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$|V_{us}|$ from kaon semileptonic form factor in $N_f = 2+1$ QCD at the physical point on (10 fm)^4

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We present a preliminary result of the kaon semileptonic form factor calculated at the smallest lattice spacing in the PACS10 configurations, whose physical volumes are more than (10 fm)^4 at the physical point. The configurations were generated using the Iwasaki gauge action and $N_f = 2+1$ stout-smeared nonperturbatively $O(a)$ improved Wilson quark action at the three lattice spacings, 0.085, 0.063, and 0.041 fm. The value of $|V_{us}|$ in the continuum limit is estimated from our results including the preliminary one. We compare our result of $|V_{us}|$ with the previous results and those through the kaon leptonic decay.

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1. Introduction

The form factor of the kaon semileptonic ($K_{\ell 3}$) decay plays an important role in determining $|V_{us}|$, which is one of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, through the $K_{\ell 3}$ decay process. The current values of the CKM matrix elements in the first row including $|V_{us}|$ give 2.2 σ violation of the CKM unitarity [1]. It could suggest the existence of physics beyond the standard model. To clarify the violation, more accurate values of the matrix elements are necessary.

So far in the various lattice QCD calculations [2–11], precise values of the $K_{\ell 3}$ form factor were reported. We also calculated the form factor [12, 13] using the $N_f = 2+1$ PACS10 configurations [14, 15], where the quark masses are tuned to be the physical ones on large spacetime volumes of more than (10 fm)^4. In this report, we present updates of those calculations, which are preliminary results of the form factor with the third PACS10 configuration at a finer lattice spacing, and also the estimated value of $|V_{us}|$ using the result.

2. Results

2.1 $K_{\ell 3}$ form factors

The simulation parameters for the three ensembles of the PACS10 configuration are tabulated in Table 1. All the configurations were generated using the Iwasaki gauge action [16] and a non-perturbative $O(a)$-improved Wilson quark action with the six-stout-smeared link [17].

The same quark action is employed in the measurements for the two- and three-point functions for the $K_{\ell 3}$ form factors. We use the random source operator spread in the spin, color, and spatial spaces proposed in Ref. [18]. The periodic boundary condition is imposed in the spatial directions in the calculation of the correlation functions, while in the temporal direction the periodic and anti-periodic boundary conditions are employed. The average of the two-point correlation functions with the different boundary conditions makes the periodicity of the temporal direction effectively doubled. Furthermore, a similar average of the three-point functions suppresses the wrapping around effect [19] in the small momentum region [12, 13].

The three-point functions are calculated using the local weak vector current and also using the conserved vector one to investigate the lattice spacing dependence of the form factors. We utilize the renormalization factor of the local vector current given by $Z_V = 1/\sqrt{F_\pi^{\rm bare}(0) F_K^{\rm bare}(0)}$, where

Table 1: Simulation parameters of the PACS10 configurations at the three lattice spacings. The bare coupling (β), lattice size ($L^3 \cdot T$), physical spatial extent (L[fm]), lattice spacing (a[fm]), the number of the configurations ($N_{\rm conf}$), and pion and kaon masses (m_π, m_K) are tabulated.

β	$L^3 \cdot T$	L[fm]	a[fm]	$N_{\rm conf}$	m_π[MeV]	m_K[MeV]
2.20	256^4	10.5	0.041	20	142	514
2.00	160^4	10.1	0.063	20	138	505
1.82	128^4	10.9	0.085	20	135	497

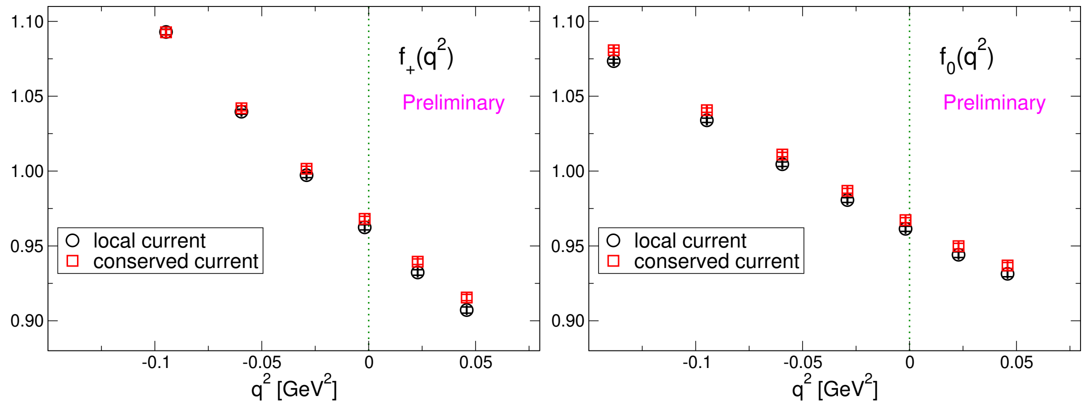

Figure 1: Preliminary results for the $K_{\ell 3}$ form factors, $f_+(q^2)$ (left) and $f_0(q^2)$ (right), as a function of q^2 at $a = 0.041$ fm. The circle and square symbols denote the data using the local and conserved vector currents, respectively.

