

Renormalization of non-local gluon operators in lattice perturbation theory

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In this study, we investigate the renormalization of a complete set of gauge-invariant non-local gluon operators up to one-loop in lattice perturbation theory. Our computations have been performed in both dimensional and lattice regularizations, using the Wilson gluon action, leading to the renormalization functions in the modified Minimal Subtraction ($\overline{\text{MS}}$) scheme, as well as conversion factors from the modified regularization invariant (RI') scheme to $\overline{\text{MS}}$.

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1. Introduction

Parton distribution functions (PDFs) provide insights into the internal composition of hadrons, involving quarks and gluons. PDFs quantify the probability of finding partons with a specific momentum fraction and spin in the infinite momentum frame. They are defined as expectation values of light-cone correlation functions in hadronic states, making it unfeasible to directly compute them on a Euclidean lattice through lattice QCD calculations.

However, over the last decade, several methods have contributed to significant progress in the direct calculation of PDFs through lattice QCD. For a recent summary of these approaches, refer to [1]. One noteworthy and commonly used approach is the quasi-distribution method, which utilizes the large momentum effective theory (LaMET) [2, 3]. Instead of computing light-cone correlation functions, this method calculates quasi-PDFs, which are defined as the matrix elements of momentum-boosted hadrons coupled to gauge-invariant non-local operators that include a finite-length Wilson line. This quasi-observable, which is usually hadron-momentum-dependent but time-independent, can be calculated on the lattice and then renormalized nonperturbatively in an appropriate scheme. Finally, the renormalized quasi-PDF is matched to the PDF through a factorization formula [4].

The gluonic contribution of hadron composition has been overlooked compared to quark counterparts, despite playing a crucial role in various physical quantities. Phenomenological data suggest that gluons contribute approximately 40% of the hadron's momentum at a scale of 6.25 GeV^2 [5]. Accurate calculations of the gluon dependent quantities are essential for J/ψ photo production, cross-section of Higgs boson production and jet production, as well as for providing theoretical input to the upcoming Electron-Ion Collider .

A crucial aspect in the direct calculation of PDFs from lattice QCD is the nonperturbative renormalization of the quasi-PDFs. As regards quark quasi-PDFs, in Ref. [6] two important features of the Wilson-line operator matrix elements were revealed on the lattice: linear divergences in addition to logarithmic divergences, and mixing among certain subsets of the original operators during renormalization. Efforts to eliminate these linear divergences have been made using various methods, however a complete nonperturbative renormalization program has only recently been developed. Similar effects are expected to be present in the renormalization of non-local gluon operators as well. A recent study [7], using the auxiliary field approach, showed that different components of non-local gluon operators have nontrivial renormalization patterns, making it challenging to evaluate accurately gluon quasi-PDFs. In addition, four gluon operators have been identified as multiplicatively renormalizable, making them suitable for defining some of the gluon quasi-PDFs.

In these proceedings, we outline the methodology for computing the renormalization of gauge-invariant non-local gluon operators in lattice perturbation theory. We calculate the renormalization functions of non-local gluon operators in the $\overline{\text{MS}}$ scheme to one loop, employing dimensional and lattice regularization, ultimately resulting in the derivation of the relevant mixing matrices. We also compute the conversion factors which are necessary in relating the renormalization functions stemming from a non-perturbative lattice scheme to $\overline{\text{MS}}$.

2. Formulation

2.1 Lattice action

We consider a non-abelian gauge theory of $SU(N_c)$ group and N_f fermions. To simplify our calculations, we employ the Wilson plaquette gauge action for gluons:

$$S_G = \frac{2}{g_0^2} \sum_{\text{plaq.}} \text{Re Tr} \{1 - U_{\text{plaq.}}\} \quad (1)$$

where

$$U_{\text{plaq.}} = U_\mu(x)U_\nu(x + \hat{\mu})U_\mu^\dagger(x + \hat{\nu})U_\nu^\dagger(x) \quad (2)$$

We expect that improved gauge actions, such as the Symanzik improved action, or the implementation of stout-smear links, will not have an impact on determining the mixing pattern under renormalization of the non-local operators.

2.2 Definition of operators

The non-local gluon operators under study are defined in the fundamental representation as:

$$O_{\mu\nu\rho\sigma}(z\hat{\tau}, 0) \equiv 2 \text{Tr} \left(F_{\mu\nu}(z\hat{\tau}) W(z\hat{\tau}, 0) F_{\rho\sigma}(0) W(0, z\hat{\tau}) \right) \quad (3)$$

where $F_{\mu\nu}$ is the gluon field strength tensor and $W(x, x + z\hat{\tau})$ is the path-ordered (\mathcal{P}) straight Wilson line inserted for gauge invariance:

$$W(x, x + z\hat{\tau}) = \mathcal{P} e^{ig_0 \int_0^z A_\mu(x + \zeta \hat{\tau}) d\zeta} \quad (4)$$

Without loss of generality, the Wilson line is chosen to lie along the z direction.

