

$B_s \rightarrow K\ell\nu$ form factors from lattice QCD with domain-wall heavy quarks

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We report on our on-going study of the $B_s \rightarrow K\ell\nu$ decay in $N_f = 2 + 1$ lattice QCD. We employ fully relativistic setup in which the Möbius domain-wall action is used for all quark flavors. The lattice cutoff is $a^{-1} \sim 2.5$ GeV, where we take bottom quark masses up to $m_Q < 0.7a^{-1}$ in order to control discretization errors. We present preliminary results for the relevant form factors extracted from correlator ratios by inspecting their ground state saturation.

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1. Introduction and Motivation

The exclusive $B \rightarrow \pi\ell\nu$ decay has been conventionally used to determine the Cabibbo-Kobayashi-Maskawa matrix element $|V_{ub}|$. However, about 2σ tension with the inclusive analysis [1] suggests that theoretical and/or experimental uncertainties have not yet been fully understood. In our previous study [2], largest theoretical uncertainties came from the statistics and chiral extrapolation. The $B_s = K\ell\nu$ decay provides an alternative determination of $|V_{ub}|$ with advantages that i) the statistical fluctuation of relevant correlation functions is suppressed as discussed in Refs. [3, 4] and ii) the chiral extrapolation may be better controlled without the valence pions. The relevant form factors have been, therefore, calculated by several groups [5–9] using bottom quark actions based on effective theories. However, recent LHCb measurement of $B_s^0 \rightarrow K^- \mu^+ \nu_\mu$ and their analysis together with $B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu$ obtained inconsistent results for $|V_{ub}|/|V_{cb}|$ depending on the form factor inputs, namely those from lattice QCD and light-cone sum rule [10]. Since a tension among the form factors from previous lattice QCD studies has also been suggested in Ref. [7], independent calculations are highly welcome. In this article, we report on our on-going study employing fully relativistic setup.

2. Gauge Ensemble and Simulation Parameters

We simulate $N_f = 2 + 1$ QCD using the Möbius domain-wall quark action [11]. Details about our ensembles can be found in Ref. [2]. This article reports preliminary results obtained on a $32^3 \times 64 \times 12$ lattice at the lattice cutoff of $a^{-1} = 2.453(44)$ GeV fixed from the Yang-Mills gradient flow. The degenerate up and down quark mass m_{ud} corresponds to an unphysically large pion mass $M_\pi = 499(1)$ MeV, whereas the strange quark mass m_s is close to the physical value leading to the kaon mass $M_K = 618(1)$ MeV. The statistics are 5,000 Hybrid Monte Carlo trajectories. The $B_s \rightarrow K$ form factors are calculated in fully relativistic setup with bottom quark masses $m_Q = m_c, 1.25m_c, 1.25^2m_c < 0.7a^{-1}$, where m_c represents the physical charm quark mass fixed from the spin-averaged charmonium mass $(M_{\eta_c} + 3M_{J/\psi})/4$, so that discretization errors remain under control.

3. Correlation Functions and Form Factors

In order to extract the form factors, we make use of the following three and two-point functions

$$C_{3,\mu}^{K \rightarrow B_s}(t, T, \vec{q}) = \frac{1}{N_{t_0}} \sum_{\vec{x}, \vec{y}, \vec{z}, t_0} \langle O_{B_s}(\vec{x}, T + t_0) V_{bl}^\mu(\vec{y}, t + t_0) O_K(\vec{z}, t_0)^\dagger \rangle \exp[-i\vec{q}(\vec{y} - \vec{z})], \quad (1)$$

$$C_{3,\mu}^{B_s \rightarrow K}(t, T) = \frac{1}{N_{t_0}} \sum_{\vec{x}, \vec{y}, \vec{z}, t_0} \langle O_K(\vec{x}, T + t_0) V_{lb}^\mu(\vec{y}, t + t_0) O_{B_s}(\vec{z}, t_0)^\dagger \rangle, \quad (2)$$

