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Renormalons in Large-Momentum Effective Theory

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In recent years, significant progress has been made in calculating the partonic structure of hadrons from lattice QCD. Such calculations have now reached a stage that calls for precision. Therefore, understanding the role of renormalons and power corrections becomes increasingly important. Here I briefly review how the concept of renormalons can give valuable insights and be used to obtain predictions with power accuracy.

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1. Introduction

The partonic structure of hadrons plays a crucial role in mapping out their 3D image and in describing the experimental data collected at high energy colliders such as the Large Hadron Collider (LHC) or the Electron-Ion Collider in the US and China. The simplest quantities that characterize such structure are the leading-twist collinear parton distribution functions (PDFs). They are defined by collinear parton operators and are nonperturbative in nature. A lot of efforts have been devoted to determining them from fitting to various experimental data (see, e.g., [1–4] and references therein).

In the past decade, there have also been significant progress on extracting the PDFs from lattice QCD (see [5–8] for a recent review) based on various proposals [9–17]. Among them, one of the widely used options is to start from the so-called quasi-light-front (quasi-LF) correlators [6, 11, 12], which are equal-time quark and gluon correlators defined on a Euclidean space interval. It can be connected to the light-front (LF) correlators defining the collinear parton distributions, either through a short-distance factorization [15] in coordinate space or through a large-momentum factorization [6, 11, 18] in momentum space, where the latter is formulated in the framework of large-momentum effective theory (LaMET) [6, 11, 12]. Recently, such an approach has been extended beyond the single parton PDFs to study multiple parton distributions that reveal the correlated partonic structure of hadrons [19, 20].

At the current stage, the calculations in LaMET have reached a level where precision control becomes important. In order to have precise predictions for the PDFs, one needs to properly renormalize the quasi-LF correlators calculated on the lattice so that a reliable continuum limit can be obtained and then matched to the physical PDFs. On the one hand, the quasi-LF correlator contains a linear power divergence arising from the Wilson line self-energy, whose renormalization leads to an ambiguity of $O(\Lambda_{\rm QCD})$ due to long-range non-perturbative effects. In addition, the perturbative hard matching kernel connecting the quasi-LF correlators to the LF correlators has renormalon ambiguities which shall be canceled by higher-twist contributions. Both ambiguities have to be properly handled. On the other hand, it has been shown that the appropriate perturbative scale in LaMET matching is $\sim 2xP_z$ with P_z being the hadron momentum and x the momentum fraction carried by the parton [21]. Therefore, there can be potentially large logarithms of the form $\ln^n 4x^2P_z^2/\mu^2$ which shall be resummed by the renormalization group equation.

Recently, it has been shown how to achieve PDF predictions from quasi-LF correlators calculated on the lattice to twist-three accuracy [22], by using the similarity between a spatial Wilson line and an infinitely heavy quark and the existing results on the pole mass of the latter to large perturbative orders in the literature. In addition, there have also been estimates on the relevant twist-four contributions [23, 24] in the LaMET calculation of PDFs, based on the requirement of renormalon cancellation between different twists. Due to the cancellation requirement, the existence of renormalon ambiguities in the leading-twist expressions can be used to estimate the size of power-suppressed corrections. Conceptually, this is similar to the estimation of the accuracy of fixed-order perturbative results by the logarithmic scale dependence.

In the following, I'll briefly review these procedures and their implementation, by taking the isovetor quark PDFs as an example.

2. Renormalons and twist-three accuracy

The quasi-LF correlator [11] used in LaMET takes the following form (for quarks)

$$\tilde{h}^B(z, P_z, a) = \langle P|\bar{\psi}(z)\Gamma W(z, 0)\psi(0)|P\rangle, \tag{1}$$

where $W(z, 0) = \mathcal{P}\exp[-ig\int_0^z A_z(z')dz']$ denotes a gauge link along the z-direction. The correlator above has a linear divergence associated with the self-energy of the Wilson line, which can be removed by a multiplicative renormalization factor $e^{\delta m(a)|z|}$ [25–29] with a denoting the lattice spacing. However, there is an ambiguity in specifying the finite piece of the mass renormalization factor $\delta m(a)$. One can choose the renormalization factor either with a "short-distance mass" defined by an infrared-regulated "pole mass" [30], or with a non-perturbatively defined mass parameter in terms of a physical matrix element [31–33]. Different mass subtractions differ by an intrinsic non-perturbative correction of $O(\Lambda_{\rm QCD})$. Thus, in addition to the usual renormalization scale dependence arising from the renormalization of logarithmic divergences, the quasi-LF correlators also have a scheme dependence from the renormalization of the linear divergence (labeled as τ-scheme dependence in Ref. [22]). Such a scheme dependence is also reflected in the self renormalization procedure proposed in [32], where one determines the ultraviolet (UV) divergences and the renormalization factor by fitting the bare quasi-LF correlators at multiple lattice spacings to a physics-dictated functional form. In the implementation of self renormalization, multiple sets of parameters have been found that correspond to the same minimal χ^2 fit. The freedom in choosing the set of parameters can be regarded as a scheme dependence.

