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R Measurements: Recent Results and Perspectives for the Evaluation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment.

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The anomalous magnetic moment of the muon a_μ represents one of the most promising quantity to search for hints of new physics beyond the Standard Model (SM) in the precision frontier. The latest experimental result from the *Muon* $g - 2$ Experiment at Fermilab has pushed the tension with the SM prediction to more than $5\,\sigma$. However, the interpretation of such discrepancy in terms of new physics is challenged by incompatibilities in some of the data employed for the dispersive evaluation of the Hadronic Vacuum Polarization (HVP) contribution to a_μ. In the following, the latest developments about the experimental inputs required to evaluate HVP dispersively are described and the perspectives for the next future summarized.

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The Anomalous Magnetic Moment of the Muon and the R Value

The anomalous magnetic moment of the muon a_μ is one of the most promising quantity where to indirectly search for physics Beyond the Standard Model (BSM). Both the experimental result $a_\mu^{\text{exp}} = (116\,592\,059 \pm 22) \times 10^{-11}$ [1] and the theoretical prediction $a_\mu^{\text{SM}} = (116\,591\,810 \pm 43) \times 10^{-11}$ [2] have achieved an accuracy well below 1 ppm and exhibit a significant tension. However, the interpretation of such discrepancy – which exceeds five standard deviations – as a BSM effect is presently questionable, due to conflicts between different evaluation of the theoretical predictions.

The muon anomaly arises from higher-order quantum effects related to the electromagnetic, weak, and strong interactions. While the first two contributions can be evaluated perturbatively within quantum field theory – thus resulting in a negligible contribution to the uncertainty – the nature of QCD does not allow to follow the same procedure for the last component. Two classes of hadronic processes come to play in the evaluation of a_μ: the Hadronic Vacuum Polarization (HVP) and the Hadronic Light-by-Light scattering (HLbL). While recent theoretical and experimental advances [4–6] have allowed to reliably quantify the contribution of HLbL to the muon anomaly – where consistent results have been obtained by data-driven and Lattice QCD (LQCD) based estimates –, the situation is more confused for HVP. The HVP contribution to the muon anomaly, $a_\mu^{\text{HVP,LO}}$, is historically evaluated (at the leading order) through a dispersive relation [7], employing experimental measurement of the R value (R_{had}) – the ratio between hadrons and two-muons production cross sections in electron-positron collisions – weighted by a kernel function $K(s)$:

$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{R_{\text{had}}(s) K(s)}{s^2}, \tag{1}$$

where s stands for the e^+e^- center-of-mass energy. In 2020 the Muon g-2 Theory Initiative published an updated prediction of a_μ^{SM} (i.e. the previously reported value of a_μ^{SM}), including a dispersive evaluation of $a_\mu^{\text{HVP,LO}}$ with 0.5% accuracy [2]. At the same time, the BMWc Collaboration has published the first – and until now the only – LQCD evaluation of $a_\mu^{\text{HVP,LO}}$ with sub-percent precision [8]. The a_μ^{SM} value obtained using the lattice determination of HVP is well in agreement with the experimental measurement and shows a clear tension (about $2\,\sigma$) with respect to including the dispersive estimate, as shown in the left of Fig. 1. The observed tension between LQCD and dispersive evaluations of the HVP contribution to a_μ calls for a careful verification on the one hand

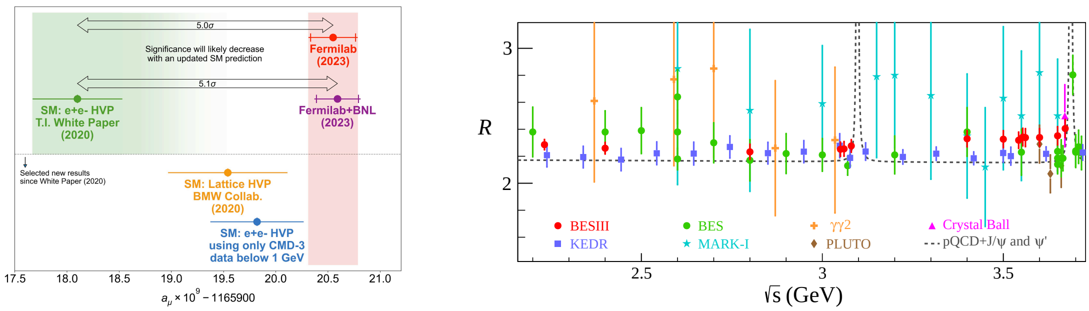

Figure 1: Left: Comparison of experimental values and SM predictions of a_μ [1]. Right: Comparison of the BESIII inclusive R measurement with previous results and pQCD predictions [3].

of the lattice calculation and, on the other, of the experimental measurements entering the dispersive relation, thus the R measurements.

