PROCEEDINGS OF SCIENCE

QED corrections to meson masses

Joshua Swaim*

University of Connecticut,
Storrs, Connecticut, United States

E-mail: joshua.swaim@uconn.edu

We present our progress on calculating leading-order QED corrections to meson masses and bare quark masses. As lattice QCD calculations become more precise, these QED corrections are becoming more important. However, one of the challenges in adding QED effects to QCD calculations is avoiding power-law suppressed finite-volume effects. By using the recently introduced infinite-volume reconstruction method for QED, we are able to avoid this problem and perform calculations with exponentially-suppressed finite-volume effects.

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*Speaker

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1. Introduction

As lattice QCD continues to progress, precision calculations increasingly need to account for QED and isospin breaking corrections [1]. For example, QED corrections are needed for precise calculations of the muon anomalous magnetic moment [2, 3]. These corrections are important not only because they directly affect observables, but also because they change the definition of the physical point [4]. Since meson masses are frequently used when defining the physical point, it is important to understand QED corrections to meson masses. Several groups have recent work on this topic [5–8].

Introducing QED to lattice calculations is challenging. One problem is that QED contains unconfined massless degrees of freedom: the photons. Photon propagators, unlike propagators for massive particles, do not decay exponentially at large distances. This means that introducing QED naively leads to large finite-volume errors. Furthermore, combining QED with periodic boundary conditions leads to technical complications. There are several formulations which address these difficulties in various ways, such as QED_{TL} [9], QED_{L} [10], QED with massive photons [11], and QED with C^* boundary-conditions [12].

In 2018, a new method was introduced that avoids the challenges posed both by volume and periodic boundary conditions. This method is called the infinite-volume reconstruction method [13]. In this method, QED corrections are calculated semi-analytically in infinite volume. Contributions from outside the lattice volume are reconstructed with exponentially-suppressed systematic errors. In this paper, we present our progress on using this method to calculate QED corrections to meson and quark masses.

In section 1.1, we discuss how QED corrections are calculated based on QCD correlation functions. In section 1.2, we explain the infinite-volume reconstruction method. In section 2.1, we demonstrate that this method works for calculating QED corrections to meson masses. Finally, in section 2.2, we briefly discuss our work so far in extracting the QED corrections to the quark mass renormalization constants.

1.1 Adding QED Corrections to QCD

In the infinite-volume reconstruction method [13], QED is introduced perturbatively. By expanding the path integral in the electric charge e, we get

$$\begin{aligned}\langle O(T)O(-T)\rangle_{\text{QCD+QED}} &= \langle O(T)O(-T)\rangle_{\text{QCD}} \\ &+ \frac{e^2}{2}\int d^4x d^4y \langle O(T)J_\mu(x)J_\nu(y)O(-T)\rangle_{\text{QCD}} S_{\mu\nu}(x-y) + O(e^4),\end{aligned} \tag{1}$$

where $S_{\mu\nu}(x-y)$ is the photon propagator. $\langle\rangle_{\text{QCD+QED}}$ represents the vacuum expectation value of operators in the full theory of QCD+QED, while $\langle\rangle_{\text{QCD}}$ represents the vacuum expectation value of operators computed using only QCD. If O is an operator that creates a hadron, the order e^2 correction can be represented diagrammatically, as shown in Figure 1. Based on equation 1, the leading-order QED correction to the mass of a hadron is given by

$$\Delta m = \frac{e^2}{2}\int d^4x \mathcal{H}_{\mu\nu}(x) S_{\mu\nu}(x), \tag{2}$$

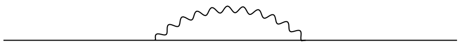

Figure 1: A diagramatic representation of the leading-order QED correction to a hadronic propagator. The straight lines represent hadronic propagators computed non-perturbatively using only QCD, and the other line represents a free photon propagator. The points where the lines connect represent current insertions.

where, on the lattice,

$$\mathcal{H}_{\mu\nu}(x) = L^3 \frac{\langle O(t+T) J_\mu(x) J_\nu(0) O(-T)\rangle_{\text{QCD}}}{\langle O(t+T) O(-T)\rangle_{\text{QCD}}}, \tag{3}$$

and O is an operator that creates the desired hadronic state. In infinite volume, assuming for example that O creates a pion, this definition would correspond to

$$\mathcal{H}_{\mu\nu}(x) = \frac{1}{2m} \langle \pi | J_\mu(x) J_\nu(0) | \pi \rangle_{\text{QCD}}. \tag{4}$$