Given the ambiguity in choosing the finite term of the mass renormalization factor, the short-distance expansion of the renormalized quasi-LF correlator must contain a twist-three non-perturbative parameter $m_0(\tau) \sim O(\Lambda_{\rm QCD})$ and thus becomes [22] (assume $z > 0$ from now on)

$$\begin{aligned} h^R(z, P_z, \mu, \tau) &= \Big(1 + m_0(\tau)z\Big)\sum_{k=0}^{\infty} C_k\left(\alpha_s(\mu), \mu^2 z^2\right)\lambda^k a_{k+1}(\mu) + O(z^2) \\ &= \sum_{k=0}^{\infty}\left[C_k\left(\alpha_s(\mu), \mu^2 z^2\right) + z m_0(\tau)\right]\lambda^k a_{k+1}(\mu) + O(z\alpha_s, z^2), \end{aligned} \tag{2}$$

where $\lambda = zP^z$, a_k are spin-k twist-2 matrix elements, and the perturbative series C_k are the associated Wilson coefficients. The twist-three contribution is universal in the sense that it multiplies the leading-twist term in the same manner independent of the spin of the local operators. On the other hand, $m_0(\tau)$ does depend on the external states in which the correlator matrix elements are taken. In the LaMET expansion, the above twist-three term translates to a linear term in the inverse hadron momentum $1/P_z$.

The parameter $m_0(\tau)$ can be determined by fitting to a non-perturbative matrix element, which can be conveniently chosen as the quasi-LF correlator at zero momentum, $h^R(z, P_z = 0, \mu, \tau)$, where only the $k = 0$ term in the above relation contributes, and both a_1 and the $\overline{\rm MS}$ series $C_0(\alpha_s(\mu), \mu^2 z^2)$ are known.

After fitting $m_0(\tau)$, one can absorb it into an additional finite renormalization so that the renormalization factor $\sim \exp[\delta m(a)z] = \exp[\delta m(a, \tau)z + m_0(\tau)z]$ becomes τ-independent. The matrix element renormalized by such a renormalization factor can then be used in the standard LaMET matching with the twist-three ambiguity from renormalization removed.

In practice, the determination of $m_0(\tau)$ can be summarized as follows. Let $h^R(z, P_z = 0, \mu)$ denote the renormalized zero momentum matrix element in the $\overline{\mathrm{MS}}$ scheme at scale μ. It satisfies a renormalization group equation

$$\frac{\partial h^R(z, P_z = 0, \mu)}{\partial \ln \mu^2} = \gamma(\mu) h^R(z, P_z = 0, \mu)\,, \tag{3}$$

where the anomalous dimension $\gamma(\alpha(\mu))$ of $h^R(z, 0, \mu)$ has been calculated up to 3-loop order for the quasi-PDF operator [34]. When z is a perturbative distance, the above equation can be solved by evolving from the initial scale $\mu_0 = z^{-1}$ which is a proper scale for the correlator in coordinate space. The solution at scale μ is then given by

$$\begin{aligned} h^R(z, P_z = 0, \mu) =& h^R(z, P_z = 0, z^{-1}) \exp\left(\int_{\alpha(z^{-1})}^{\alpha(\mu)} \frac{\gamma(\alpha')}{\beta(\alpha')} d\alpha'\right) \\ =& h^R(z, P_z = 0, z^{-1}) \exp\left[-\mathcal{I}(z^{-1})\right] \exp\left[\mathcal{I}(\mu)\right], \end{aligned} \tag{4}$$

where $\mathcal{I}(\mu)$ is an analytic function of $\alpha(\mu)$ because both $\gamma(\alpha)$ and $\beta(\alpha)$ are polynomials of α when truncated at a certain order in perturbation theory.

Similarly, for the lattice scheme with explicit linear divergence, the matrix elements now have the following form:

$$e^{\delta m(a) z} h^B(z, P_z = 0, a^{-1}) = h^{\mathrm{lat}}(z, P_z = 0, z^{-1}) e^{-\mathcal{I}^{\mathrm{lat}}(z^{-1})} e^{\mathcal{I}^{\mathrm{lat}}(a^{-1})}, \tag{5}$$

