced and its calibration via the van-der-Meer scan is biased. Currently, this effect is estimated using Gaussian models for particle densities in colliding beams. However, for more precise calculations, models of beam-beam force that account for the beam shape deviations from Gaussianity should be found. In this article, the model for beam-beam interactions of Q-Gaussian colliding beams is presented. It is believed that this model adequately describes non-Gaussian tail shapes. For LHC conditions, a comparison with the regular Gaussian model is performed. The evolution of the effect during the van-der-Meer scan is demonstrated.

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1. Introduction

The luminosity of a collider is proportional to the particle collision intensity and depends on the intensities and overlap integral of the colliding beams. Collider experimental programs aim for high luminosity by increasing beam intensities or reducing beam size, resulting in more intense beams. When high-density beams collide, strong electromagnetic interactions cause beam perturbations by the non-linear beam-beam force, which causes orbit shift of the whole bunch (coherent effect) and particle redistribution inside the bunch (incoherent effect). This effect impacts collider luminosity in two ways. First, the number of collisions is changed from expected values; second, the luminosity calibration via van-der-Meer scan [1] is biased [2, 3]. For example the bias for LHC is more than 1% [4, 5]. Thus, beam-beam interaction influences the collider performance [6].

The current estimation of the beam-beam effect is based on the Gaussian assumption for the particle distribution in the colliding beams [2]. In [7, 8], it was demonstrated that the particle distribution is deviated from Gaussian. Thus, for more precise calculations, the non-Gaussianity of the particle distribution should be considered. The Q-Gaussian, see [9], describes the bunch profile for LHC and HL-LHC more realistically [8, 10]. In [11] the effect of the non-Gaussian tails on the emittance evolution and intra-beam scattering was investigated using Q-Gaussian beams. In [12], we investigated the beam overlap of Q-Gaussian beams. In this work, the beam-beam interaction of Q-Gaussian beams is considered, model for the total beam-beam kick is presented and the beam-beam effect on luminosity is estimated. The evolution of the effect during van-der-Meer scan is demonstrated.

2. Model

As two bunches of charged particles approach each other, they interact electromagnetically, resulting in two types of momentum kick. This section describes the total beam-beam kick of two Q-Gaussian bunches with equal RMS beam size and tail density in their respective horizontal and vertical directions. The Q-Gaussian distribution is defined as:

$$QG(u; q, \sigma) = \frac{1}{\sqrt{5-3q}\, C^{qG}\, \sigma} e_q \left[-\frac{x^2}{(5-3q)\,\sigma^2} \right], \tag{1}$$

where q is the tail density, σ is the standard deviation i.e. RMS beam size, and C^{qG} is a constant resulted from the normalization of the Q-Gaussian. Fig. 1 shows how tail density q affects the tail-to-core distribution. The distribution is light-tailed for $q < 1$. As q increases, the distribution get heavier tails. For $q = 1$, the Q-Gaussian distribution represents the usual Gaussian. For more on Q-Gaussian, see [11, 12].

When an ultra-relativistic charged particle, with relativistic factor $\beta \approx 1$, and charge Q_2 passes through (or near) a charged particle density ρ_1 at a zero crossing angle, it receives a total transverse momentum kick, which is defined as [2, 13]:

$$\Delta \boldsymbol{p}_2(x, y) = \frac{Q_2}{c} \int \boldsymbol{E}_1^{\perp}(x, y, z)\, dz = \frac{Z_2 e}{c} \boldsymbol{E}_1(x, y), \tag{2}$$

where $Z_2 e$ is the charge of the kicked particle, $\boldsymbol{E}_1^{\perp}(x, y, z)$ is the electric field produced by the kicker bunch ρ_1 perpendicular to the trajectory of the kicked particle, and $\boldsymbol{E}_1(x, y)$ is the total

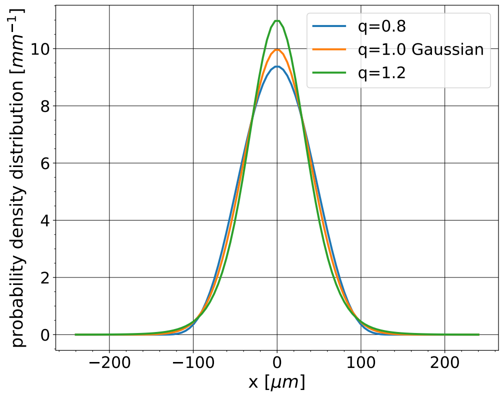


Figure 1: Q-Gaussian bunch profile with RMS $\sigma_x = 40\ \mu m$ and tail density $q = 0.8$, 1 "Gaussian" and 1.2

transverse field from the transverse distribution $\rho_1(x, y)$ of the kicker. The field from a round Gaussian bunch can be simply found by applying Gauss' law, whereas the field from an elliptical bunch is obtained from the potential using Green function with Gaussian form, as demonstrated by Bassetti and Erskine in [14]. The task is more complex for Q-Gaussian due to its complex form Eq. (1). Since $\boldsymbol{E}(x_p, y_p)$ produced by the bunch ρ_1 at a certain point (x_p, y_p) is equal to the sum of electric fields produced by the individual particles forming it, $\boldsymbol{E}(x_p, y_p) = \sum_i \boldsymbol{E^i}(x_p, y_p)$, and since ρ_1 is continuous, the electric field can be approximated by the following integral:

$$\boldsymbol{E}_1(x_p, y_p) = \frac{N_1 Z_1 e}{2\pi\epsilon_o} \int\int \frac{(\boldsymbol{x_p} - \boldsymbol{x}) + (\boldsymbol{y_p} - \boldsymbol{y})}{(x_p - x)^2 + (y_p - y)^2} QG(x; q_x, \sigma_x)\, QG(y; q_y, \sigma_y) dx\, dy, \quad (3)$$