Due to anti-symmetry of $F_{\mu\nu}$, for a fixed choice of the Wilson line there are 36 non-local operators in total by selecting the indices of $O_{\mu\nu\rho\sigma}$ to be in any direction. However, only gluon operators that exhibit multiplicative renormalizability are appropriate for defining the gluon quasi-PDF [4]. Suitable candidates for the unpolarized gluon quasi-PDF can be provided by [8]:

$$\tilde{f}_{g/H}^{(n)}(x, P^z) = \mathcal{N}^{(n)} \int \frac{dz}{2\pi x P^z} e^{izxP^z} \langle H(P) | O^{(n)}(z, 0) | H(P) \rangle \quad (5)$$

where $\mathcal{N}^{(n)}$ is a renormalization factor, x is the longitudinal momentum fraction carried by the gluon, $P^\mu = (P^0, 0, 0, P^z)$ is the hadron momentum, and $H(P)$ is the momentum boosted hadron states. Potential candidates for the gluon operator are denoted here as $O^{(n)}(z, 0)$.

3. Perturbative Calculation - Results

To study the renormalization of the non-local gluon operators, we choose, for convenience, to calculate the following Green's functions:

$$\Lambda_O = \langle A_{v_1}^{\alpha_1}(q_1) A_{v_2}^{\alpha_2}(q_2) O_{\mu\nu\rho\sigma} \rangle \quad (6)$$

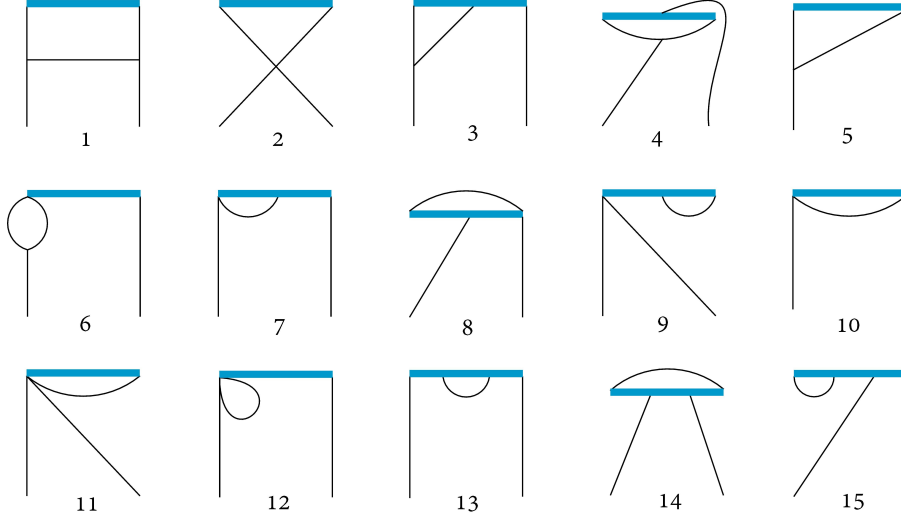


Figure 1: Feynman diagrams contributing to the one-loop calculation of the Green's functions of the non-local operators. Solid lines represent gluons. The operator insertion is denoted by a solid box.

where operator $O_{\mu\nu\rho\sigma}$ is defined by Eq. (3), and $A_{\nu_1}^{\alpha_1}(q_1)$, $A_{\nu_2}^{\alpha_2}(q_2)$ are two external gluons. The main task in this work is the evaluation of the above bare amputated Green's functions up to one-loop using dimensional and lattice regularization in the the modified minimal subtraction ($\overline{\text{MS}}$) scheme. The Feynman diagrams of $\Lambda_O^{1\text{-loop}}$ are shown in Fig. (1).

We define the renormalization factors which relate the bare quantities with the renormalized counterparts through:

$$O_{\mu\nu\rho\sigma}^R = Z_{O_{\mu\nu\rho\sigma}}^{-1} O_{\mu\nu\rho\sigma} \quad , \quad A_\mu^R = Z_A^{-1/2} A_\mu \quad (7)$$

where A_μ (A_μ^R) is the bare (renormalized) gluon field. In the presence of operator mixing, the relationship between the bare and the renormalized operators is appropriately generalised.