$$C_{3,\mu}^{K \rightarrow K}(t, T) = \frac{1}{N_{t_0}} \sum_{\vec{x}, \vec{y}, \vec{z}, t_0} \langle O_K(\vec{x}, T + t_0) V_{ll}^\mu(\vec{y}, t + t_0) O_K(\vec{z}, t_0)^\dagger \rangle, \quad (3)$$

$$C_{3,\mu}^{B_s \rightarrow B_s}(t, T) = \frac{1}{N_{t_0}} \sum_{\vec{x}, \vec{y}, \vec{z}, t_0} \langle O_{B_s}(\vec{x}, T + t_0) V_{bb}^\mu(\vec{y}, t + t_0) O_{B_s}(\vec{z}, t_0)^\dagger \rangle, \quad (4)$$

$$C_2^K(t, \vec{p}_K) = \frac{1}{N_{t_0}} \sum_{\vec{x}, \vec{z}, t_0} \langle O_K(\vec{x}, t + t_0) O_K(\vec{z}, t_0)^\dagger \rangle \exp[-i\vec{p}_K(\vec{x} - \vec{z})], \quad (5)$$

$$C_2^{B_s}(t) = \frac{1}{N_{t_0}} \sum_{\vec{x}, \vec{z}, t_0} \langle O_{B_s}(\vec{x}, t + t_0) O_{B_s}(\vec{z}, t_0)^\dagger \rangle, \quad (6)$$

where $V_{bl}^\mu(x) = \bar{b}(x)\gamma^\mu l(x)$, $V_{lb}^\mu(x) = \bar{l}(x)\gamma^\mu b(x)$, $V_{ll}^\mu(x) = \bar{l}(x)\gamma^\mu l(x)$ and $V_{bb}^\mu(x) = \bar{b}(x)\gamma^\mu b(x)$. We apply Gaussian smearing to the interpolating fields O_{B_s} and O_K to enhance their overlap with the ground state. The three-point functions are measured with five different values of the source-sink separation $T = 12, 16, 20, 24$ and 28 in order to study their ground state saturation. The statistical accuracy of both three- and two-point functions is improved by averaging over the source point (\vec{z}, t_0) . A momentum-projected volume source with Z_2 noise is used for the average over the spatial source point \vec{z} . We repeat the measurement for four values of the source timeslice $t_0 = 0, 16, 32$ and 48 for the average over t_0 . These three- and two-point functions can be expressed as, for instance,

$$C_{3,4}^{K \rightarrow B_s}(t, T, \vec{q}) = \sum_{n,m} A_n^{B_s} (A_m^K)^* D_{4,nm}^{K \rightarrow B_s} \exp[-E_n^{B_s}(T-t)] \exp[-E_m^K t], \quad (7)$$

$$C_2^K(t, \vec{p}_K) = \sum_n A_n^K (A_n^K)^* (\exp[-E_n^K t] + \exp[-M_n^K(T-t)]), \quad (8)$$

where $A_n^{B_s} = \frac{\langle 0 | O_{B_s} | B_s^n \rangle}{\sqrt{2E_n^{B_s}}}$ and $A_n^K = \frac{\langle 0 | O_K | K^n \rangle}{\sqrt{2E_n^K}}$ are the overlaps of the interpolating fields with the physical states, and $D_{4,nm}^{K \rightarrow B_s} = \frac{\langle B_s^n | V_{bl}^4 | K^m \rangle}{2\sqrt{E_n^{B_s} E_m^K}}$ involves the matrix element of interest.

