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Parton Distributions from Boosted Fields in the Coulomb Gauge

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We propose a new method to calculate parton distribution functions (PDFs) from lattice correlations of boosted quarks and gluons in the Coulomb gauge, within the framework of Large Momentum Effective Theory. Compared to the widely used gauge-invariant Wilson-line operators, these correlations greatly simplify the renormalization thanks to the absence of linear power divergence. Besides, they enable access to larger off-axis momenta under preserved 3D rotational symmetry, as well as enhanced long-range precision that facilitates the Fourier transform. We verify the factorization formula that relates this new observable to the quark PDF at one-loop order in perturbation theory. Moreover, through a lattice calculation of the pion valence quark PDF, we demonstrate the aforementioned advantage and features of the Coulomb gauge correlation and show that it yields consistent result with the gauge-invariant method. This opens the door to a more efficient way to calculate parton physics on the lattice.

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Recent years have seen significant development in the lattice QCD calculation of parton distribution functions. One of the most widely used methods is the *large-momentum effective theory* (LaMET) [1–3], which was proposed a decade ago. In this approach, one starts from the quasi-PDF (qPDF) defined as Fourier transform of an equal-time correlation at large proton momentum, and relates it to the PDF through power expansion and effective theory matching [4].

At the core of LaMET is the simulation of nonlocal bilinear operators such as $O_\Gamma(z) \equiv \bar\psi(z)\Gamma W(z,0)\psi(0)$, where Γ is a Dirac matrix, and $W(z,0)$ is a spacelike Wilson line that connects 0 to $z^\mu = (0,\vec z)$ to make $O_\Gamma(z)$ gauge invariant. By construction $O_\Gamma(z)$ must approach the light-cone $t + |\vec z| = 0$ under a Lorentz boost along the $\vec z$-direction, which can be achieved on the lattice by simulating a boosted hadron. One major challenge here is to reach large momentum which controls the power accuracy. To ensure a smooth Wilson line both $\vec z$ and the momentum $\vec p$ must be along one spatial axis, which leaves out all the off-axis directions that can be used to reach higher momenta. Another important issue is the renormalization of $O_\Gamma(z,a)$ under lattice regularization with spacing a, as it includes a linear power divergence $\propto \exp(-\delta m(a)|\vec z|)$ with $\delta m(a) \sim 1/a$, which originates from the Wilson-line self-energy. In order to calculate the x-dependence of PDFs, such a divergence must be subtracted at all $\vec z$ [5], and a nontrivial matching onto the $\overline{\rm MS}$ scheme [6, 7] is required to cancel the associated renormalon $\exp(-m_0|\vec z|)$ with $m_0 \sim \Lambda_{\rm QCD}$ [8].

In this work we propose to calculate the PDFs from pure quark and gluon correlations in the Coulomb gauge (CG), within the framework of LaMET. Without the Wilson line, the CG correlation is free from the linear divergence and renormalon, which greatly simplifies the renormalization. Besides, the computation and storage cost for Wilson lines can be reduced in lattice simulations, and one can reach larger off-axis momenta by taking advantage of the 3D rotational symmetry of CG. Moreover, since the renormalization factor is independent of z, the exponential decaying bare correlation at large $|\vec z|$ is unaffected, which will enhance the precision and facilitate the Fourier transform. At last, one can do the momentum smearing [9, 10] in the CG and compute both GI and CG qPDFs simultaneously, as they share the same quark propagators.

The CG quark qPDF is defined as

$$\tilde f(x,P^z,\mu) = P^z\int_{-\infty}^{\infty}\frac{dz}{2\pi}e^{ixP^z z}\tilde h(z,P^z,\mu)\,,\qquad \tilde h(z,P^z,\mu) = \frac{1}{2P^t}\langle P|\bar\psi(z)\gamma^t\psi(0)\Big|_{\vec\nabla\cdot\vec A=0}|P\rangle\,, \tag{1}$$

where $z^\mu = (0,0,0,z)$, $|P\rangle$ is a hadron state with $P^\mu = (P^t,0,0,P^z)$ normalized to $\langle P|P\rangle = 2P^t\delta^{(3)}(0)$, and μ is the $\overline{\rm MS}$ scale. The GI qPDF follows a similar definition except that the quark correlator is replaced with $O_{\gamma^t}(z)$. The CG condition $\vec\nabla\cdot\vec A = 0$ is fixed so that the quark correlation can have a nonvanishing matrix element.

Meanwhile, the quark PDF $f(x,\mu)$ is defined as

$$f(x,\mu) = \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda x}h(\lambda,\mu)\,,\qquad h(\lambda,\mu) = \frac{1}{2P^+}\langle P|\bar\psi(\xi^-)W(\xi^-,0)\gamma^+\psi(0)|P\rangle\,, \tag{2}$$

where $\lambda = P^+\xi^-$ and $\xi^- = (t-z)/\sqrt2$. Under an infinite Lorentz boost, the CG reduces to the light-cone gauge $A^+ = (A^t + A^z)/\sqrt2 = 0$ with a proper boundary condition, where $W(\xi^-,0) = P\exp\left[-ig\int_0^{\xi^-} d\eta^-\, A^+(\eta^-)\right]$ vanishes, so the qPDF becomes equivalent to the PDF.