$F_H^{\rm bare}(0)$ for $H = \pi, K$ is the bare electromagnetic form factor evaluated using the local vector current at the zero momentum transfer squared, $q^2 = 0$.

From the three-point function in each q^2, the $K_{\ell 3}$ decay matrix element is extracted, which is expressed by the two form factors, $f_+(q^2)$ and $f_-(q^2)$, given by

$$\langle \pi(\vec{p}_\pi) \left| V_\mu \right| K(\vec{p}_K) \rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2), \tag{1}$$

where V_μ is the weak vector current. Using the two form factors, another form factor $f_0(q^2)$ is defined as,

$$f_0(q^2) = f_+(q^2) + \frac{-q^2}{m_K^2 - m_\pi^2} f_-(q^2). \tag{2}$$

The preliminary results for the $K_{\ell 3}$ form factors, $f_+(q^2)$ and $f_0(q^2)$, at $a = 0.041$ fm are presented in Fig. 1. The figure shows that the data from the two vector currents have clear signals in both the form factors, and the data near $q^2 = 0$ can be computed thanks to the large volume of the PACS10 configuration even with the periodic boundary condition in the spatial directions. The conserved current data are systematically larger than the local ones in almost all the q^2 regions. We, however, observe that the discrepancy between the conserved and local data becomes smaller than those at the coarser lattice spacings [13].

The preliminary results for the two form factors with the local current are compared with the data at the different lattice spacings in Fig. 2. Although there are tiny discrepancies among the data at the three different lattice spacings in the smaller and larger q^2 regions, the data near $q^2 = 0$ can be expressed by a monotonic function of q^2. It suggests that the local current data seems to have a smaller lattice spacing effect. On the other hand, it is observed that the conserved current data have larger lattice spacing dependence than those in the local data.

2.2 $f_+(0)$ in the continuum limit and $|V_{us}|$

A q^2 interpolation is carried out to obtain the value of $f_+(0)$, which is essential to determine the value of $|V_{us}|$ through the $K_{\ell 3}$ decay. In the interpolation we employ fit functions for $f_+(q^2)$ and

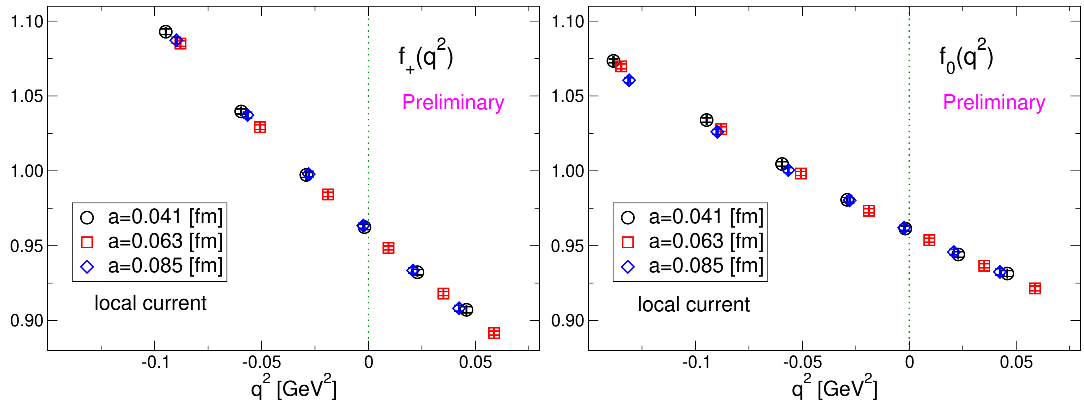

Figure 2: Lattice spacing dependences for $f_+(q^2)$ (left) and $f_0(q^2)$ (right) as a function of q^2 with the local vector current. The data at $a = 0.041$ fm are preliminary, while other two data are presented in Ref. [13].

