PROCEEDINGS OF SCIENCE

Theory overview of muon g-2 and EDM

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We present a concise review of the new physics sensitivity of leptonic dipole moments and their interrelationship. In particular, focusing on the current muon g-2 anomaly, we analyse both low-energy and high-energy tests to confirm or to falsify it.

Muon4Future Workshop (Muon4Future2023)
29–31 May, 2023
Venezia, Italy

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1. Introduction

The anomalous magnetic moment of the muon, $a_\mu \equiv (g_\mu - 2)/2$, has provided an enduring hint of new physics (NP) for many years. The recent a_μ measurement by the Muon g-2 collaboration at Fermilab [1] has confirmed the earlier result by the E821 experiment at Brookhaven [2], yielding the average $a_\mu^{\rm EXP} = 116592061(41)\times 10^{-11}$. The comparison of this result with the Standard Model (SM) prediction $a_\mu^{\rm SM} = 116591810(43)\times 10^{-11}$ of the Muon g-2 Theory Initiative [3] leads to an intriguing 4.2σ discrepancy [1]

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 251\,(59)\times 10^{-11}\,. \qquad (1)$$

On the theory side, the only source of sizable uncertainties in $a_\mu^{\rm SM}$ stems from the non-perturbative contributions of the hadronic sector, which have been under close scrutiny for several years. The SM prediction $a_\mu^{\rm SM}$ in Eq. (1) has been derived using $(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-}$, the leading hadronic vacuum polarization (HVP) contribution to the muon g-2 based on low-energy $e^+e^- \to$ hadrons data obtained by the Muon g-2 Theory Initiative [3]. Alternatively, the HVP contribution has been computed using a first-principle lattice QCD approach [3]. Recently, the BMW lattice QCD collaboration (BMWc) computed the leading HVP contribution to the muon g-2 with sub per-cent precision, finding a value, $(a_\mu^{\rm HVP})_{\rm BMW}$, larger than $(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-}$ [4]. If $(a_\mu^{\rm HVP})_{\rm BMW}$ is used to obtain $a_\mu^{\rm SM}$ instead of $(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-}$, the discrepancy with the experimental result is reduced to 1.6σ only. The above results are respectively

$$(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-} = 6931\,(40)\times 10^{-11}\,, \qquad (a_\mu^{\rm HVP})_{\rm BMW} = 7075\,(55)\times 10^{-11}\,. \qquad (2)$$

The difference between these two values has been referred to as the *new muon g-2 puzzle* [5]. In [5], it was investigated the possibility to solve this tension invoking NP in the hadronic cross-section. It was argued that the most plausible scenario requires the presence of a light NP mediator that modifies the experimental cross-section $\sigma_{\rm had}$. However, this non-trivial setup, where NP hides in $e^+e^- \to$ hadrons data, is excluded by a number of experimental constraints [5]. Alternative confirmations of the e^+e^- determinations of the HVP contribution to the muon g-2, based on either additional lattice QCD calculations or direct experimental measurements, as proposed by the MUonE experiment [6], will be crucial to solve this intriguing puzzle. Interestingly, the muon g-2 discrepancy of eq. (1) can be solved by a NP effect of the same order as the SM weak contribution $\approx 2\times 10^{-9}$ [3]. In principle, NP scenarios entailing weakly coupled particles at the electroweak scale could provide a natural explanation of eq. (1), see e.g. [7]. In practice, however, the experimental bounds by LEP and LHC highly disfavours this possibility. Therefore, the scenarios preferred by data include either very light and feebly coupled particles, see e.g. [8], or very heavy and strongly coupled particles [9].

Heavy NP contributions to the muon g-2 stem from the dipole operator $\left(\overline{\mu}_L\sigma_{\mu\nu}\mu_R\right) HF^{\mu\nu}$ where $H = v + h/\sqrt{2}$ contains both the Higgs boson field h and its vacuum expectation value $v = 174$ GeV while $F^{\mu\nu}$ is the electromagnetic field strenght tensor. After electroweak symmetry breaking, $\Delta a_\mu^{\rm NP} \sim (g^2_{\rm NP}/16\pi^2)\times(m_\mu v/\Lambda^2)$, where $g_{\rm NP}$ is a representative NP coupling. Therefore, the chiral enhancement $v/m_\mu \sim 10^3$, together with the assumption of a new strong dynamics ($g_{\rm NP} \sim 4\pi$), bring the sensitivity of the muon g-2 to NP scales of order $\Lambda \sim 100$ TeV [9].