The B_s meson is at rest throughout our measurement. Then parallel and perpendicular form factors in the parametrization

$$\langle K(p_K) | V^\mu | B(p_{B_s}) \rangle = \sqrt{2M_{B_s}} \left\{ v^\mu f_{\parallel}(E_K) + p_{K,\perp}^\mu f_{\perp}(E_K) \right\} \quad (9)$$

are given by the following simple expressions

$$f_{\parallel}(E_K) = \frac{\langle K | V^0 | B_s \rangle}{\sqrt{2M_{B_s}}}, \quad f_{\perp}(E_K) = \frac{1}{p_K^i} \frac{\langle K | V^i | B_s \rangle}{\sqrt{2M_{B_s}}}. \quad (10)$$

In order to study the E_K dependence, namely momentum transfer dependence, of the form factors, we vary the kaon momentum as $|\vec{p}_K|^2 = 0, 1, 2, 3$ (in units of $(2\pi/L)^2$), and calculate $C_{3,\mu}^{K \rightarrow B_s}$ and $C_2^K(t, \vec{p}_K)$ as indicated in Eqs. (1) and (5). For the latter, we used the local kaon sink, which is useful for canceling overlap factors in a correlator ratio for non-zero recoils (see Eq. (15) below). Other correlators (2)–(4) and (6) are used only for zero recoil.

4. Form Factors Extraction at Zero Recoil

We test two correlator ratios to extract the form factor at zero recoil. Figure 1 shows the following ratio of three- to two-point functions

$$R_{3p2p} = \frac{C_{3,4}^{K \rightarrow B_s}(t, T)}{C_2^K(t) C_2^{B_s}(T-t)} \rightarrow \frac{D_{4,00}^{K \rightarrow B_s}}{A_0^K A_0^{B_s}} \quad (11)$$

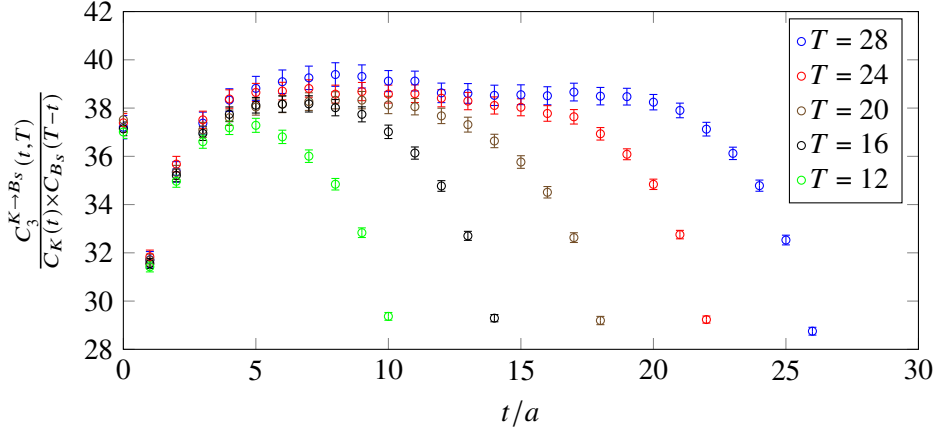


Figure 1: Correlator ratio R_{3p2p} as a function of the temporal location of the vector current t . We plot data with $m_Q = m_c$.

as a function of the temporal location of the vector current. We observe reasonable ground state saturation particularly around the temporal mid-point $t \sim T/2$. To estimate the ground state contribution in the rightmost side of Eq. (11), we carry out a simultaneous fit of the three- and two-point function to the expressions (7)–(8) including the ground ($n, m = 0$) and first excited ($n, m = 1$) states. This ratio, however, needs the renormalization constant of the heavy-light current $Z_{V_{bl}}$, which we estimate as $Z_{V_{bl}} = \sqrt{Z_{V_{ll}}Z_{V_{bb}}}$. For the determination of $Z_{V_{bb}}$ we use the vector current conservation $Z_{V_{bb}}C_{3,4}^{B_s \rightarrow B_s}(t, T)/C_2^{B_s}(T) = 1$ and $Z_{V_{ll}}$ is determined in our study [12].