$f_0(q^2)$ based on the next-to-leading order formulas in the SU(3) chiral perturbation theory [20, 21] with some additional terms and a constraint of $f_+(0) = f_0(0)$. The fit formulas are given in Ref. [12]. Figure 3 presents that the fit in each current data at $a = 0.041$ fm works well. The values of $f_+(0)$ are obtained from the fits of each current data. As shown in Table 1, the values for m_π and m_k are slightly different from the physical ones. A short chiral extrapolation of $f_+(0)$ to the physical meson masses is carried out using the same formulas as the fit forms.

The lattice spacing dependence of $f_+(0)$ is plotted in Fig. 4. The magenta symbols are preliminary results at $a = 0.041$ fm obtained from the above fit. The other data were determined with the same q^2 interpolations at each lattice spacing in Ref. [12, 13]. As discussed in the last subsection, the local current data has an almost flat behavior against the lattice spacing, while the conserved current data has a clear slope. In order to estimate $f_+(0)$ in the continuum limit, we investigate three fit forms using the two current data. In the fit we assume the results from the two currents coincide in the continuum limit. The one fit form utilizes constant and linear functions of a for the local and conserved vector current data, respectively, whose result is denoted by Fit A in Fig. 4. Fit B and C in the figure adopt quadratic and linear functions of a, respectively, for both the current data. The two results obtained from Fit A and B differ beyond their statistical errors, while the result from Fit C covers the other two fit results within its error. Thus, we take the result from Fit C as our preliminary result in this calculation. The result is consistent with our previous result estimated from the two coarser lattice spacing data [13].

In the left panel of Fig. 5, our preliminary result of $f_+(0)$ is compared with the previous calculations in Refs. [2–11] including our previous one [13]. Our result is reasonably consistent with the previous results within about $2\,\sigma$, although systematic errors in our result are not estimated yet.

Combining our $f_+(0)$ and the experimental value, $|V_{us}|f_+(0) = 0.21654(41)$ [22], the value of $|V_{us}|$ is estimated. Our result is plotted in the right panel of Fig. 5 together with the previous results [6, 8–11, 13]. Again, our result reasonably agrees with the previous ones determined from $f_+(0)$ through the $K_{\ell 3}$ decay process. The figure also shows that our preliminary result is consistent with $|V_{us}|$ obtained from the $K_{\ell 2}$ decay using $|V_{us}|F_K/|V_{ud}|F_\pi = 0.27599(37)$ [22] and the ratio of the decay constants F_K/F_π. In the figure, we plot $|V_{us}|$ using F_K/F_π in PDG22 [1] and also our

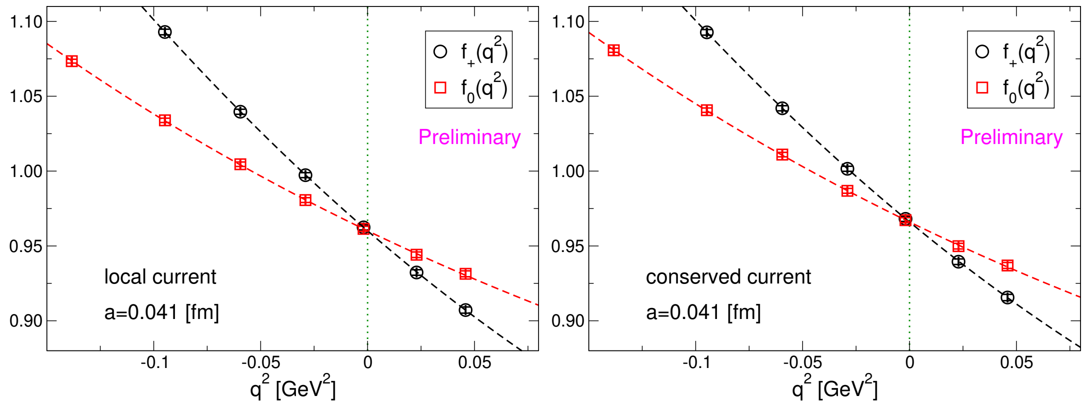

Figure 3: Fit results for the form factors using the local (left) and conserved (right) vector currents at $a = 0.041$ fm expressed by dashed curves. The circle and diamond symbols represent $f_+(q^2)$ and $f_0(q^2)$, respectively.

