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Rare K decays off and on the lattice

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The importance of rare K decays especially in the context of a kaon unitarity triangle (KUT) is emphasized. The decay $K_L \to \pi^0 \nu\bar{\nu}$ is theoretically very clean but experimentally extremely challenging. The Standard Model prediction $\mathcal{B} \sim 3 \times 10^{-11}$ is still about two orders of magnitude away from the current experimental upper bound. One way to continue to make progress towards the construction of a KUT is by improving the accuracy in the calculation of ε'. Another way which is the primary focus here is via studies of $K^0 \to \pi^0\mu^+\mu^-$. LHCb, J-PARC, the proposed HIKE project, phenomenology, and in fact precision studies on the lattice can all play a very important role in this context.

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1. Introduction and Motivation

The experiment of Christenson, Cronin, Fitch and Turlay [1] showed for the first time that K_L does decay to a two-pion final state. Thereby the experiment gave the first evidence for CP violation. It is actually indirect CP violation since the mixing between K^0 and $\bar{K}^0$ is primarily responsible for the seen effect. Nevertheless, its observation has the profound consequence that CP is not a symmetry of nature. Naturalness arguments then suggest that new physics is likely to entail new CP-odd phases. Searching for such phases is perhaps one of the most promising ways to probe for new physics.

Our primary focus here is to probe for new physics by the construction of the kaon unitarity triangle (KUT) [2] and below we give a brief review of the ongoing and future efforts towards constructing it with rare kaon decays. The current construction of the unitarity triangle is dominated by B physics, and a construction only from kaon decays would represent a crucial intergenerational test of our understanding of flavor physics and thereby the Standard Model (SM). Any deviation between the triangle constructions with K and B decays, respectively, would be a clear sign for new physics. The opportunities at future kaon facilities obtained a lot of recent attention [3], and include the potential to discover a light dark sector [4]. A detailed summary of SM predictions can be found in Ref. [5].

The rare decay $K_L \to \pi^0 \nu\bar{\nu}$ is an important target for the construction of the KUT and other purposes. It violates CP [6] and is theoretically very clean; it is dominated by the top quark and therefore it is a safe probe for short distance physics. However, its measurement is unfortunately experimentally exceedingly challenging. After many years and a lot of progress, the KOTO experiment at J-PARC has obtained an impressive upper bound of $3.0 \cdot 10^{-9}$ [7]. This is still about two orders of magnitude above the SM [8], and a lot more work is anticipated [9].

The challenges of measuring $K_L \to \pi^0 \nu\bar{\nu}$ are why it would be extremely useful to improve constraints also on the related modes involving lepton pairs: $K^0 \to \pi^0 l^+ l^-$. These are theoretically less “clean”, so theoretical input becomes rather important but their experimental detection can be much less challenging.

Below, in sections 2–6 we give an overview over current developments in key rare decay channels including the above. We conclude in Sec. 7.

2. $K^+ \to \pi^+ \nu\bar{\nu}$

This rare decay of the charged kaon is an important pillar of the KUT [2]. While a large fraction of the amplitude is dominated by short-distance physics, in particular the top-quark contribution, giving sensitivity to $V_{ts}^* V_{td}$, unfortunately it does also receive some long-distance contributions from charm- and up-quark intermediate states. These have been calculated in chiral perturbation theory [10] and are subject to on-going lattice [11] and phenomenological [12] studies. BNL experiments [13] saw a few candidate events and obtained a result,

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})^{\mathrm{BNL}} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}\,, \tag{1}$$

which is $\sim 1\sigma$ above the SM prediction [8]

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})^{\mathrm{SM}} = (8.4 \pm 1.0) \times 10^{-11}\,. \tag{2}$$

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The experimental effort continued at CERN with the NA62 experiment, and with several years of improvements and progress, NA62 observed $K^+ \to \pi^+ \nu\bar{\nu}$ with a significance of 3.4σ, resulting in the branching ratio measurement [14]

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})^{\text{NA62}} = \left(10.6^{+4.0}_{-3.4} \pm 0.9\right) \cdot 10^{-11}\,, \tag{3}$$

in agreement with the SM. Fig. 4 in Ref. [15] summarizes the history of theoretical predictions and experimental measurements of $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$. It is interesting to observe that improvements of the bound on $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$ from around 10^{-9} to around 10^{-11} took ~20 years. This underscores the importance of continuous efforts on long time scales needed in kaon physics.

3. The gold-plated decay $K_L \to \pi^0 \nu\bar{\nu}$

The decay $K_L \to \pi^0 \nu\bar{\nu}$ violates CP [6]. In the SM, this is a consequence of all three generations of up-type quarks u, c, and t entering the electroweak loop inducing the $s \to d$ FCNC transition. However, the top quark contribution dominates completely and therefore the process is clearly dominated by short-distance physics. Thus, a measurement of its rate would cleanly give the Kobayashi-Maskawa phase necessary for CP violation to occur. However, from the experimental point of view, $K_L \to \pi^0 \nu\bar{\nu}$ is a very challenging process. After many years of progress, the KOTO experiment at J-PARC has achieved a (90% CL) upper bound of [7]

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})^{\text{KOTO}} \le (3.0 \times 10^{-9})\,. \tag{4}$$

This upper bound is still some two orders of magnitude larger than the SM prediction [8]

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})^{\text{SM}} = (3.4 \pm 0.6) \times 10^{-11}\,, \tag{5}$$

and consequently it could take some time before we reach the discovery of this mode. It is important therefore to tighten the constraints on this process as much as possible and a lot more experimental work is planned in the future [9].