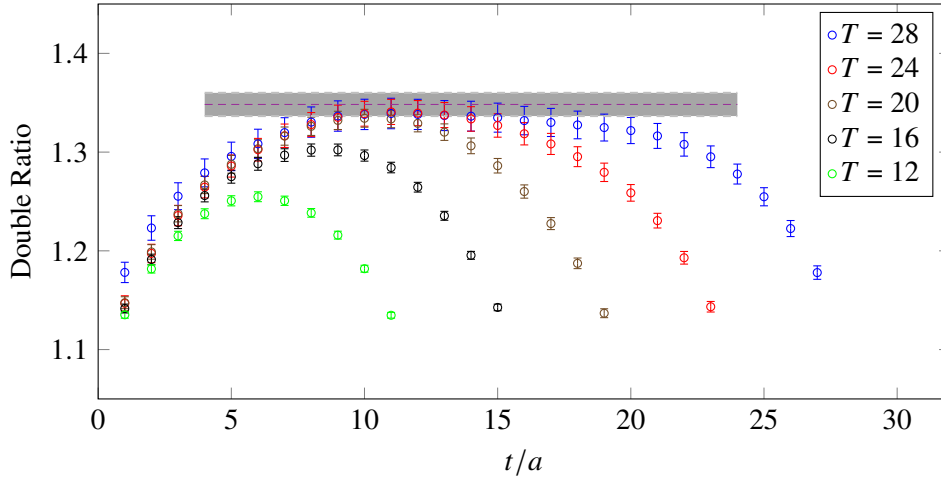


Figure 2: Double ratio at $m_Q = m_c$. Symbols are simulation results for five different values of the source-sink separation T , whereas the gray band shows coefficient C_{00} in Eq. (13) representing the ground state contribution.

We also test the double ratio of three-point functions

$$\text{Double Ratio} = \frac{C_{3,4}^{K \rightarrow B_s}(t, T) C_{3,4}^{B_s \rightarrow K}(t, T)}{C_{3,4}^{K \rightarrow K}(t, T) C_{3,4}^{B_s \rightarrow B_s}(t, T)} \rightarrow \frac{D_{4,00}^{K \rightarrow B_s} D_{4,00}^{B_s \rightarrow K}}{D_{4,00}^{K \rightarrow K} D_{4,00}^{B_s \rightarrow B_s}}, \quad (12)$$

where the renormalization and overlap factors get canceled [13]. We carry out a simultaneous fit of this ratio to the form

$$\frac{C_{3,4}^{K \rightarrow B_s}(t, T) C_{3,4}^{B_s \rightarrow K}(t, T)}{C_{3,4}^{K \rightarrow K}(t, T) C_{3,4}^{B_s \rightarrow B_s}(t, T)} = C_{00} \left(1 + A' \left[\exp[-\Delta E_K t] + \exp[-\Delta E_K (T - t)] \right] + B' \left[\exp[-\Delta E_{B_s} t] + \exp[-\Delta E_{B_s} (T - t)] \right] \right) \quad (13)$$

and two-point functions to Eq. (8) to estimate the coefficients C_{00} , A' , B' and energy differences ΔE_{B_s} and ΔE_K . Simulation data of the ratio and its ground state contribution C_{00} are plotted in Fig. 2, where we again observe reasonable ground state saturation around the mid-point $t \sim T/2$.

m_Q	m_c	$1.25m_c$	1.25^2m_c
$R_{3p2p} A_0^K A_0^{B_s} \sqrt{Z_{V_{bl}}}$	1.154(12)	1.177(12)	1.209(13)
$\sqrt{\text{Double Ratio}}$	1.1546(10)	1.1802(16)	1.2061(25)

Table 1: Comparison of R_{3p2p} and Double Ratio

The ground state contributions of R_{3p2p} and the double ratio are related as

$$R_{3p2p} A_0^K A_0^{B_s} \sqrt{Z_{V_{bl}}} = \sqrt{\text{Double Ratio}}. \quad (14)$$

Good numerical consistency shown in Table 1 suggests that the ground state contribution is reliably extracted for both of the two ratios. The same table also shows that the result with R_{3p2p} has much larger uncertainty, which mainly comes from the measurement of $Z_{V_{bl}}$. Hence we have used double ratio in our analysis.