where h^{lat} is the perturbation series evaluated at scale z^{-1} in the lattice scheme. Note that the z-dependence is now completely factorized in both schemes. In the lattice matrix element, the mass dependence with the linearly-divergent mass correction $\delta m(a)$ has been manifestly separated out. The z-dependence is physical, and thus should not depend on scheme choices. So one can identify $h^{\mathrm{lat}}(z, P_z = 0, z^{-1}) e^{-\mathcal{I}^{\mathrm{lat}}(z^{-1})}$ in the lattice scheme and $h(z, P_z = 0, z^{-1}) \exp\left[-\mathcal{I}(z^{-1})\right]$ in the $\overline{\mathrm{MS}}$ scheme. The above expressions allow one to express the lattice matrix element in terms of the $\overline{\mathrm{MS}}$ perturbation series

$$h^B(z, P_z = 0, a^{-1}) = h^R(z, P_z = 0, z^{-1}) e^{-\mathcal{I}(z^{-1})} e^{\mathcal{I}^{\mathrm{lat}}(a^{-1})} e^{-\delta m(a) z}. \tag{6}$$

From Eq. (6), one can write

$$\begin{aligned} m_0(\tau) z (1 + O(z\Lambda_{\mathrm{QCD}})) =& \ln\left[h^R(z, P_z = 0, z^{-1}) e^{-\mathcal{I}(z^{-1})}\right] \\ & - \ln\left[h^B(z, P_z = 0, a^{-1}) e^{-\mathcal{I}^{\mathrm{lat}}(a^{-1})} e^{\delta m(a,\tau) z}\right]. \end{aligned} \tag{7}$$

This allows one to extract $m_0(\tau)$ as a slope of such a quantity. Note that for a single lattice spacing, $I_0 = e^{-\mathcal{I}^{\mathrm{lat}}(a^{-1})}$ is a constant and thus is absorbed into the interception and not affecting the slope $m_0(\tau)$.

In Ref. [22], the authors fitted $m_0(\tau)$ with short distance pion PDF matrix elements, and the results are shown in Fig. 1. As shown by cyan and orange bands in the top panel in Fig. 1, the fixed-order (NLO or NNLO) C_0 introduces a large uncertainty from the variation of the scale μ from 1 GeV to 4 GeV, corresponding to all possible relevant physics scales ($2xP^z$) in the problem.

After resumming the large logarithmic terms $\alpha_s^n(\mu)\ln^n(z^2\mu^2)$ in C_0 to reduce the μ dependence, the fitting yields the hatched green band on the top panel, which has a strong dependence on z, and becomes unreasonably large at $z > 0.2$ fm. This is an indication that there is a significant contamination effect from unaccounted higher-order terms in twist-two C_0, which has logarithmic dependence in z. The z-dependence is altered by the truncation in $\alpha_s(z^{-1})$.

The large uncertainty in the twist-three mass parameter m_0 can be translated to that for light-cone PDFs in LaMET calculations. The extracted isovector light-cone PDF is shown in the bottom plot of Fig. 1. With the fixed order C_k, the uncertainty in m_0 yields $> 30\%$ error near $x \sim 0.3$. With renormalization group improvement, significant uncertainty in the extracted PDF still exists, shown as the hatched green band with NLO+RGR label in the same plot. These large uncertainties indicate that improving calculations up to twist-three accuracy is crucially important for lattice data at $P_z \sim 2$ GeV. To achieve this, one also needs to understand the renormalon effect in the perturbative Wilson coefficients, or in other words, in $h^R(z, P_z = 0, \mu)$ above, which have not been taken into account in the discussion so far.

It is well-known that the perturbative Wilson coefficients $C_k(\alpha_s) = \sum_n c_{kn}\alpha_s^n$ are asymptotic series because of infrared contributions at large orders, a phenomenon called infrared renormalons (IRR) [35]. For the quasi-LF correlator, the leading IRR comes from the long-distance contributions to the self-energy of the Wilson line, making c_{kn} grow factorially $\propto n!(\beta_0/2\pi)^n$ at large-n orders [23], where $\beta_0 = 11 - 2n_f/3$ (n_f is the number of active quark flavors) is the first term of the QCD beta function. This behavior is the same as the perturbation series for the “pole" mass of a heavy quark [30, 36].

The perturbative runaway infrared contributions at large orders must be regularized, or equivalently, one has to specify a way to resum the perturbative series. Different methods of resummation/regularization yield results differing by the order of the minimal term in the series, at $n \sim 2\pi/\beta_0\alpha_s$ [37]. A simple estimation shows that this is a twist-three contribution $O(z\Lambda_{\rm QCD})$, a linear power in z (other renormalon poles corresponding to higher-power-z/twist terms). Therefore, the twist-two contributions themselves are ambiguous up to higher-twist contributions, and the twist-three parameter $m_0(\tau)$ depends on the resummation/regularization method for the leading renormalon series in $C_k(\alpha_s)$. There have been a number of proposals for renormalon regularization in the literature [38]. For example, one can define subtraction for infrared contributions at every order in perturbation series. One can also calculate the series in usual $\overline{\rm MS}$ or lattice method and define an all-order sum through a Borel transformation. In this regard, it has been advocated to use the principal value prescription to regulate the renormalon poles in the Borel integral [39, 40]. In Ref. [22], the authors have taken a prescription for $C_k(\alpha_s)$ together with the UV regularization of the correlator as the complete τ-scheme.

