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The distribution amplitude of the η_c meson

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We report on the first lattice determination of the pseudoscalar meson η_c light-cone distribution amplitude, using a set of three CLS $N_f = 2$ ensembles at a pion mass $m_\pi \sim 270$ MeV and lattice spacings $a \sim 0.076$ fm, 0.066 fm and 0.049 fm. Employing Short Distance Factorization, we extract the pseudo-DA on the lattice for Ioffe times $\nu \leq 4.5$, and the various lattice spacings allow us to take the continuum limit. We employ a basis of Jacobi polynomials to parametrize the distribution amplitude, which allows to express the matching to the pseudo distribution in closed form, and we observe a strong effect which we attribute to the heavy charm-quark mass.

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Introduction

The discovery of the J/ψ state at SLAC and BNL in 1974 corroborated the existence of the charm quark and validated the GIM mechanism, which explains the suppression of flavor-changing neutral currents in experiments. Ever since, more quarkonium states have been discovered composed of a $Q\bar{Q}$ pair of heavy quarks. However, their rate of production is poorly understood even today, and several models trying to capture the essential features of the problem have emerged. For example, the Color Evaporation Model is based on the principle of quark-hadron duality, and so it expects the production cross section of quarkonia and the $Q\bar{Q}$ pair to be directly connected below the open Q threshold. It owes its name to the assumption that a number of soft gluon emissions occur after the $Q\bar{Q}$ creation, decorrelating the initial and final states. Quite in the opposite direction, the Color Singlet Model assumes the initial $Q\bar{Q}$ pair is already a color singlet and carries the same spin as the final hadron, and assumes that no meaningful emission of gluons occurs. Finally, non-relativistic QCD builds the physical quarkonium as a linear combination of Fock states ordered by α_s and the relative velocity v_Q. Being an effective theory, however, it introduces a number of new parameters. See [1] for an overview of these methods.

In our project, we study the distribution amplitude (DA) of the pseudoscalar meson η_c, which appears in processes like the Higgs boson decay to charmonium states [2]. Besides, we plan to extend the methodology underlined here to the J/ψ DA, which will serve to probe the charm Yukawa coupling at the High-Luminosity LHC. We employ a set of CLS $N_f = 2$ lattice simulations to compute the pseudo-DA, and we employ the Short Distance Factorization (SDF) method [3] to relate the latter to the light-cone DA in the continuum limit.

From Euclidean space to the light cone

Let us start establishing the relation between the light-cone DA and the pseudo-DA [3–5] in the continuum limit. In Euclidean space, we compute the matrix element

$$M^{\mu}(p,z) = \left\langle \eta_c(1s) \middle| \bar{c}(z)\gamma^{\mu}\gamma_5 W(z,0)c(0) \middle| 0 \right\rangle, \tag{1}$$

where W is a Wilson line, $\langle \eta_c(1s)|$ is the final pseudoscalar meson state, $|0\rangle$ is the QCD vacuum, and c, $\bar{c}$ are the charm-quark lines. The matrix element M^{μ} is evaluated at particular values of the momentum p and the Wilson line extension $|z|$. We decompose M^{μ} in a purely twist > 2 contribution $\mathcal{M}'$ and a term $\mathcal{M}$ which gives access to the leading-twist distribution amplitude [3],

$$M^{\mu}(p,z) = 2p^{\mu}\mathcal{M}(p,z) + z^{\mu}\mathcal{M}'(p,z). \tag{2}$$

To extract the leading-twist part of $\mathcal{M}$, we set the momentum $p^{\mu} = (0,0,p^3,E)$ and the equal-time separation $z^{\mu} = (0,0,z^3,0)$. Choosing $\mu = 4$, we get rid of the main high-twist contamination $\mathcal{M}'$. And to remove the prefactor $2E$ and obtain a renormalized quantity, we form the RGI ratio [5, 6]

$$\phi(\nu,z) \equiv \frac{M^4(p,z)M^4(0,0)}{M^4(0,z)M^4(p,0)}, \tag{3}$$

where $\nu = pz$ is the Ioffe time. For DAs, we can shift ϕ by a phase to obtain a real quantity, $\tilde{\phi}$, which we call the reduced Ioffe-time pseudo-distribution amplitude (rpITD) and define as