Our accuracy of the form factor at zero recoil is typically 0.5%. In accordance with the discussions in Refs. [3, 4], this is significantly better than that for $B \rightarrow \pi\ell\nu$ as shown in Fig. 3.

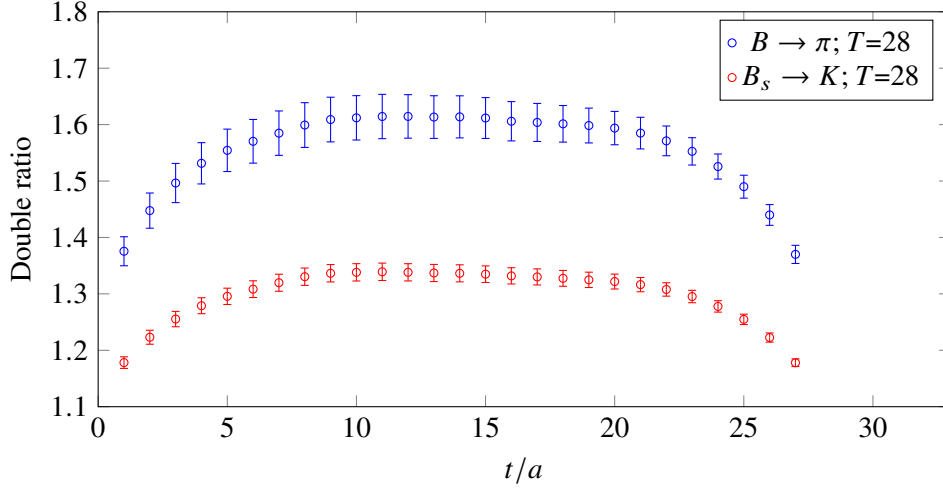


Figure 3: Double ratios of $B \rightarrow \pi$ and $B_s \rightarrow K$ at $m_Q = m_c$. We plot data for the source sink separation $T = 28$.

5. Form Factor Extraction for Non-zero Recoil

In order to extract $f_{\parallel}(E_K)$ and $f_{\perp}(E_K)$ at non-zero recoils, we consider the following ratios respectively

$$R_4(\vec{p}_K) = \frac{C_{3,4}^{K \rightarrow B_s}(t, T, \vec{p}_{B_s} = \vec{0}, \vec{p}_K)}{C_{3,4}^{K \rightarrow B_s}(t, T, \vec{p}_{B_s} = \vec{0}, \vec{p}_K = \vec{0})} \times \frac{C_2^K(t, \vec{p}_K = \vec{0})}{C_2^K(t, \vec{p}_K)}, \quad (15)$$

$$R_i(\vec{p}_K) = \frac{C_{3,i}^{K \rightarrow B_s}(t, T, \vec{p}_{B_s} = \vec{0}, \vec{p}_K)}{C_{3,4}^{K \rightarrow B_s}(t, T, \vec{p}_{B_s} = \vec{0}, \vec{p}_K)}, \quad (16)$$

where three- and two-point functions are averaged over appropriate momentum configurations to improve their statistical accuracy. We then carry out a combined fit of these ratios to the following forms

$$R_4(\vec{p}_K) = C_{44} \left(1 + A \exp[-\Delta E_{B_s}(\vec{p}_{B_s} = \vec{0})(T-t)] + B \exp[-\Delta E_K(\vec{p}_K = \vec{0})t] + F \exp[-\Delta E_K(\vec{p}_K)t] \right), \quad (17)$$

$$R_i(\vec{p}_K) = C_{4i} \left(1 + G \exp[-\Delta E_{B_s}(\vec{p}_{B_s} = \vec{0})(T-t)] + H \exp[-\Delta E_K(\vec{p}_K)t] \right), \quad (18)$$

and two-point functions, $C_2^K(t, \vec{p}_K)$ and $C_2^{B_s}(t)$ to Eq. (8). The coefficients A, B, F, G and H are originated from the excited state contributions. The overall factors, C_{44} and C_{4i} , encode the ground state contribution and can be used to determine the form factors as

$$f_{\parallel}(E_K) = C_{44} \sqrt{2M_K C_{00}}, \quad f_{\perp}(E_K) = \frac{C_{4i} C_{44} \sqrt{2M_K C_{00}}}{p_K^i}. \quad (19)$$