Inclusive R Value Measurements: BESIII

The R value is defined as total hadron production cross section in electron-positron annihilations $\sigma(e^+e^- \to q\bar{q})$, normalized by $\sigma^{\rm LO}(e^+e^- \to \mu^+\mu^-)$ – the leading order QED evaluation of the di-muon production cross section. The most natural experimental approach to the determination of $R_{\rm had}$ is to select events with hadrons in the final state, thus measuring inclusively the hadron production cross section. The latest result of the inclusive approach to the measurement of $\sigma(e^+e^- \to q\bar{q})$ has been published by the BESIII Collaboration [3]. The measurement is based on the analysis of a small fraction – namely 14 out of 130 energy points – of the BESIII data sets collected specifically for this purpose, covering an energy range between 2 and 3.7 GeV.
Hadronic events have been selected, where at least two charged particles have been detected. Background contributions arising from the dominant QED processes $e^+e^- \to e^+e^-, \mu^+\mu^-,$ and $\gamma\gamma$, are suppressed by dedicated conditions on the total energy recorded by the electromagnetic calorimeter as well as the angles between pairs of charged particles, when less then 4 of them are detected. The latest generation of QED generators – e.g Babayaga@NLO, Ekhara, DIAG36, ... – are employed in MC simulations to evaluate and subtract residual background contributions.
A critical point in the measurement of $R_{\rm had}$ is the determination of the hadron-event selection efficiency and of radiative corrections. These are obtained from MC simulation based on the LUARLW generator [9] – which provides a purely theoretical modelling of the hadronization process, including initial- and final-state radiation (ISR, FSR) effects – after tuning to the experimental data. Differently to previous measurements, a second generator, Hybrid [10], was used to validate the prediction of LUARLW. The Hybrid generator makes use of the Phokhara and ConExc generators to simulate exclusive processes, which have been precisely measured in the past, including LUARLW only for the yet unknown contribution. The agreement between the two generators is remarkable and the systematic uncertainty due to the modelling of the hadron-production processes could be kept well below the 2.5% level.
The final result is depicted on the right of Fig. 1 in comparison with previous measurements and the theoretical prediction by perturbative QCD (pQCD). The BESIII Collaboration has achieved an unprecedented precision in the energy region below 3 GeV as well as good agreement with previous results. Above the J/Ψ resonance a clear discrepancy – of about $2\,\sigma$ – with pQCD calculations is to be observed. Despite the observed tension would in principle increase the dispersive value of $a_\mu^{\rm HVP,LO}$, the impact of such increase to the total HVP contribution is negligible, due to the strong enhancement of the low-energy region induced by the kernel function. Nevertheless, the origin of the observed tension needs to be clarified and the extension of the measurement to the full BESIII data set will provide interesting insights.

The Low Energy Region

At energies below $\approx$1.8 GeV, the Bhabha scattering process represents such an overwhelming background that no experiment has succeeded in performing an inclusive R measurement yet. Anyway, since the multiplicity of hadrons that can be produced in this region is limited, the contribution of each (hadronic) final state is determined through exclusive measurements. The R