$$\tilde{\phi}(\nu,z) = e^{-i\nu/2}\phi(\nu,z). \tag{4}$$

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The rpITD is related to the $\overline{\text{MS}}$ Ioffe-time distribution amplitude (ITD) $\tilde{\phi}(\nu, \mu)$ via [7]

$$\tilde{\phi}(\nu, z) = \int_0^1 \mathrm{d}w \, C(w, \nu, z\mu) \, \tilde{\phi}(w\nu, \mu), \tag{5}$$

where $C(w, \nu, z\mu)$ is a matching kernel known up to NLO in α_s [7], and $\mu = 3\,\text{GeV}$ is the $\overline{\text{MS}}$ renormalization scale. Throughout these proceedings we use $N_f = 2$ dynamical flavors, $N_c = 3$ colors, $\Lambda_{\text{QCD}} = 330\,\text{MeV}$ [8], and the aforementioned renormalization scale. Since the Ioffe time and the Bjorken x are conjugate variables, one can reconstruct the light-cone DA $\phi(x, \mu)$ from the real Fourier transform

$$\tilde{\phi}(\nu, \mu) = \int_0^1 \mathrm{d}x \cos\left[\nu\,(x - 1/2)\right] \phi(x, \mu). \tag{6}$$

To extract the DA from a lattice calculation, one needs to apply eqs. (5) and (6) consecutively after taking the continuum limit. This requires evaluating $\tilde{\phi}(\nu, z)$ at each (ν, z) for several lattice spacings. Unfortunately, this is not practical or possible at the moment. Instead, we follow an approach already used in the study of parton distribution functions [9], and parametrize the light-cone DA in terms of a basis of Jacobi polynomials,

$$\phi(x, \mu) = (1 - x)^{\alpha} x^{\alpha} \sum_{n=0}^{\infty} d_{2n}^{(\alpha)} \tilde{J}_{2n}^{(\alpha)}(x), \tag{7}$$

where $\tilde{J}_{2n}^{(\alpha)}(x) \equiv J_{2n}^{(\alpha)}(2x - 1)$ and d_{2n} are free coefficients which should be determined by the data. Usually, one specifies a basis of Jacobi polynomials via two parameters, α and β, but in the case of charmonium DAs without electromagnetism, one expects $\phi(x, \mu)$ to be symmetric around $x = 1/2$. Therefore, we set $\alpha = \beta$ and use only even ns, which have the correct symmetry. In this case, the Jacobi polynomials are proportional to Gegenbauer polynomials, which are commonly used in the study of DAs [10]. Then, the relation between $\tilde{\phi}(\nu, z)$ and $\phi(x, \mu)$ given by eqs. (5) and (6) together can be expressed as a series with the coefficients $d_{2n}^{(\alpha)}$. The main strategy is to develop in a Taylor series the cosine in eq. (6) and express the powers of $x - 1/2$ using Jacobi polynomials. Employing the orthogonality relation of the latter, it is possible to find

$$\tilde{\phi}(\nu, z) = \sum_{n=0}^{\infty} d_{2n}^{(\alpha)} \sigma_{2n}^{(\alpha)}(\nu, z\mu). \tag{8}$$

The functions σ_n work as a basis in Ioffe time, just as the Jacobi polynomials in the Bjorken variable, but given their lengthy expression, we defer to write them down to a future publication. For the present work, let us suffix to say that one can divide σ_n in a LO piece $\sigma_{\text{LO},n}$, setting to zero the strong coupling in $C(w, \nu, z\mu)$, and a NLO part $\sigma_{\text{NLO},n} = \sigma_n - \sigma_{\text{LO},n}$. We plot them both in fig. 1. Looking closer at this figure, each σ_n peaks at a different value of ν, providing information on different regions. From the fact that $\tilde{\phi}(\nu = 0, z) = 1$ and that σ_0 is the only nonzero function at $\nu = 0$, we can determine that d_0 is a Beta function,