To determine the renormalon ambiguity requires, in principle, calculating C_k to all orders, which is a formidable task. Fortunately, lattice numerical calculations to perturbative large orders have become possible for certain quantities [30, 41]. In particular, the pole mass on pure gauge ensembles ($n_f = 0$) has been calculated to $n \sim 20$, $m = \mu\sum_n r_n\alpha_s^{n+1}$, and confirmed the conjecture on the leading IR renormalon form at large n [30],

$$r_n = N_m\left(\frac{\beta_0}{2\pi}\right)^n\frac{\Gamma(n+1+b)}{\Gamma(1+b)}\left[1+\frac{c_1 b}{b+n}+...\right], \quad (8)$$

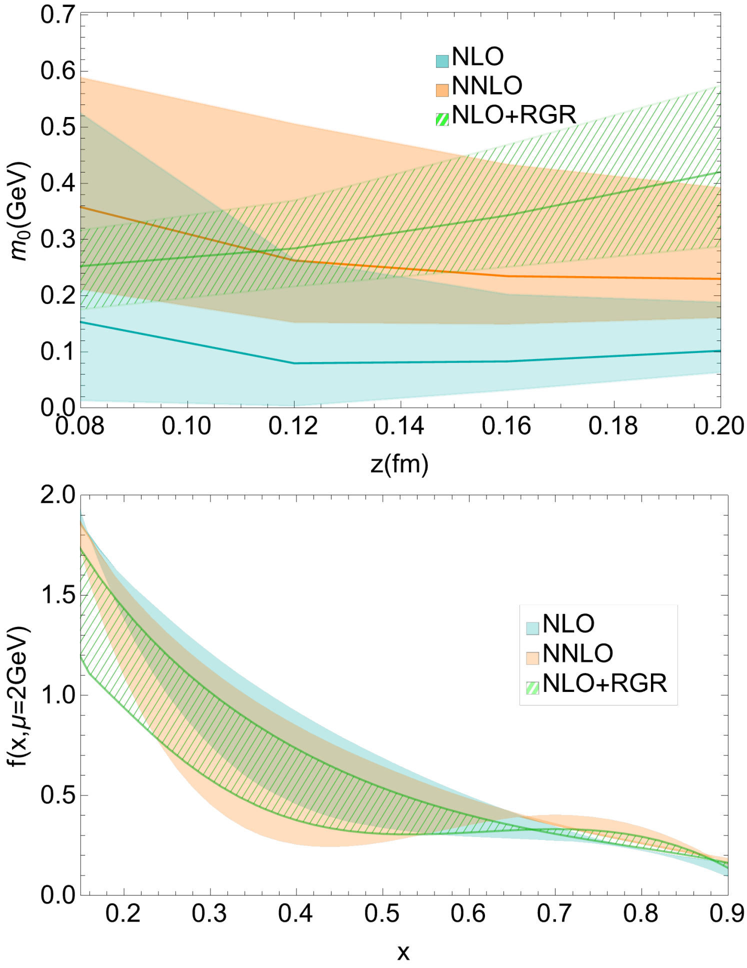

Figure 1: These plots are taken from [22]. Top: Uncertainty in $m_0(\tau)$ extracted from the pion $P_z = 0$ matrix element and fixed-order Wilson coefficients with and without RG resummation. The band width shows the renormalizaiton scale μ dependence. Bottom: The uncertainties in the pion light-cone PDF extracted from LaMET expansion with the above m_0. The overlap region exhibits a darker color.

where $b = \beta_1/2\beta_0^2$ and $c_1 = (\beta_1^2 - \beta_0\beta_2)/(4b\beta_0^4)$ are from higher orders in the QCD beta function. Moreover, the mass renormalon strength has been determined to be $N_m(n_f = 0) = 0.660(56)$ in $\overline{\text{MS}}$ scheme. With dynamic fermions, $N_m(n_f = 3) = 0.575$ has been taken in Ref. [22] using an analytical method in Ref. [36].