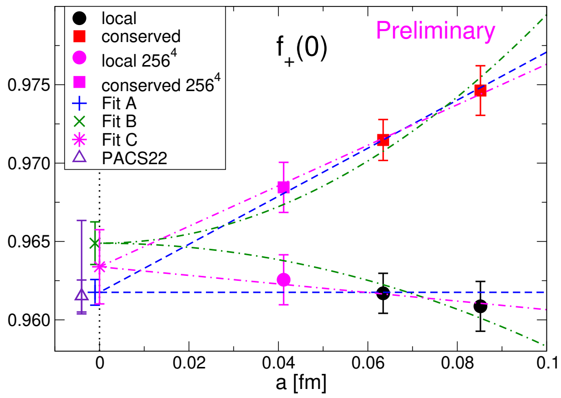

Figure 4: Lattice spacing dependence of $f_+(0)$. The circle and square symbols represent the data with the local and conserved vector currents, respectively. Different curves correspond to fit results using different fit forms for the continuum extrapolation. The triangle symbol denotes our previous result in the continuum limit [13].

preliminary result of F_K/F_π in the continuum limit calculated with the PACS10 configurations. In contrast to the consistencies, our preliminary result differs from the estimated value of $|V_{us}|$ through the unitarity of the CKM matrix using $|V_{ud}|$ by 2–3 σ depending on the error of $|V_{ud}|$ [23, 24]. To clarify the discrepancy, it is important to decrease the uncertainties in the lattice QCD calculations and also the ones in the $|V_{ud}|$ determinations.

3. Summary

We have presented the preliminary result of the $K_{\ell 3}$ form factors calculated with the PACS10 configuration at the lattice spacing of $a = 0.041$ fm, where the pion and kaon masses are physical ones and the spatial extent is more than 10 fm. The form factors are calculated using the local and conserved vector currents. We have observed they have different lattice spacing dependence. Using the results calculated on the other two PACS10 configurations at the coarser lattice spacings, we

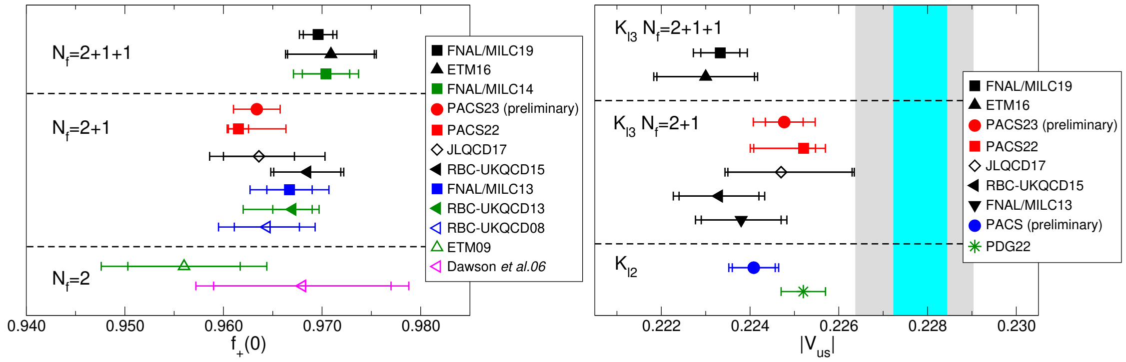

Figure 5: Left : Comparison of our preliminary result of $f_+(0)$ with the previous results [2–11, 13]. The inner and outer errors express the statistical and total errors. The total error is evaluated by adding the statistical and systematic errors in quadrature. **Right :** Comparison of $|V_{us}|$ using our preliminary result of $f_+(0)$ with the previous results [6, 8–11, 13]. $|V_{us}|$ determined from the $K_{\ell 2}$ decay are also plotted using F_K/F_π of our preliminary result and PDG22 [1]. The closed (open) symbols represent results in the continuum limit (at a finite lattice spacing). The inner and outer errors express the lattice QCD and total errors. The total error is evaluated by adding the errors in the lattice QCD and experiment in quadrature. The value of $|V_{us}|$ determined from the unitarity of the CKM matrix using $|V_{ud}|$ in Refs. [23] and [24] are presented by the light blue and gray bands, respectively.

have obtained the preliminary result of $f_+(0)$ in the continuum limit. The value of $|V_{us}|$ estimated using our preliminary $f_+(0)$ reasonably agrees with the previous results determined through the $K_{\ell 3}$ decay and also the ones through the $K_{\ell 2}$ decay. On the other hand, our preliminary result differs from the ones through the CKM unitarity using $|V_{ud}|$ by 2–3 σ. In future to finalize our calculation, we will estimate systematic errors of $f_+(0)$, *e.g.*, fit forms for a q^2 interpolations and continuum extrapolations, isospin breaking effects, and a dynamical charm quark effect. Furthermore, we plan to evaluate the phase space integral of the $K_{\ell 3}$ decay from our form factors as in our previous work.

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