$$d_0^{(\alpha)} = \frac{1}{B(1/2, 1 + \alpha)}. \tag{9}$$

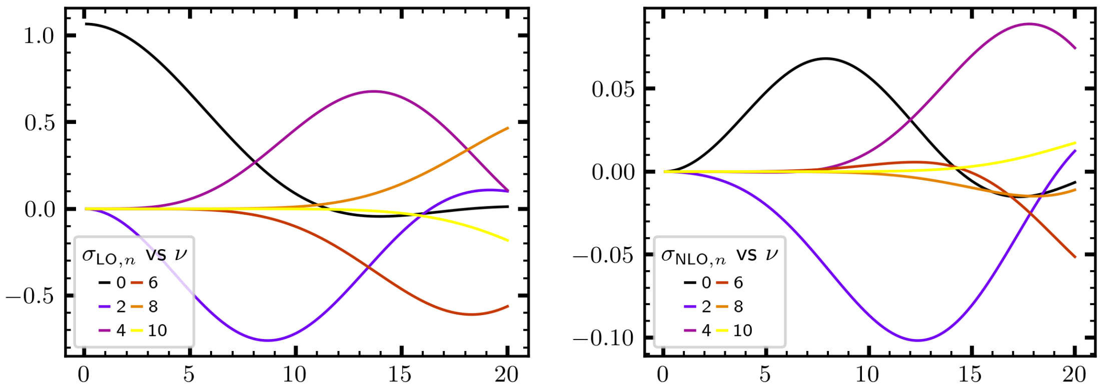

Figure 1: Left: The LO contribution to σ_n versus Ioffe time. Right: The NLO contribution $\sigma_{\mathrm{NLO},n} = \sigma_n - \sigma_{\mathrm{LO},n}$ versus Ioffe time. As representative values, we use $\alpha = 2$, $|z|/a = 5$, and $a = 0.0658\,\mathrm{fm}$. Only the n-even coefficients are nonzero.

id	β	a [fm]	m_π [MeV]	m_{η_c} [MeV]	κ_ℓ	κ_c	# cnfgs	# hits
B6	5.2	0.0755(9)(7)	281	2929	0.13597	0.12529	118	10
F7	5.3	0.0658(7)(7)	265	2955	0.13638	0.12713	200	10
O7	5.5	0.0486(4)(5)	268	2972	0.13671	0.13022	164	10

Table 1: CLS ensembles used on our analysis. The various columns indicate, from left to right, the ensemble label, its bare strong coupling constant, the lattice spacing, the approximate π and η_c mass, the light- and charm-quark κ, and the number of gauge configurations and hits.

The lattice calculation

We employ a set of three $N_f = 2$ CLS ensembles [11, 12] with parameters and statistics gathered in table 1. Their action is formed by a Wilson plaquette in the gauge sector and two mass-degenerate flavors of non-perturbatively $O(a)$ improved Wilson quarks. Their pion mass is $m_\pi \sim 270\,\mathrm{MeV}$, κ_c is fixed by the physical mass of the D_s meson, $m_{D_s} = m_{D_s,\mathrm{phy}} = 1969(1)$ GeV [13], and the strange quark is set to its physical value. All ensembles employ the locally deflated Schwarz preconditioned generalised conjugate residual (GCR) solver, but B6 and F7 use the domain decomposition hybrid Monte Carlo (DD-HMC) algorithm, while O7 employs mass preconditioned hybrid Monte Carlo (MP-HMC). The coupling constant was defined via the Schrödinger functional, and the scale was determined using the kaon decay constant. Regarding the quark-connected charm propagators, we compute two sets of data: the matrix elements in eq. (1), and pseudoscalar correlators for η_c states in the A_1^{-+} cubic representation. Both sets include four different Gaussian smearings, six momenta computed via twisted boundary conditions, and ten U(1) random sources defined in a random time slice, diluted in spin, and employing the one-end trick. For eq. (1) alone, we compute ten different Wilson-line extensions, and we employ the η_c correlators to solve a 4×4 GEVP and find the optimal interpolator to the pseudoscalar meson at each momentum. On a lattice simulation, we start from $M^\mu(p, z, t, a, m_\pi, m_{\eta_c})$, where besides the momentum and Wilson line, there are dependencies in time, lattice spacing, and π and η_c masses. The time dependence vanishes in the double ratio eq. (3), while the imaginary part of the matrix elements disappears due to the extra phase in eq. (4). Then, we extract a real value for each $\tilde{\phi}(p, z, a, m_\pi, m_{\eta_c})$ fitting the time dependence to a constant minimizing a chi-square function. We repeat the fit for ensembles B6, F7, and O7 individually to obtain the plot on the lhs of fig. 2, which shows the rpITD with different markers for each ensemble and different colors for each Wilson line.