4. $K^0 \to \pi^0 e^+ e^-$

This charged counterpart of the gold-plated mode discussed in the last section has been of interest for a very long time [16–21]. For the case of a $\pi^0 e^+ e^-$ final state, the experimental detection faces a daunting challenge by the so-called “Greenlee background” [22]. The problem in the actual detection of the π^0 is that [23]

$$\mathcal{B}(K_L \to e^+ e^- \gamma\gamma) \approx (5.95 \pm 0.33) \cdot 10^{-7}\,, \tag{6}$$

which is some four orders of magnitude larger than the SM prediction [18]

$$\mathcal{B}(K_L \to \pi^0 e^+ e^-) = \left(3.2^{+1.2}_{-0.8}\right) \cdot 10^{-11}\,. \tag{7}$$

Moreover, even if one could detect $\pi^0 e^+ e^-$ then the portion of the amplitude that comes from one virtual photon (or Z) going to e^+e^- is CP violating but if the pair comes from two virtual photons the contribution is CP-conserving. So a good theoretical understanding becomes mandatory. And that is where modern lattice methods become very relevant along with, of course, phenomenology.

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5. $K^0 \to \pi^0\mu^+\mu^-$

This mode is very interesting as LHCb has a very large flux of K_S which they can use to study decays to final states like $\mu^+\mu^-$ and $\pi^0\mu^+\mu^-$ [24]. K_L could be studied, for example, at J-PARC and at the proposed HIKE experiment at CERN [25, 26]. In this case, the Greenlee background is three (rather than four) orders of magnitude larger than the potential signal for K_L decays, as [23]

$$\mathcal{B}(K_L \to \mu^+\mu^-\gamma\gamma) = \left(1.0^{+0.8}_{-0.6}\right) \cdot 10^{-8}\,, \tag{8}$$

compared to the SM prediction [27]

$$\mathcal{B}(K_L \to \pi^0\mu^+\mu^-)^{\mathrm{SM}} = (1.5 \pm 0.3) \cdot 10^{-11}\,. \tag{9}$$

On the phenomenological front considerable work has already been done [27, 28] and more is in progress [29]. Prospects for precise lattice studies are quite good given that the RBC-UKQCD collaboration has and is doing several related studies [11, 30]. In particular, this means that lattice methods may be used to calculate the CP violating and CP conserving contributions to $\mu^+\mu^-$ or e^+e^- from one photon and two photons.

From the results in Ref. [27] one obtains the ratio of the CP-conserving contribution over the total branching ratio as

$$\mathcal{B}(K_L \to \pi^0\mu^+\mu^-)^{\mathrm{CPC}}/\mathcal{B}(K_L \to \pi^0\mu^+\mu^-) \sim 0.3\,. \tag{10}$$

In order to distinguish CP-conserving and CP-violating contributions to $K_L \to \pi^0 l^+ l^-$ experimentally, one can employ the Dalitz distribution and asymmetry observables [16, 18, 31–33].

It is known that the decays $K_L \to \pi^0 e^+e^-$ and $K_L \to \pi^0\mu^+\mu^-$ are complementary regarding their constraints on new physics models, as together they can be used to distinguish between (pseudo)scalar and (axial)vector new physics operators [34]. Furthermore, they allow the test of lepton flavor universality in kaon decays [28, 35]. The effect of future measurements of $\mathcal{B}(K_L \to \pi^0 l^+ l^-)$ on the constraints of Wilson coefficients of new physics models are shown in Ref. [36].

6. $K \to \mu^+\mu^-$

Recently, it has been shown that through time-dependent measurements of $K \to \mu^+\mu^-$ it would in principle be possible to separate short- and long-distance physics, thereby enabling a new technique for the determination of the unitarity triangle height η [37–40], see for a short summary Ref. [41]. Experimentally, this method is very challenging, see for first ideas Ref. [42]. However, already time-integrated measurements of $\mathcal{B}(K_S \to \mu^+\mu^-)$ [43] are constraining new physics models [44]. The prospects for future lattice results are promising [45, 46].

7. Conclusions

In the upcoming era of precision flavor physics, it will be important to construct the unitarity triangle primarily from kaon decays only, in order to further test our understanding of CP violation in the SM. This has special importance also due to the on-going tension in V_{cb} and V_{ub} which

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make a unitarity triangle construction independent of those quantities desirable, see also Ref. [47]. Any deviations from the current construction of the unitarity triangle, which is dominated by B physics, will be a strong sign of new physics. We reiterate that the gold-plated channel $K_L \to \pi^0 \nu\bar{\nu}$ can be very powerfully constrained by the semileptonic decays $K^0 \to \pi^0 l^+ l^-$ through their shared dependence on the height η of the unitarity triangle, along with $K^+ \to \pi^+ \nu\bar{\nu}$ through the Grossman-Nir bound [48].

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