Using the above knowledge on the mass renormalon, the leading renormalon contribution for C_k after a Borel transformation reads

$$C_k(\alpha_s(z^{-1}), 1)_{\text{PV}} = N_m \frac{4\pi}{\beta_0} \int_{0,\text{PV}}^{\infty} du\, e^{-\frac{4\pi u}{\alpha_s(z^{-1})\beta_0}} \frac{1}{(1-2u)^{1+b}} \big(1 + c_1(1-2u) + \ldots\big), \tag{9}$$

where a PV prescription has been chosen for regulating poles on the Borel u-plane. The leading renormalon resummation (LRR) result can then be defined by resumming the leading divergent contributions to all orders at $\mu = z^{-1}$, while keeping the lower-order expansion of $C^{\text{LRR}}(\alpha_s)$ the

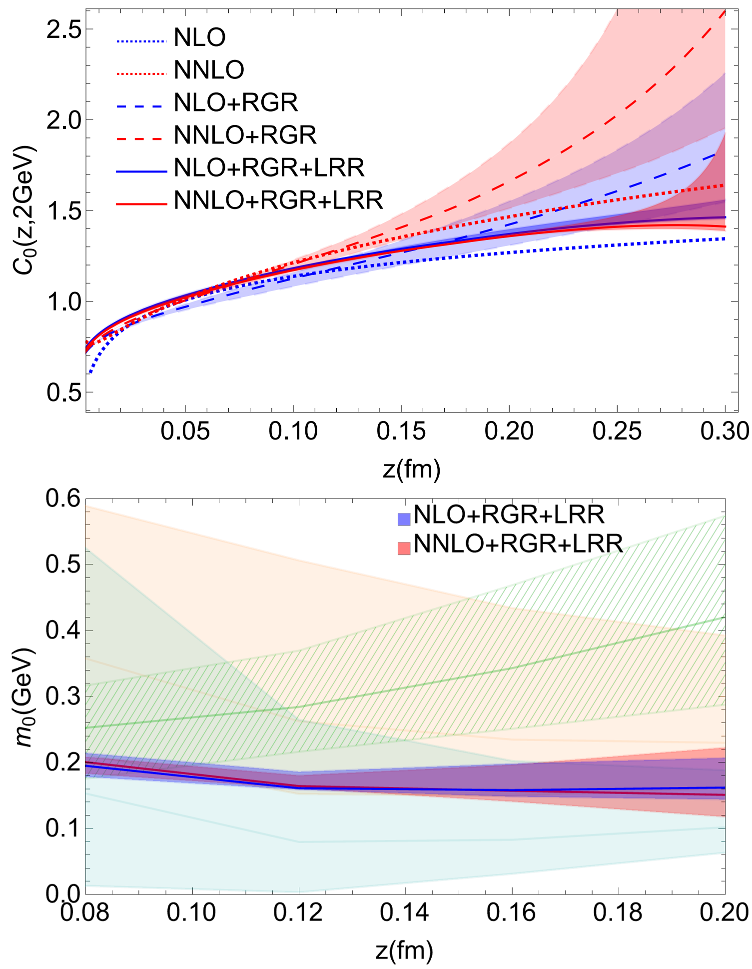

Figure 2: These plots are taken from [22]. Top: The comparison of $C_0(\alpha_s(\mu), z^2\mu^2)$ from the fixed-order (dotted), renormalization group resummation (dashed), and the leading renormalon resummation (solid). Bottom: $m_0(\tau)$ extracted from leading renormalon resummation with PV as an IR regulator.

same as $C_0(\alpha_s(\mu), z^2\mu^2)$,

$$C^{\text{LRR}}(\alpha_s(z^{-1}), 1) = C_k(\alpha_s(z^{-1}), 1) + \left[C_k(\alpha_s(z^{-1}), 1)_{\text{PV}} - \sum_i \alpha_s^{i+1}(z^{-1}) r_i \right]. \tag{10}$$

Now $C^{\text{LRR}}(\alpha_s)$ contains not only the fixed-order results calculated explicitly, but also higher-order (twist-two) perturbative terms contributing to the leading factorial growth.

The comparison of the $k = 0$ Wilson coefficient $C_0(\alpha_s(\mu), z^2\mu^2)$ for the fixed-order (NLO+NNLO), fixed order (NLO+NNLO) with RGR, and the LRR-improved formalism is shown in the top panel of Fig. 2. The error bands in RGR are obtained by varying the resummation scale from $0.75z^{-1}$ to $1.5z^{-1}$, corresponding to about 30% change in the strong coupling. While there is a large difference from NLO to NNLO in fixed-order calculations with or without renormalization group improvement, the LRR results show much better convergence in the perturbative region $z < 0.3$ fm, and much smaller dependence on the scale variation, indicating that NNLO term is already dominated by the leading renormalon.

The lower panel of Fig. 2 shows the LRR-improved m_0 result. By including the leading renormalon in the perturbative Wilson coefficients, there is now a clear window near $z = 0.12$ fm for a constant $m_0(\tau) = 0.161^{+0.025}_{-0.002}$ GeV for NLO (blue band) with much smaller uncertainty. Thus Eq. (2) achieves the linear-z accuracy when the leading renormalon series is resummed. The NNLO renormalon-resummed results $m_0(\tau) = 0.164^{+0.016}_{-0.003}$ GeV (red band) clearly demonstrates the good convergence of the method. The difference between the non-perturbative lattice result and the perturbation series is well described by the linear dependence in z in the perturbatively reliable region. This indicates that twist-three power accuracy has been reached for describing the $P_z = 0$ matrix element.