Finally, the continuum extrapolation involves not only removing the regulator, but also estimating the high-twist contamination and the impact of the small difference in the meson masses between the three ensembles. The basic piece of the extrapolation model is eq. (8), which intends to describe the leading-twist, continuum ITD. Given the interval of Ioffe times in our study, we observe that we are only sensitive to the d_0 coefficient, such that the DA at this stage is determined by the α parameter alone. Regarding the lattice artifacts, they start at $\mathcal{O}(a)$ because there is no Symanzik improvement available for the bilocal operators in eq. (1). Besides, we can use CP symmetry to determine that $\tilde{\phi}(p, z) = \tilde{\phi}(-p, -z)$, such that odd powers of a must be functions of $a|p|$ or $a/|z|$. Furthermore, together with Wilson-line-dependent lattice artifacts, there can also be global lattice artifacts $a\Lambda_{\text{QCD}}$ —note that we use Λ_{QCD} to render all quantities dimensionless. The lattice artifacts may also depend on the Ioffe time, and to model such dependence we can exploit the same basis of Jacobi polynomials used for the ITD. Following the example of [9], we define nuisance functions

$$A_r^{(\alpha)}(x) = (1-x)^\alpha x^\alpha \sum_{s=1}^{S_{a,r}} a_{r,2s}^{(\alpha)} \tilde{J}_{2s}^{(\alpha)}(x), \tag{10}$$

where $a_{r,2s}^{(\alpha)}$ are the nuisance fit parameters. We are interested in the Ioffe time dependence,

$$A_r^{(\alpha)}(\nu) = \int_0^1 \mathrm{d}x\, A_r^{(\alpha)}(x) \cos\left[\left(x - \frac{1}{2}\right)\nu\right] = \sum_{s=1}^{S_{A,r}} a_{r,2s}^{(\alpha)} \sigma_{\text{LO},2s}^{(\alpha)}(\nu). \tag{11}$$

Note that the term $s = 0$ vanishes due to the condition $\tilde{\phi}(\nu = 0, z) = 1$. From examination, we consider it is enough to include $r = 1$ only and set $S_{A,1} = 1$, such that the nuisance function reduces to one single term with one fit parameter. In the fit, each term will have a similar nuisance function with its own free parameter. With regard to higher-twist effects, their leading behavior goes as $z^2\Lambda_{\text{QCD}}^2$, and we include another nuisance function for them. Finally, the mass corrections are the least understood. Since we work with heavy quarks, it is possible that the charm-quark mass affects significantly not only the nuisance functions, but also the leading twist ITD. Unfortunately, no mass corrections are included in the calculation of the matching kernel $C(w, \nu, z\mu)$ [7]. Some results do exists for other distributions but are not applicable here. They can only guide us to create an Ansatz, which is also informed by the fit quality of the extrapolation. We introduce an identical term for the pion masses, although for the latter we only wish to model the mistunings in the pion mass, not account for any physics. Then, the fit model we employ at this stage is

$$\begin{aligned}\tilde{\phi}(\nu, z, a) = \tilde{\phi}_{\text{lt}}(\nu, z) + \frac{a}{|z|}\left(A_1(\nu) + \log\left(\frac{m_{\eta_c}}{m_{\eta_c,\text{phy}}}\right) D_1(\nu)\right) \\ + a\Lambda_{\text{QCD}} B_1(\nu)x + z^2\Lambda_{\text{QCD}}^2 C_1(\nu) + \frac{a}{|z|}\log\left(\frac{m_\pi}{m_{\pi,\text{phy}}}\right) E_1(\nu).\end{aligned} \tag{12}$$