value is, therefore, determined as the sum of the single cross sections.
At low energies, the hadron-production cross section is dominated by the formation of strong resonances – $\rho(770)$, $\omega(782)$, and $\Phi(1020)$ – and their subsequent decay channels: $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, and $K\bar{K}$. The contribution of low-energy processes to $a_\mu^{\mathrm{HVP,LO}}$ is, furthermore, enhanced by the kernel function, resulting in the $e^+e^- \to \pi^+\pi^-$ transition to account for about 70% of both the total HVP contribution and uncertainty. A precise and reliable determination of the pion form factor (FF) in the time-like region – i.e. the cross section of the process $e^+e^- \to \pi^+\pi^-$ – is, therefore, crucial to the correct interpretation of the experimental measurement of a_μ.
The presently most accurate measurements of the pion FF have been performed by the KLOE [11] and BaBar [12] Collaborations with uncertainties of 0.5% and 0.6%, respectively. The two results were obtained employing the so-called Initial-State Radiation (ISR) technique, namely exploiting events where a photon is emitted by one of the colliding particles (before annihilation), thus reducing the effective center-of-mass energy $\sqrt{s}$ of the collision. The measurements show a significant tension of about $3\,\sigma$ (right of Fig. 2), which is currently still unexplained. Both Collaborations have performed detailed studies and cross checks of their results, showing good agreement e.g. in the analogous QED process $e^+e^- \to \mu^+\mu^-$ between experimental measurement and theoretical predictions. Additional results from various experiment [13–17] – performed either through "traditional" energy scan or with the ISR method – distribute in between the two most accurate measurements, but none of them have a sufficient precision to discriminate between the two. As possible explanation for the long-standing tension, an underestimation of the systematic uncertainty in at least one of the results has been proposed. In the SM prediction of a_μ evaluated by the Muon g-2 Theory Initiative, a conservative merging procedure has been followed in order to account for the tensions in the pion FF measurement and to obtain a constisten uncertainty [2].
Recently, the CMD3 Collaboration has presented a new measurement of the pion FF with 0.8% precision [18].

Pion Form Factor and CMD3

The measurement of the pion FF performed by the CMD3 Collaboration is based on the analysis of data samples collected in three different acquisition campaigns between 2013 and 2020.
In the analysis events are retained where two oppositely-charged tracks are recorded, which resemble a back-to-back kinematics. Both tracks are required to be in-time – with a detection time difference smaller than 20 ns –, to originate from the interaction point, and to carry a momentum (p) in the range $0.45E_{\mathrm{beam}} < p < min(E_{\mathrm{beam}} + 100\,\mathrm{MeV}, \frac{4}{3}E_{\mathrm{beam}})$, where $E_{\mathrm{beam}} = \sqrt{s}/2$ stands for the beam energy. Additionally, for energy points beyond the K^+K^- production threshold, the momenta are required to exceed by at least 15% those arising from the production of a pair of charged kaon. The sample of selected events is mainly contributed by the $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, and $\pi^+\pi^-$ – processes with a small contamination from the $e^+e^- \to e^+e^-l^+l^-, \pi^+\pi^-\pi^0$ $(l = e, \mu)$ transitions.

The pion form factor $|F_\pi|^2$ is determined as

$$|F_\pi|^2 = \frac{\sigma^0_{ee}}{\sigma^0_{\pi\pi}} \left(\frac{N_{\pi\pi}}{N_{ee}} - \Delta^{bg} \right) \frac{\varepsilon_{ee}(1+\delta_{ee})}{\varepsilon_{\pi\pi}(1+\delta_{\pi\pi})} \tag{2}$$

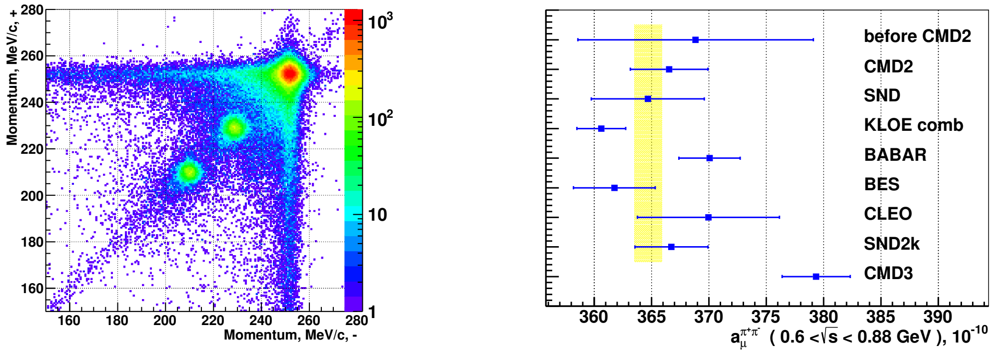

Figure 2: Left: CMD3, distribution of the track momenta at 0.25 GeV. The contribution of Bhabha, di-muon, and di-pion events are clearly visible as blobs on the diagonal. Right: comparison of pion FF contribution $a_\mu^{\mathrm{HVP,LO}}$ evaluated with different data sets. The yellow band corresponds to the average of all results, but CMD3. [18]