1.2 The Infinite-Volume Reconstruction Method

We could use equation 2 to estimate Δm by evaluating $\mathcal{H}_{\mu\nu}(x)$ using lattice QCD and replacing the integral over space with a sum over the lattice volume. However, this would result in large finite-volume errors. To see this, note that when $t >> |\vec{x}|$, $\mathcal{H}_{\mu\nu}(x)$ is order 1, even at large distances. Similarly, the photon propagator $S_{\mu\nu}(t, \vec{x})$ is only power-law (not exponentially) suppressed at large t because the photon is massless. Therefore, the finite-volume errors resulting from evaluating equation 2 on the lattice will only be power-law suppressed.

To get exponentially-suppressed finite-volume effects, we can reconstruct the large-distance contributions to the integral using the infinite-volume reconstruction method [13]. At large $|x|$, $\mathcal{H}_{\mu\nu}(x)$ is dominated by contributions from the lowest energy states. We choose some cutoff time t_s that is large enough for $\mathcal{H}_{\mu\nu}(t_s, \vec{x})$ to be dominated by the single-meson intermediate states, but small enough that we don't need to worry about around-the-world effects. Then we can reconstruct $\mathcal{H}_{\mu\nu}(t, \vec{x})$ for $t > t_s$ using

$$\mathcal{H}_{\mu\nu}(t, \vec{x}') \approx \int d^3\vec{x} \mathcal{H}_{\mu\nu}(t_s, \vec{x}) \int \frac{d^3\vec{p}}{(2\pi)^3} e^{-i\vec{p}\cdot(\vec{x}'-\vec{x})} e^{-(E_{n,\vec{p}}-m_\pi)(t-t_s)}, \tag{5}$$

with corrections to this formula exponentially suppressed (see [13] for more details).

2. Results

2.1 Calculating Meson Masses

In figure 2, we show our calculated Δm based on equation 2 as a function of the cutoff time t_s. We show both the results with the infinite-volume reconstruction and the "short" results where the integral is simply cutoff at time t_s and no infinite-volume reconstruction is performed. There is a plateau region (highlighted in the plots) where the calculated mass correction does not depend strongly on t_s. This indicates that we can indeed choose t_s sufficiently large that the infinite-volume reconstruction works well.