3. Renormalons and twist-four accuracy

To explain how the concept of renormalons can be used to get insight into the structure of twist-four power corrections, let us consider the LaMET factorization for the quasi-PDF (which is a Fourier transform of the quasi-LF correlator discussed previously with respect to z) [42, 43],

$$Q(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} C(\tfrac{x}{y}, xP_z, \mu_F) q(y, \mu_F) + \frac{1}{P_z^2} Q_4(x, P_z) + \ldots, \tag{11}$$

where the factorization scale has been denoted as μ_F, and for brevity we do not show the dependence on the renormalization scale. μ_F has to be taken of the order of $|x|P_z$ to avoid large logarithms. The coefficient function $C(x, P_z, \mu_F) = \delta(1-x) + O(\alpha_s)$ is given by the perturbative expansion.

To understand the role of renormalons, let us assume for a moment that the factorization is done using a hard cutoff $\Lambda_{\rm QCD} \ll \mu_F \ll P_z$, i.e. the contributions with loop momenta $|k| > \mu_F$ are included in the coefficient function, whereas the contributions with $|k| < \mu_F$ are included in the PDF. In this scheme, the coefficient function has the following expansion at $P_z \to \infty$

$$\begin{aligned} C(x, P_z, \mu_F) = \delta(1-x) + c_1\alpha_s + c_2\alpha_s^2 + \ldots \\ - \frac{\mu_F^2}{P_z^2} D(x) + \ldots, \end{aligned} \tag{12}$$

where $c_k = c_k(x, \ln P_z^2/\mu_F^2)$ are the perturbative coefficients depending logarithmically on the scales and the D-term represents the leading power correction. Since the l.h.s. of Eq. (11) does not depend on μ_F, any such dependence should cancel on the r.h.s.. In particular, the logarithmic dependence on the scale in $c_k(x, \ln P_z^2/\mu_F^2)$ is canceled by the scale-dependence of the PDF $q(x, \mu_F)$. The cancellation of the power dependence, on the other hand, must involve the twist-four contribution $Q_4(x, P_z)$. Thus, in this factorization scheme one expects that

$$Q_4(x, P_z, \mu_F) = \mu_F^2 \int_{-1}^{1} \frac{dy}{|y|} D(\tfrac{x}{y}) q(y, \mu_F) + \widetilde{Q}_4(x, P_z, \mu_F), \tag{13}$$

where $\widetilde{Q}_4$ depends on μ_F at most logarithmically. The appearance of the term $\sim \mu_F^2$ can be traced to quadratic UV divergence (in addition to the logarithmic UV divergence) of the twist-four operators that are responsible for the power correction, in the cutoff scheme. One can prove that the cutoff dependence $\sim \mu_F^2$ of the higher-twist operators is indeed that of Eq. (12).

In practice, perturbative calculations are usually done using dimensional regularization. In this case, power-like terms as in Eq. (12) do not appear. But the coefficients c_k computed in the $\overline{\text{MS}}$ scheme then grow factorially with the order k, inducing a renormalon ambiguity that must be compensated by adding a non-perturbative higher–twist correction. In this way, the same picture as in the cutoff scheme reappears in dimension regularization.

Returning to (13), we observe that the quadratic term in μ_F is spurious since its sole purpose is to cancel the similar contribution to the coefficient function. Therefore, it does not contribute to any physical observable. The idea of the renormalon model of the power corrections [44] is that, with a replacement of μ_F by a suitable non-perturbative scale, this contribution reflects the order and the functional form of actual power-suppressed contribution. Assuming this "ultraviolet dominance" [38, 45, 46] one obtains the following model:

$$Q_4(x, P_z, \mu_F) = \kappa\Lambda^2_{\text{QCD}} \int_{-1}^{1} \frac{dy}{|y|} D(\tfrac{x}{y})\, q(y, \mu_F)\,, \tag{14}$$

with the dimensionless coefficient $\kappa = O(1)$ which cannot be fixed within theory and remains a free parameter.

To estimate the power corrections from the renormalon model, it is convenient to start from the short distance factorization in coordinate space. The corresponding coefficient function (labeled as H in this section) have the perturbative expansion

$$H = \delta(1-\alpha) + \sum_{k=0}^{\infty} h_k a_s^{k+1}\,, \qquad a_s = \frac{\alpha_s(\mu)}{4\pi}\,, \tag{15}$$

with factorially growing coefficients $h_k \sim k!$.