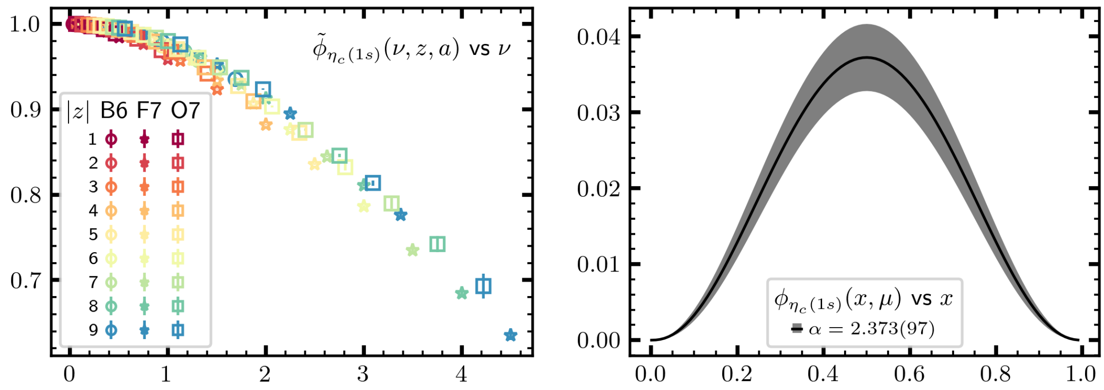

Figure 2: Left: The values of the reduced Ioffe-time pseudo-distribution amplitude used for this study. The different colors indicate the length of the Wilson line, while the markers refer to the ensembles in table 1. Right: The light-cone distribution amplitude as obtained from our extrapolation to the continuum.

Note that the product of $a/|z|$ by the nuisance functions produce terms proportional to $a|p|$ automatically, and that we normalize by the physical meson masses $m_{\pi,\text{phy}} = m_{\pi^0} = 134.9768(5)$ MeV and $m_{\eta_c,\text{phy}} = 2983.9(4)$ MeV [13]. The model includes one nonlinear fit parameter, α, and five linear ones, $a_{1,2}$, $b_{1,2}$, $c_{1,2}$, $d_{1,2}$, and $e_{1,2}$. We leverage this asset and rewrite the linear parameters in terms of α following the variable projection algorithm [14], which helps to speed up and stabilize the chi-square minimization. The model eq. (12) fits our data with $\chi^2/\text{dof} = 165/134 = 1.23$. We find $\alpha = 2.37(10)$, $a_{1,2} = 4.216(26)$, $b_{1,2} = 2.53(18)$, $c_{1,2} = -0.1005(20)$, $d_{1,2} = -47.91(6)$, and $e_{1,2} = -4.570(24)$, where errors are purely statistical at this stage. This result allows to evaluate the light-cone DA as given by eq. (7) including only $n = 0$. The result can be seen on the rhs of fig. 2.

Conclusions and outlook

We apply Short Distance Factorization to compute the distribution amplitude of the η_c meson. Our lattice dataset consists of three $N_f = 2$ CLS ensembles at a pion mass $m_\pi \sim 270$ MeV and three different lattice spacings. We describe the light-cone DA with the help of a basis of Jacobi polynomials eq. (7), and we compute in closed form the relation between the former and the pseudo-DA using the polynomials orthogonality properties. We take the continuum limit using the fit function eq. (12), which models the leading $O(a)$ lattice artifacts, the high-twist contamination, and the small differences of the meson masses in the ensembles. Besides, we observe a large effect coming from the η_c mass, which points to the need to include nonzero quark masses in the matching kernel. Finally, we extract the light-cone DA to first order in eq. (7).

The current status of the project serves as a proof of concept on how to get DAs on the continuum. Future work will focus on extending the analysis to several more CLS ensembles of similar characteristics. They will extend the Ioffe time interval, hopefully constrain more terms in the series eq. (7), and probe the pion mass dependence. Besides, we recently got access to several $N_f = 2 + 1 + 1$ ensemble at the physical pion mass, which will allow to include the missing sea-quark effects and eliminate the systematics from the unphysical pion masses. We are interested

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in recomputing the matching kernel in eq. (5), this time keeping the quark masses nonzero, which should improve our Ansatz for the continuum limit.

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