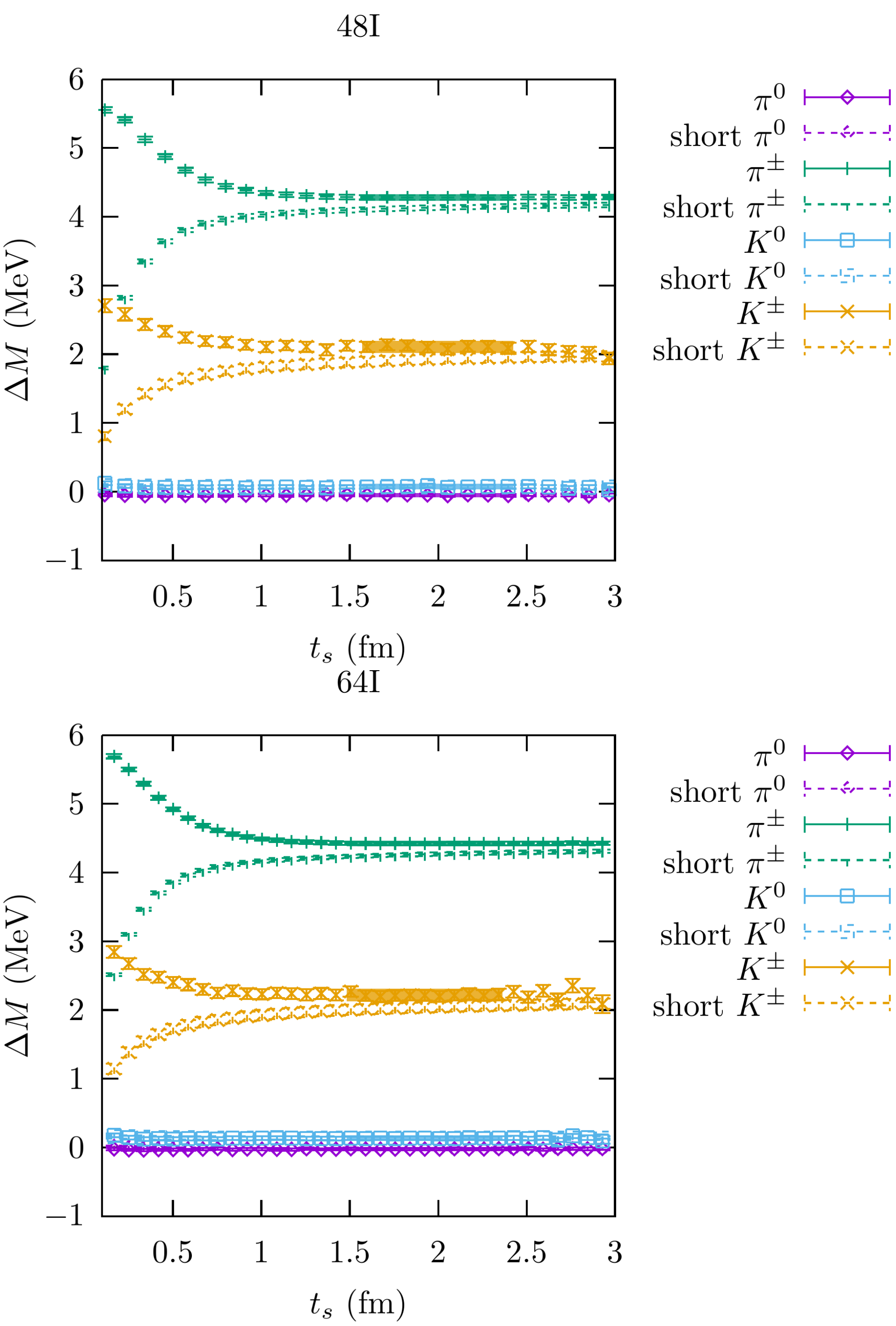

Figure 2: ΔM versus t_s on a $48^3 \times 96$ lattice (top) and $64^3 \times 128$ lattice (bottom) on ensembles from RBC/UKQCD [14] using Iwasaki gauge action and domain-wall fermions. "Short" means including only $|t| < t_s$ contributions.

2.2 Quark Masses and Renormalization

In QCD, the quark masses renormalize by a multiplicative constant

$$m_f^{\overline{\mathrm{MS}},\mathrm{QCD}} = Z_m m_f, \tag{6}$$

where m_f is the bare quark mass of flavor f and $m_f^{\overline{\mathrm{MS}},\mathrm{QCD}}$ is the renormalized quark mass in QCD (without QED corrections). Adding QED to the theory introduces additional divergences. Therefore, the renormalization constant is modified. We define Z_{QED} by

$$m_f^{\overline{\mathrm{MS}}} = Z_m(1 + e_f^2 Z_{\mathrm{QED}})m_f, \tag{7}$$

where $m_f^{\overline{\mathrm{MS}}}$ is the renormalized quark mass taking both QCD and QED into account.

To get Z_{QED}, we note that hadron masses are renormalization-invariant by definition. To see how we can use this fact, suppose that we could calculate the shift Δm_H in the mass of a hadron H caused by making a small change $\Delta m_f^{\overline{\mathrm{MS}}}$ in an $\overline{\mathrm{MS}}$ quark mass. We could then figure out the shift Δm_f in the bare lattice quark mass that would be required to produce the same Δm_H. By comparing the shift in $\overline{\mathrm{MS}}$ quark mass to the equivalent shift in bare quark mass, we could determine the renormalization constant.

To the leading order, the change in hadron mass, m_H, due to a change in the quark mass m_f and introducing an electric charge e is

$$\Delta m_H = \frac{e^2}{2}\int d^4x \mathcal{H}_{\mu\nu}(x) S_{\mu\nu}(x) + \Delta m_f \mathcal{H}_f^{3\mathrm{pt}},$$

where $\mathcal{H}$ is the four-point function, and (in the lattice normalization)

$$\mathcal{H}_f^{3\mathrm{pt}} = L^3 \frac{\langle O_H(T)\bar{\psi}_f(0)\psi_f(0)O_H(-T)\rangle}{\langle O_H(T)O_H(-T)\rangle}.$$

In $\overline{\mathrm{MS}}$, we can calculate the divergent part of the integral using the operator product expansion [15]. We can compare this with the small-distance (high-momentum) contribution to this integral from the lattice.

3. Conclusion

We demonstrated that the infinite-volume reconstruction method can be used to get QED corrections to meson masses with exponentially-suppressed finite-volume effects. To get final results, we still need to perform a continuum extrapolation and choose a scheme to match our simulation parameters to the physical world. To get quark mass corrections, we need to determine QED corrections to the renormalization constants.

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