Consider the Borel transform

$$B[H](w) = \sum_{k=0}^{\infty} \frac{h_k}{k!} \left(\frac{w}{\beta_0}\right)^k \tag{16}$$

where powers of $\beta_0 = 11/3N_C - 2/3n_f$ are inserted for later convenience. The Borel image can be used as a generating function for the fixed-order coefficients

$$h_k = \beta_0^k \left(\frac{d}{dw}\right)^k B[H](w)\Big|_{w=0}\,. \tag{17}$$

Moreover, the sum of the series can be obtained as the integral over positive values of the Borel parameter w

$$H = \delta(1-\alpha) + \frac{1}{\beta_0}\int_0^{\infty} dw\, e^{-w/(\beta_0 a_s)} B[H](w)\,. \tag{18}$$

As it stands, the integral is not defined because the Borel transform generally has singularities on the integration path, known as (infrared) renormalons. One can adopt a definition of the integral deforming the contour above or below the real axis, or as the principle value. These definitions are arbitrary, and their difference, which is exponentially small in the coupling, must be viewed as an intrinsic uncertainty of perturbation theory that has to be removed by adding power-suppressed nonperturbative corrections.

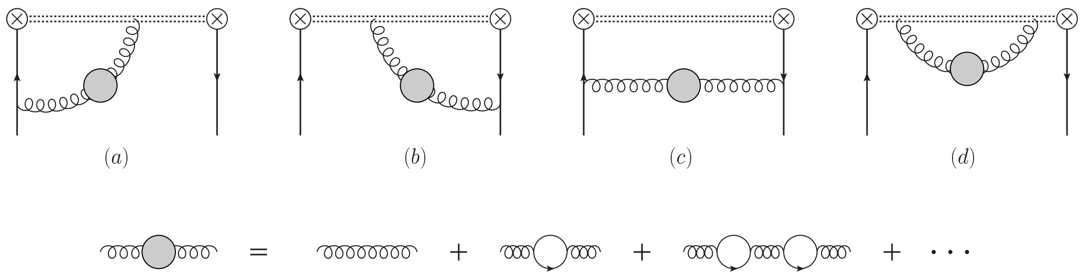

Figure 3: This plot is taken from [23]. Bubble-chain contribution to the coefficient function. The Wilson line factor is shown by the double dotted line.

To determine the renormalon ambiguity, one needs to know the coefficient function to all orders in principle. Naturally, such a full all-order calculation cannot be performed. Instead, we employ the approximation [38, 46] restricting ourselves to the perturbative series generated by the running-coupling effects in the one-loop diagrams, i.e. using QCD coupling at the scale of the gluon virtuality. Such contributions can be traced by computing the diagrams with the insertion of k fermion loops in the one-loop diagram and replacing $-\frac{2}{3}n_f \mapsto \beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$, see Fig. 3.

Using the above approximation, one obtains the following form for the Borel transform of the coefficient function (assume $\Gamma = \gamma^0$ in the quasi-LF correlator)

$$B[H](w) = \frac{2C_F}{w}\left\{\left[\frac{1+\alpha^2}{1-\alpha} - \Big(2\alpha\, {}_2F_1(1,2-w,2+w,\alpha) + \bar{\alpha}(1-w^2)\Big)\alpha^w\, h_0(w,X)\right]_+ + \delta(\bar{\alpha})\left[\frac{3(w^2-w-1)}{(w+2)(2w-1)}h_0(w,X) - \frac{3}{2}\right]\right\} + \widetilde{R}(w) - 4C_F\bar{\alpha}(1+w)\alpha^w\, h_0(w,X)\,, \tag{19}$$

where $\bar{\alpha} = 1-\alpha$,

$$h_0(w,X) = X^w\frac{\Gamma(1-w)}{\Gamma(2+w)}, \quad X = \frac{z^2\mu^2 e^{5/3}}{4}, \tag{20}$$

and the function $\widetilde{R}(w)$ is defined as the series expansion in terms of another function

$$R(w) = 2C_F\left\{\left[\frac{1+\alpha^2}{1-\alpha}\frac{\alpha^w G_0(w)-1}{w} + \alpha^w\bar{\alpha}(2+w)G_0(w)\right]_+ + \frac{\delta(\bar{\alpha})}{w}\left[\frac{3}{2} - \frac{2w+3}{(w+2)(w+1)}G_0(w)\right]\right\},$$
$$G_0(w) = \frac{\Gamma(4+2w)}{6\Gamma(1-w)\Gamma(1+w)\Gamma^2(2+w)}, \tag{21}$$

such that

$$R(w) = \sum_n w^n R_n\,, \qquad \widetilde{R}(w) = \sum_n \frac{w^n}{(n+1)!}R_n\,. \tag{22}$$

The Taylor expansion of the Borel transform at $w = 0$ gives the perturbative expansion for the coefficient function in terms of the coupling constant.