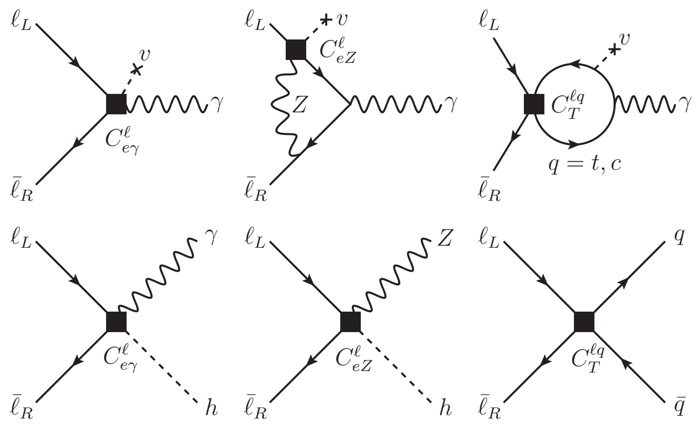

Figure 1: *Upper row:* Feynman diagrams contributing to the leptonic g-2 up to one-loop order in the Standard Model EFT. *Lower row:* Feynman diagrams of the corresponding high-energy processes. Dimension-6 effective interaction vertices are denoted by a square (from [9]).

A direct detection of new particles at a so high scales is beyond the capabilities of any foreseen collider. Furthermore, the discovery of new particles by their direct production [10] couldn't be unambiguously associated to Δa_μ. In other words, we need to test the muon g-2 anomaly model-independently. Our goal is to outline possible directions for such a model-independent test.

2. High-energy tests of the muon g-2 anomaly

In ref. [9], it was argued that a muon collider (MC) running at energies E of several TeV would represent the only machine able to probe NP in the muon g-2 model-independently. In fact, the same dipole operator generating Δa_μ unavoidably induces also a NP contribution to the process $\mu^+\mu^- \to h\gamma$. Focusing on the leptonic g-2, the relevant effective Lagrangian reads

$$\mathcal{L} = \frac{C^\ell_{eB}}{\Lambda^2}\left(\bar\ell_L\sigma^{\mu\nu}e_R\right)HB_{\mu\nu} + \frac{C^\ell_{eW}}{\Lambda^2}\left(\bar\ell_L\sigma^{\mu\nu}e_R\right)\tau^I H W^I_{\mu\nu} + \frac{C^\ell_T}{\Lambda^2}(\bar\ell^a_L\sigma_{\mu\nu}e_R)\varepsilon_{ab}(\overline{Q}^b_L\sigma^{\mu\nu}u_R) + h.c. \tag{3}$$

where $\Lambda \gtrsim 1$ TeV is assumed. In figure 1, we show the Feynman diagrams contributing to the leptonic g-2 as well as to correlated high-energy processes. An explicit one-loop calculation of Δa_ℓ provides the following result

$$\Delta a_\ell \simeq \frac{4m_\ell v}{e\Lambda^2}\left(C^\ell_{e\gamma} - \frac{3\alpha}{2\pi}\frac{c^2_W - s^2_W}{s_W c_W}C^\ell_{eZ}\log\frac{\Lambda}{m_Z}\right) - \sum_{q=c,t}\frac{4m_\ell m_q}{\pi^2}\frac{C^{\ell q}_T}{\Lambda^2}\log\frac{\Lambda}{m_q}, \tag{4}$$

where s_W (c_W) is the sine (cosine) of the Weinberg angle while $C_{e\gamma}$ and C_{eZ} are linear combinations of C_{eB} and C_{eW}. From eq. (4), one can find [9]

$$\Delta a_\mu \approx 3\times 10^{-9}\left(\frac{250\,\text{TeV}}{\Lambda}\right)^2\left(C^\mu_{e\gamma} - 0.2C^{\mu t}_T - 0.001C^{\mu c}_T - 0.05C^\mu_{eZ}\right).$$

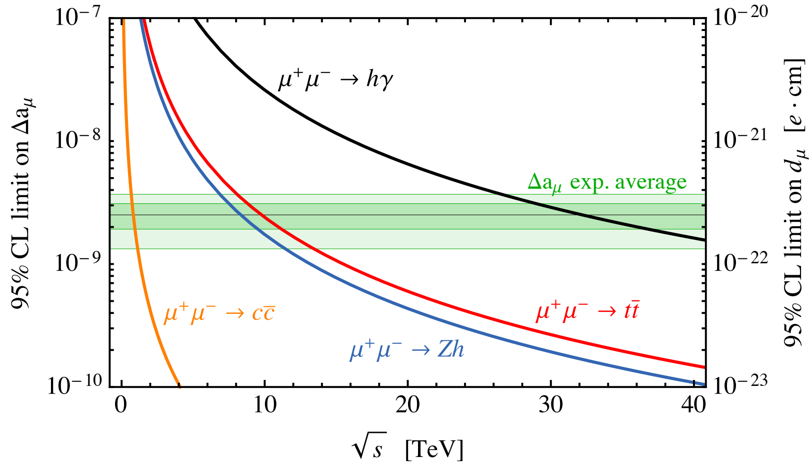

Figure 2: 95% C.L. reach on Δa_μ as a function of $\sqrt{s}$, from the processes $\mu^+\mu^- \to h\gamma$ (black), $\mu^+\mu^- \to hZ$ (blue), $\mu^+\mu^- \to t\bar{t}$ (red), and $\mu^+\mu^- \to c\bar{c}$ (orange) from [9].