where N_i, ε_i, σ_i^0, and $(1+\delta_i)$ are the number of selected events, the selection efficiency, the lowest order (point-like) cross section, and the radiative corrections for the final state $i = e^+e^-, \pi^+\pi^-$, respectively, and Δ^{bg} is the contribution from background contaminations. While efficiencies, cross sections, and radiative corrections are derived from Monte Carlo (MC) simulations, the number of selected events is obtained through a 2D maximum likelihood fit to the distribution of the track momenta or of the energy releases within the first (thin) electromagnetic calorimeter (LXe). The Probability Density Functions (PDFs) for the fit to the track momenta are extracted by MC simulations, including free parameters to allow for eventual discrepancies between simulated and actual detector response. Dedicated studies on data have provided the basis to the construction of the PDFs for the fit to the energy releases in the LXe calorimeter. The two methods provide best performance at different $\sqrt{s}$: at lower energies the mass difference between electrons, muons, and pions allows for a clear separation in the momenta of the corresponding tracks (see Fig. 2 left), what becomes more and more difficult with the increasing of the energy. Oppositely, the differentiation by the energy releases in the calorimeter become clearer and clearer the larger the center-of-mass energy – as the separation between minimum ionizing particles (muons), hadronic (pions), and electromagnetic (electrons) showers increases. The final measurement is, hence, obtained as the combination of the results from the two different fits, the first applied only upto 0.9 GeV and the latter becoming available starting from 0.54 GeV. Moreover, the comparison of the two methods in the overlapping region provides a consistency check of the measurement. Additionally, the analysis of the average track angle – defined as the mean of the two track polar angles after reversing the negative particle momentum – provides a third evaluation of the cross sections in the region of the ρ resonance. Here the three techniques show an agreement at the 0.2% level. A similar consistency has also been verified between measured cross section and QED predictions of di-muon production below 0.75 GeV.

As a result of detailed studies, the pion FF measured by CMD3 reaches an accuracy of 0.7-0.8% in the crucial region below the ρ resonance. The result shows a considerable tension with all previous measurements obtained with the ISR technique as well as with the energy scan methodology as shown on the right of Fig. 2. When using this result for the contribution of the pion FF to dispersive evaluation the HVP, a value of a_μ is found, which is compatible with both the BMWc LQCD prediction and the experimental measurement (see left of Fig. 1).

Perspectives

The CMD3 measurement of the pion FF has sparked intense discussions on the origin of the tension with the previous results – particularly important would be a clarification on the discrepancy with CMD2, the predecessor of CMD3. Further studies – including dedicated data acquisition periods – are been considered by the CMD3 Collaboration to further corroborate the evaluated systematic uncertainty. Similarly, new measurements are presently on going within the BaBar, Belle II, KLOE, BESIII, and SND Collaborations, previously employed analysis techniques are being investigated and new one developed, with the aim to provide an extended set of accurate (and reliable) determinations of the pion FF.

At higher energies, new results are to be expected by the BESIII Collaboration, following the analysis of the full data sample dedicated to the inclusive R measurement. A persistence of the presently observed tension between pQCD predictions and measured R value would clearly require detailed investigations both on the theory and on the experimental perspective.

In conclusion, the recent development in the pion FF measurements have highlighted the need to clarify and investigate the reliability of the presently available data. Despite the perspective of considering the latest result by CMD3 as the only correct measurement could be temping – it would solve all the existing tensions in the muon $g-2$ sector –, such an approach would be against any good scientific practice. It would mean excluding more than 20 years of precision measurements on the basis of matching a desired outcome, rather than any evidence of flaws in the analysis procedure.

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