As in our study of $B \rightarrow \pi\ell\nu$ [2], we convert f_{\parallel} and f_{\perp} to HQET motivated definition of form factors f_1 and f_2 as

$$f_1(E_K) + f_2(E_K) = \frac{f_{\parallel}(E_K)}{\sqrt{2}}, \quad f_2(E_K) = \frac{E_K f_{\perp}(E_K)}{\sqrt{2}}. \quad (20)$$

In Figs. 4a and 4b, we plot these form factors as a function of E_K . The typical statistical accuracy is 7% for $f_1 + f_2$ and 8% for f_2 , which is better than the $B \rightarrow \pi\ell\nu$ case.

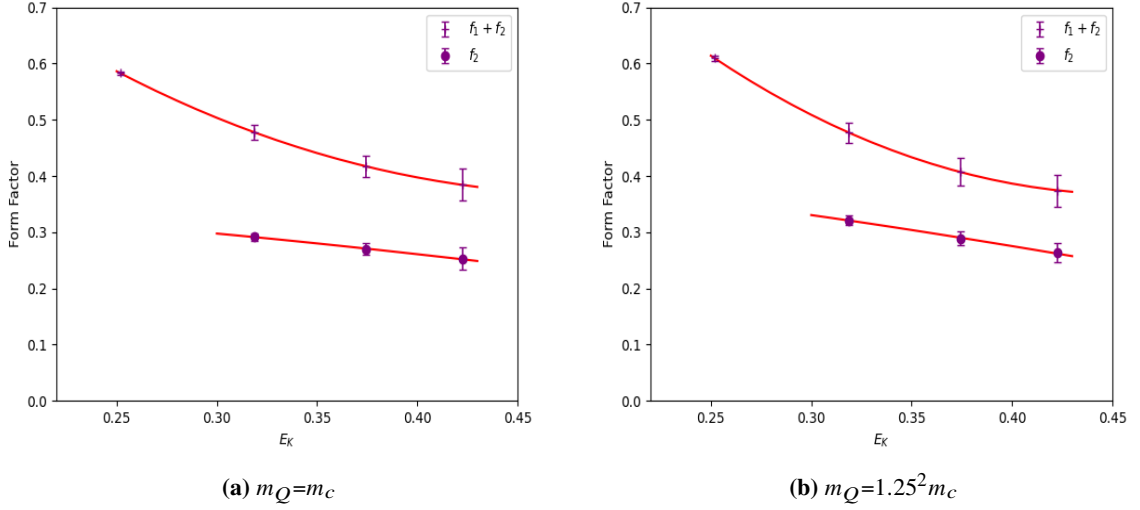


Figure 4: Form factors $f_1 + f_2$ (pluses) and f_2 (circles) as a function of kaon energy E_K . The left and right panels show results at $m_Q = m_c$ and $1.25^2 m_c$, respectively. The red lines are fit curves polynomial in E_K .

6. Summary and Outlook

We report on JLQCD’s on-going study of the $B_s \rightarrow K\ell\nu$ decay with the Möbius domain-wall bottom quarks. We present preliminary results at $a^{-1} \sim 2.5$ GeV and $M_\pi \sim 500$ MeV with bottom quark masses taken up to $0.7a^{-1}$ in order to control discretization effects. The ground state saturation has been carefully confirmed by simulating five values of the source-sink separation and testing two correlator ratios to extract the form factors. The statistical accuracy is improved by averaging relevant correlators over four source timeslices, and turns out to be better than our previous study of $B \rightarrow \pi\ell\nu$ using the same gauge ensemble. The extension of this study to pion masses down to $M_\pi \gtrsim 230$ MeV and two larger lattice cutoffs $a^{-1} \lesssim 4.5$ GeV is on-going.

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