The Borel transform above has singularities at $w = 1/2, 1, 2, 3\ldots$. The singularity at $w = 1/2$ is generated by the contribution of large momenta in the self-energy insertions in the Wilson line and is part of the renormalization factor

$$B[H] \overset{w\to 1/2}{=} \frac{-4C_F}{w-1/2}\sqrt{X}. \tag{23}$$

This singularity is well-known [47] and is in the one-to-one correspondence to the linear UV divergence in the Wilson line self-energy. It can be removed when renormalizing in the ratio scheme [48], but will be present in the hybrid [29] or self renormalization [32] and is linked to the renormalon ambiguity discussed in the previous section.

The renormalon singularity at $w = 1$ is determined by

$$B[H](w) \overset{w\to 1}{=} \frac{-4C_F}{1-w}\Big[\alpha + \bar{\alpha}\ln\bar{\alpha} + \alpha\bar{\alpha}\Big]X\,. \tag{24}$$

Renormalon singularities at $w = n$ ($n = 2, 3 \ldots$) have a generic form

$$B[H](w) = \frac{2C_F}{n-w}\Big[\alpha^n p_{n-1}(\alpha) + \frac{(-1)^n\delta(\bar{\alpha})}{n!(n-2)!n^2(2n-1)}\Big]X^w, \tag{25}$$

where $p_n(\alpha)$ is a polynomial of order n, e.g. $p_1(\alpha) = (5\alpha-3)/6$, $p_2(\alpha) = (\alpha^2 - 25\alpha + 20)/180$, etc.

A singularity on the integration path in Eq. (18) means that the perturbation theory is incomplete and the sum of the series is ill-defined. It is customary [38] to estimate the corresponding ambiguity as

$$\delta H(w_0) = -\pi\frac{1}{\beta_0}e^{-w_0/(\beta_0 a_s)}\operatorname*{Res}_{w=w_0}\big[B[H](w)\big]\,, \tag{26}$$

where w_0 is the position of the singularity and $\operatorname*{Res}_{w=w_0}\big[B[H](w)\big]$ is the corresponding residue. Note that $e^{-w_0/(\beta_0 a_s)} = (\Lambda^2/\mu^2)^{w_0}$. Following the standard logic [38, 46] we assume that this ambiguity must be canceled by adding a non-perturbative correction of the same order of magnitude.

Considering $\delta H(1)$, we obtain the leading power correction to the quasi-LF correlators ($\lambda = zP^z$)

$$\mathcal{I} = h(\lambda)\Big\{1 + \kappa(z^2\Lambda^2_{\rm QCD})\mathcal{R}_{\mathcal{I}}(\lambda)\Big\}, \tag{27}$$

where

$$\mathcal{R}_{\mathcal{I}}(\lambda) = \frac{1}{h(\lambda)}\int_0^1 d\alpha\,(\alpha + \bar{\alpha}\ln\bar{\alpha} + \alpha\bar{\alpha})h(\alpha\lambda)\,, \tag{28}$$

$h(\lambda)$ denotes the leading-twist matrix element and κ is a number of $O(1)$.

After taking the Fourier transform with respect to z, one then obtains the result for the quasi-PDF

$$Q(x, P_z) = q(x)\Big\{1 + \frac{\Lambda^2_{\rm QCD}}{x^2\bar{x}P_z^2}\mathcal{R}_Q(x)\Big\} \tag{29}$$

with

$$R_Q(x) = \frac{\bar{x}}{q(x)}\Big\{\int_x^1 \frac{dy}{1-y}\Big[y(2y-1)q(\tfrac{x}{y}) - q(x)\Big] + 2q(x) - xq'(x)\Big\}, \tag{30}$$

where $\bar{x} = 1 - x$ and $q(x)$ is the leading-twist PDF. Note that we have extracted the prefactor $1/(x^2\bar{x})$ for the power correction anticipating that it is enhanced as $1/x^2$ and $1/(1-x)$ in the regions of small $x \to 0$ and large $x \to 1$ Bjorken variable, respectively.

Power corrections for the pseudo-PDF (Fourier transform with respect to λ with z fixed) can be obtained easily as

$$\mathcal{P}(x, z, \mu) = q(x)\Big\{1 - (z^2\Lambda_{\rm QCD}^2)\theta(|x| < 1)\mathcal{R}_{\mathcal{P}}(x)\Big\}, \tag{31}$$

with

$$\mathcal{R}_{\mathcal{P}}(x) = \frac{1}{q(x)}\int_{|x|}^{1}\frac{dy}{y}\,(y + \bar{y}\ln\bar{y} + y\bar{y})q(\tfrac{x}{y})\,. \tag{32}$$

4. Summary

To summarize, we have briefly reviewed the concept of renormalons and their connection to power suppressed contributions that appear in the calculation of PDFs from lattice QCD. They are expected to play an increasingly important role as we strive for the precise determination of the partonic structure of hadrons in the future.

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