The corresponding renormalized amputated Green's functions are expressed as (for simplicity, we omit Lorentz indices whenever they can be understood from the context):

$$\Lambda_O^R = Z_A Z_O^{-1} \Lambda_O \quad (8)$$

3.1 Dimensional Regularization

The computations are performed in D -dimensional Euclidean spacetime, where $D = 4 - 2\epsilon$, and ϵ is the regularization parameter. In contrast to 2-point Green's functions involving local operators, the integration results become significantly more complicated due to the presence of both the external momentum and the length of the Wilson line in the integrands. Additionally, there is a nontrivial dependence on the preferred direction of the Wilson line, leading to further complexity. We apply new techniques, similar to [9], for assessing one-loop tensor integrals with an exponential factor in D dimensions. For the elimination of the poles in ϵ , we adopt the $\overline{\text{MS}}$ scheme.

3.1.1 Renormalizations Functions

We start by considering the amputated tree-level Green's functions, as presented in Eq. (6), which read:

$$\begin{aligned} \Lambda_O^{\text{tree}} = \delta^{\alpha_1 \alpha_2} & \left(+ q_{1\mu} q_{2\rho} \delta_{\nu_1 \nu} \delta_{\nu_2 \sigma} e^{-izq_1 \hat{\tau}} + q_{1\mu} q_{2\rho} \delta_{\nu_1 \sigma} \delta_{\nu_2 \nu} e^{izq_1 \hat{\tau}} \right. \\ & - q_{1\nu} q_{2\rho} \delta_{\nu_1 \mu} \delta_{\nu_2 \sigma} e^{-izq_1 \hat{\tau}} - q_{1\nu} q_{2\rho} \delta_{\nu_1 \sigma} \delta_{\nu_2 \mu} e^{izq_1 \hat{\tau}} \\ & - q_{1\mu} q_{2\sigma} \delta_{\nu_1 \nu} \delta_{\nu_2 \rho} e^{-izq_1 \hat{\tau}} - q_{1\mu} q_{2\sigma} \delta_{\nu_1 \rho} \delta_{\nu_2 \nu} e^{izq_1 \hat{\tau}} \\ & \left. + q_{1\nu} q_{2\sigma} \delta_{\nu_1 \mu} \delta_{\nu_2 \rho} e^{-izq_1 \hat{\tau}} + q_{1\nu} q_{2\sigma} \delta_{\nu_1 \rho} \delta_{\nu_2 \mu} e^{izq_1 \hat{\tau}} \right) \end{aligned} \quad (9)$$

where z and $\hat{\tau}$ is the length and direction of the Wilson line. Notice that the above expression is antisymmetric in $\{\mu, \nu\}$ and $\{\rho, \sigma\}$ as expected.

Subsequently, we proceed to the 1-loop calculations. We find that only diagrams 3, 6 and 13, contribute to the $1/\epsilon$ terms, and therefore the renormalization function of the operators in $\overline{\text{MS}}$ is not affected by the remaining diagrams. However, they contribute to the renormalized Green's functions. Below we present the $\mathcal{O}(1/\epsilon)$ contributions of the perturbative calculation:

$$\Lambda_O^{1\text{-loop}}|_{\mathcal{O}(1/\epsilon)} = \frac{g^2 N_c}{16\epsilon\pi^2} \left(\delta_{\mu\hat{\tau}} + \delta_{\nu\hat{\tau}} + \delta_{\rho\hat{\tau}} + \delta_{\sigma\hat{\tau}} - \frac{1}{2}\beta \right) \Lambda_O^{\text{tree}} \quad (10)$$

where β , the gauge fixing parameter, is defined such that $\beta = 0(1)$ corresponds to the Feynman (Landau) gauge. The computation was carried out in an arbitrary covariant gauge, allowing for a direct verification of the gauge invariance of the renormalization functions. It should be noted that, at the one-loop level in dimensional regularization (DR), the pole terms are proportional to the tree-level values for each one of the operators, indicating no mixing with operators of equal or lower dimension.