The main contribution to Δa_μ comes from the coefficient $C_{e\gamma}$ related to the photonic dipole operator which also induces a contribution to the process $\mu^+\mu^- \to h\gamma$ (see figure 1). In particular, the total cross-section of $\mu^+\mu^- \to h\gamma$ is given by [9]

$$\sigma_{h\gamma} = \frac{s}{48\pi}\frac{|C^{\mu}_{e\gamma}|^2}{\Lambda^4} \approx 0.7\,\mathrm{ab}\left(\frac{\sqrt{s}}{30\,\mathrm{TeV}}\right)^2\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2 \tag{5}$$

where we kept only the dominant $C^{\mu}_{e\gamma}$ contribution to Δa_μ. In figure 2, we show as a black line the 95% C.L. reach from $\mu^+\mu^- \to h\gamma$ on Δa_μ as a function of the collider energy.

Thanks to the growth with energy of $\sigma_{h\gamma}$ as well as of the reference integrated luminosity $\mathcal{L} = (\sqrt{s}/30\,\mathrm{TeV})^2 \times 10\,\mathrm{ab}^{-1}$, we see that a muon collider with $\sqrt{s} \gtrsim 30$ TeV would have the sufficient sensitivity to test the muon g-2 anomaly.

3. Low-energy tests of the muon g-2 anomaly

The dipole operators of eq. (3) generally have a non-trivial flavour and CP structure. As a result, a NP contribution to Δa_μ is typically accompanied by lepton flavor violating (LFV) and CP violating effects [11]. Below the electroweak scale dipole transitions $\ell \to \ell'\gamma$ in the leptonic sector are described by the effective Lagrangian

$$\mathcal{L} = e\frac{m_\ell}{2}\left(\bar{\ell}_R\sigma_{\mu\nu}A_{\ell\ell'}\ell'_L + \bar{\ell}'_L\sigma_{\mu\nu}A^{\star}_{\ell\ell'}\ell_R\right)F^{\mu\nu} \tag{6}$$

where $\ell, \ell' = e, \mu, \tau$. Starting from eq. (6), we can evaluate LFV processes, such as $\mu \to e\gamma$,

$$\frac{\mathrm{BR}(\ell \to \ell'\gamma)}{\mathrm{BR}(\ell \to \ell'\nu_\ell\bar{\nu}_{\ell'})} = \frac{48\pi^3\alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right). \tag{7}$$

The effective Lagrangian of eq. (6) generates also flavor conserving processes such as the anomalous magnetic moments of leptons, Δa_ℓ, as well as the leptonic electric dipole moments (EDMs, d_ℓ) which read

$$\Delta a_\ell = 2m_\ell^2\,\mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell\,\mathrm{Im}(A_{\ell\ell}). \tag{8}$$

In concrete NP scenarios, one would generally expect that Δa_ℓ, d_ℓ and $\mathrm{BR}(\ell \to \ell'\gamma)$ are correlated. However, these connections depend on the unknown flavor and CP structures of the underlying NP sector and therefore are model-dependent.

Parametrizing the amplitude $A_{\ell\ell'}$ as $A_{\ell\ell'} = c_{\ell\ell'}/\Lambda^2$, where Λ refers to the NP scale, we can evaluate which are the values of Λ probed by $\mu \to e\gamma$. We find that

$$\mathrm{BR}(\mu \to e\gamma) \approx 10^{-12} \left(\frac{500\ \mathrm{TeV}}{\Lambda}\right)^4 \left(|c_{\mu e}|^2 + |c_{e\mu}|^2\right) . \tag{9}$$

Combining Δa_ℓ and $\mathrm{BR}(\ell \to \ell'\gamma)$, one can find that

$$\begin{aligned} \mathrm{BR}(\mu \to e\gamma) &\approx 10^{-12} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{e\mu}}{2 \times 10^{-5}}\right)^2 , \\ \mathrm{BR}(\tau \to \ell\gamma) &\approx 10^{-8} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{\ell\tau}}{5 \times 10^{-3}}\right)^2 , \end{aligned} \tag{10}$$

where $\theta_{\ell\ell'} = \sqrt{|c_{\ell\ell'}|^2 + |c_{\ell'\ell}|^2}/c_{\mu\mu}$. As a result, it is found that the solution of the muon g-2 anomaly requires highly suppressed flavor mixing angles $\theta_{e\mu}$ [12]. We also find that

$$d_e \approx 10^{-24} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) \varphi^e_{\mathrm{CP}} \quad e\,\mathrm{cm}, \tag{11}$$

and therefore also the electron EDM exceeds the current experimental bound by several orders of magnitudes unless the CP violating phase $\varphi^e_{\mathrm{CP}} = [\mathrm{Im}(c_{ee})/\mathrm{Re}(c_{\mu\mu})] \lesssim 10^{-5}$ [12].