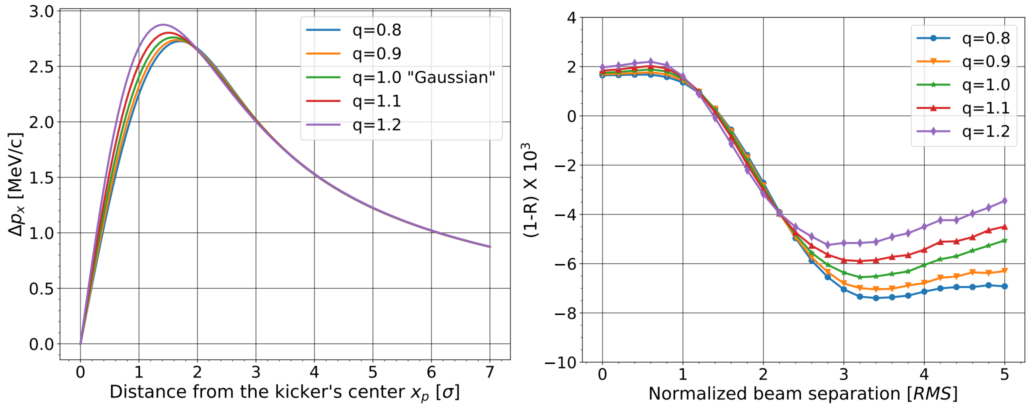
where N_1 is the number of particles in the kicker bunch and $Z_1 e$ is the charge. Fig. 2a shows the horizontal kick Δp_x produced by Q-Gaussian bunch, where different tail-weight q is considered. The dependence of the kick on the tail weight is divided into three regions: region–1 near the bunch center for $|x| \lesssim 1.8\,\sigma$, the heavier the tails is the stronger $|\Delta p_x|$; region–2 at medium range from the bunch center for $1.8\,\sigma \lesssim |x| \lesssim 3.5\,\sigma$, the lighter the tails the stronger $|\Delta p_x|$; region–3 at a further distance from the bunch center for $|x| \gtrsim 3.5\,\sigma$, $|\Delta p_x|$ of Q-Gaussian tends to that of Gaussian with a deviation up to 0.02%.

3. Simulation and Results

The luminosity variations due to the beam-beam effect of Q-Gaussian beams were estimated using a modification of V. Balagura's B*B simulation code [15]. The B*B code calculates beam-beam luminosity corrections R, the ratio of beam overlap with and without beam-beam interaction, $\Omega_{bb}/\Omega_{no-bb}$[1], based on the exact nonlinear electrostatic force for round and elliptical Gaussian and multi-Gaussian charge densities. To estimate the effect for Q-Gaussian beams, the Q-Gaussian distribution and its related kicks has been implemented in B*B.

The dependence of correction R on tail weight q has been investigated for one of ATLAS van-der-Meer scan bunch parameters, taken as in [2] from [16], listed in table 1. Two Q-Gaussian

[1]The beam overlap represents the convolution of the transverse particle densities of the colliding bunches, it is defined as: $\Omega(\Delta x, \Delta y) = \int \rho_1(x, y)\, \rho_2(x + \Delta x, y + \Delta y)\, dxdy$, where Δx and Δy are the horizontal and vertical beam separations.



(a) **(b)**

Figure 2: The horizontal beam-beam kick Δp_x gained by the particle at the distance $(x_p, 0)$ from the center of Q-Gaussian kicker bunch with parameters as in table 1 and tail weights q = 0.8, 0.9, 1 "Gaussian", 1.1, and 1.2 (2a), and the resultant beam-beam correction R during the horizontal van-der-Meer scan of two of these q-Gaussian bunches with the same parameters (2b)

Momentum p [GeV]	$Z_{1,2}$	$\beta_{x,y}$ [m]	Tune $Q_{x,y}$	RMS size [μm]	$N_{1,2}$
3500	1	1.5	64.31, 59.32	40	8.5×10^{10}

Table 1: Bunch parameters used for simulation from one of ATLAS van-der-Meer scan data [16]

beams with equal RMS beam size σ and tail weights q in their respective horizontal and vertical directions is investigated. Five different cases were simulated for different $q = 0.8, 0.9, 1$ "Gaussian" , 1.1, and 1.2. Fig. 2b shows the evolution of the beam-beam correction during van-der-Meer scan. At small separations, the heavy-tailed beams experience stronger beam-beam effects, which results in a higher luminosity variation, while at large separations, the light-tailed beams have the higher variation due to the higher sensitivity of their beam overlap Ω to small variations in separation.

4. Summary

The particle distribution in the beam has a significant impact on the beam-beam force. The evolution of the beam-beam effect of Q-Gaussian bunch during van-der-Meer scan is demonstrated. As one can see from Fig. 2b the beam-beam bias calculated for Gaussian and Q-Gaussian beams can differ up to 0.5% at van-der-Meer scan. Bearing in mind that the target precision of luminosity measurements is below 1% in HL-LHC, it means the beam-beam interaction as well as its influence on beam overlap should be considered carefully with the proper beam distribution.

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