The renormalization factor of the gluon field in DR is given by:

$$Z_A^{\text{DR}, \overline{\text{MS}}} = 1 + \frac{g^2}{16\epsilon\pi^2} \left(\frac{13N_c}{6} - \frac{N_c}{2} (1 - \beta) - \frac{2}{3}N_f \right) \quad (11)$$

Using the $\overline{\text{MS}}$ condition and Eqs. (8), (10), and (11), we find the renormalization function of the operators:

$$Z_O^{\text{DR}, \overline{\text{MS}}} = 1 + \frac{g^2}{16\epsilon\pi^2} \left(\left(\frac{5}{3} + \delta_{\mu\hat{\tau}} + \delta_{\nu\hat{\tau}} + \delta_{\rho\hat{\tau}} + \delta_{\sigma\hat{\tau}} \right) N_c - \frac{2}{3}N_f \right) \quad (12)$$

We observe that this result depends on the choice of indices for the operators, specifically whether they align with the direction of the Wilson line or not.

As expected from gauge invariance in $\overline{\text{MS}}$, the β dependence disappears in the renormalization function of the operators, upon taking into account the gluon field renormalization function. Gauge invariance cannot be ensured in all schemes due to the presence of gauge-dependent renormalized external fields in the Green's functions.

It is worth mentioning that the renormalization function of the operators is independent of the length of the Wilson line (z). There is no dimensionless factor dependent on z that could emerge in the pole part, because the leading pole at each loop cannot depend on external momenta or the renormalization scale. Consequently, z independence is expected to persist at all orders in perturbation theory.

The $\overline{\text{MS}}$ -renormalized Green's functions, which equal the finite part of Λ_O , are very lengthy expressions involving integrals over Bessel functions; their explicit expression will appear in our forthcoming [10]. They are the fundamental ingredient for the construction of the conversion factors between non-perturbatively applicable renormalization schemes and $\overline{\text{MS}}$.

3.2 Lattice Regularization

We now focus on computing the bare Green's functions, as given by Eq. (6), using lattice regularization. The tree-level Green's functions yield the same result as in dimensional regularization (DR), shown in Eq. (9).

Following that, we move on to the 1-loop calculations. This computation is considerably more complicated than dimensional regularization due to the subtleties involved in extracting divergences from lattice integrals. To begin, we write the lattice expressions in the form of a sum of continuum integrals and additional lattice corrections. Noteworthy, these additional terms, although they have a simple quadratic dependence on the external momentum q , are expected to have a nontrivial dependence on z as seen in non-local fermion operators [6].

Several diagrams (d1,d2,d4,d5,d8,d10, and d14) give precisely the same contributions as in DR. This aligns with expectations, considering that these contributions are finite as $\epsilon \rightarrow 0$. Consequently, the limit $a \rightarrow 0$ can be applied right from the beginning, without inducing any lattice corrections. However, we must ensure that we eliminate the overall factor of $1/a^2$, attributed to the presence of the external gluons in the Green's functions, by extracting two powers of the external momentum, (aq) .

We present the one-loop expression for diagram 13, as seen in Fig. (1), in lattice regularization:

$$\Lambda_O^{\text{d13}} = \frac{g^2 N_c}{16\epsilon\pi^2} \left(c_1 + c_2 \beta - c_3 \frac{|z|}{a} + 8 \log \frac{|z|}{a} (2 + \beta) \right) \Lambda_O^{\text{tree}} \quad (13)$$

where $c_1 = 32.2481(2 \pm 2)$, $c_2 = 14.2405(9 \pm 4)$, and $c_3 = 79.8193(6 \pm 8)$. Note here the presence of both linear divergence and logarithmic divergence in a , features revealed in the non-local fermion operators as well.

The non-local gluon operators exhibit a rotational symmetry of the octahedral point group with respect to the three transverse directions to the preferred direction of the Wilson line. Considering also parity arguments, given an appropriate choice of a renormalization scheme, there is no requirement of mixing among operators that do not share the same transformations under these symmetries. In particular, we find that mixing is possible only between pairs of operators. An example of such a pair is represented by $\left[\frac{1}{2} (O_{z_1 z_2} + O_{z_2 z_1}), \frac{1}{2} (O_{4142} + O_{4241}) \right]$, where the transverse directions to the Wilson line, z direction, are denoted by 1, 2, and 4.

Our findings of the possible mixing pairs due to symmetries, along with the one-loop bare and renormalized Green's functions in lattice regularization will be included in [10].

4. Summary

In this work, we obtain the renormalization functions of non-local gluon operators in $\overline{\text{MS}}$ scheme using both dimensional and lattice regularizations. In addition, using the one-loop result of the renormalized functions of the operators, we find the conversion factors between the RI' and

$\overline{\text{MS}}$ scheme. This will enable to convert the corresponding lattice non-perturbative results to the $\overline{\text{MS}}$ scheme. Furthermore, we find that rotational and parity symmetry allow a finite mixing among operators in pairs. These results can help the non-perturbative studies of the gluon quasi-PDF calculation.

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