4. Conclusion

The muon g-2 discrepancy is one of most interesting hints of new physics emerged so far in particle physics, which has recently been reinforced by the E989 experiment at Fermilab. However, the low-energy determination of Δa_μ requires that systematic and hadronic uncertainties are under control at the level of $\Delta a_\mu \sim 10^{-9}$. Needless to say, an independent test of Δa_μ, not contaminated by the above uncertainties, would be very desirable. Interestingly, a multi-TeV muon collider can achieve this goal, providing a model-independent test of new physics in the muon g-2 through the high-energy processes $\mu^+\mu^- \to h\gamma, hZ, q\bar{q}$. These results rely on measurements with $\mathcal{O}(1)$ accuracy, therefore not requiring a precise control of systematic or theoretical uncertainties. These findings are model-independent, as they are formulated in terms of the same effective operators controlling the lepton dipole moments. Should the muon g-2 anomaly be confirmed in the future, this would constitute a *no-lose* theorem for a multi-TeV muon collider, guaranteeing the discovery of new physics in high-energy collisions.

From the low-energy side, the same dipole operator generating a new physics contribution to Δa_μ is expected to generate also other low-energy processes including lepton flavour violating (LFV) decays such as $\mu \to e\gamma$ and CP violating processes like the electron EDM.

We hope that, with the expected sensitivities of next-generation experiments, NP will show up in some of the processes analysed in this contribution. In this case, the interrelationship among leptonic $g-2$, EDMs and LFV will be of outmost importance to disentangle among different NP scenarios.

Acknowledgements

This work received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement n. 101086085 – ASYMMETRY, by the INFN Iniziative Specifica APINE, and by the Italian Ministry of University and Research (MUR) via the PRIN 2022 project n. 2022K4B58X – AxionOrigins.

References

[1] B. Abi *et al.* [Muon g-2], Phys. Rev. Lett. **126** (2021) no.14, 141801.

[2] G. W. Bennett *et al.* [Muon g-2], Phys. Rev. D **73** (2006), 072003 doi:10.1103/PhysRevD.73.072003 [arXiv:hep-ex/0602035 [hep-ex]].

[3] T. Aoyama, *et al.* Phys. Rept. **887** (2020), 1-166.

[4] S. Borsanyi, *et al.* Nature **593** (2021) no.7857, 51-55.

[5] L. Di Luzio, A. Masiero, P. Paradisi and M. Passera, Phys. Lett. B **829** (2022), 137037.

[6] C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni, Phys. Lett. B **746** (2015), 325-329; G. Abbiendi, *et al.* Eur. Phys. J. C **77** (2017) no.3, 139; A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D **102** (2020) no.7, 075013.

[7] P. Athron, C. Balázs, D. H. J. Jacob, W. Kotlarski, D. Stöckinger and H. Stöckinger-Kim, JHEP **09**, 080 (2021); A. Broggio, E. J. Chun, M. Passera, K. M. Patel and S. K. Vempati, JHEP **11**, 058 (2014); G. Barenboim, P. Paradisi, O. Vives, E. Lunghi and W. Porod, JHEP **04**, 079 (2008); G. Isidori and P. Paradisi, Phys. Rev. D **73** (2006), 055017; S. Mihara, J. P. Miller, P. Paradisi and G. Piredda, Ann. Rev. Nucl. Part. Sci. **63**, 531-552 (2013).

[8] W. J. Marciano, A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D **94**, no.11, 115033 (2016); C. Cornella, P. Paradisi and O. Sumensari, JHEP **01**, 158 (2020).

[9] D. Buttazzo and P. Paradisi, Phys. Rev. D **104**, no.7, 075021 (2021); P. Paradisi, O. Sumensari and A. Valenti, Phys. Rev. D **106** (2022) no.11, 115038.

[10] R. Capdevilla, D. Curtin, Y. Kahn and G. Krnjaic, Phys. Rev. D **103** (2021) no.7, 075028.

[11] G. F. Giudice, P. Paradisi and M. Passera, JHEP **11**, 113 (2012);

[12] L. Calibbi, P. Paradisi and R. Ziegler, Eur. Phys. J. C **74**, no.12, 3211 (2014); F. Feruglio, P. Paradisi and A. Pattori, Eur. Phys. J. C **75**, no.12, 579 (2015).