$\lvert\vec{p}\rvert$ (GeV)	$\vec{n}$	$\vec{k}$	t_s/a	(#ex,#sl)
0	(0,0,0)	(0,0,0)	8,10,12	(1, 16)
1.72	(0,0,4)	(0,0,3)	8	(1, 32)
			10	(3, 96)
			12	(8, 256)
2.15	(0,0,5)	(0,0,3)	8	(2, 64)
			10	(8, 256)
			12	(8, 256)
2.24	(3,3,3)	(2,2,2)	8	(2, 64)
			10	(8, 256)
			12	(8, 256)

Table 1: Details of lattice setup, where $\vec{p} = (2\pi)/(L_s a)\vec{n}$, $\vec{k}$ is the momentum-smearing parameter [16], t_s is the source-sink separation, and (#ex,#sl) are the numbers of exact and sloppy inversions per configuration.

According to LaMET [3], when $P^z \gg \Lambda_{\rm QCD}$ the CG qPDF can be perturbatively matched onto the PDF through a factorization formula [11],

$$\tilde{f}(x, P^z, \mu) = \int \frac{dy}{|y|}\, C\Big(\frac{x}{y}, \frac{\mu}{|y|P^z}\Big) f(y, \mu) + O\Big(\frac{\Lambda^2_{\rm QCD}}{x^2 P_z^2}, \frac{\Lambda^2_{\rm QCD}}{(1-x)^2 P_z^2}\Big), \tag{3}$$

where C is the matching coefficient. By calculating the NLO corrections to the quark CG qPDF and PDF in a free quark state, we find out that their collinear divergences are identical [12], which confirms Eq. (3) at the same order. With a double Fourier transform of Eq. (3) [11], we also derive a short-distance factorization:

$$\tilde{h}(z, P^z, \mu) = \int du\, C(u, z^2\mu^2) h(u\tilde{\lambda}, \mu) + O(z^2\Lambda^2_{\rm QCD}). \tag{4}$$

To test the CG method, we calculate the pion valence quark PDF on a gauge ensemble in 2+1 flavor QCD generated by the HotQCD collaboration [13] with Highly Improved Staggered Quarks [14], where the lattice spacing $a = 0.06$ fm and volume $L_s^3 \times L_t = 48^3 \times 64$. We use tadpole-improved clover Wilson valence fermions on the hypercubic (HYP) smeared [15] gauge background, with a valence pion mass $m_\pi = 300$ MeV. To improve the signal of boosted pions at $\vec{p} = (2\pi)/(L_s a)\vec{n}$, we utilize the momentum-smeared [16] pion source with optimized quark boost $\vec{k}$ [9, 10]. We employ 109 gauge configurations and perform multiple exact and sloppy Dirac operator inversions on each of them using All Mode Averaging [17]. Since the quark propagators are the same, we calculate the GI qPDF with 1-step HYP-smeared Wilson lines and the CG qPDF during contraction at no additional cost. More details of the statistics are shown in Table 1.

For a 4D lattice of spatial volume V, we fix QCD in the CG by finding the gauge transformation Ω of link variables $U_i(t, \vec{x})$ that minimizes the criterion [18, 19]

$$F[U^\Omega] = \frac{1}{9V} \sum_{\vec{x}} \sum_{i=1,2,3} \big[-{\rm Re\,Tr}\, U_i^\Omega(t, \vec{x}) \big] \tag{5}$$

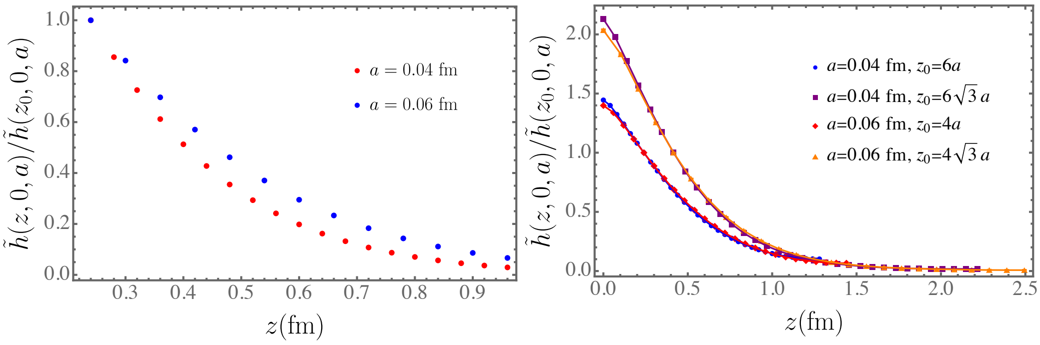

Figure 1: Comparison of CG ratios $\tilde{h}(z,0,a)/\tilde{h}(z_0,0,a)$ at $a = 0.04$ and 0.06 fm, with z_0 fixed in the physical unit. Left panel shows the GI matrix elements (with linear divergence), whereas the right panel shows the CG matrix elements. In the right panel, we choose $\vec{z}$ to be along $(0,0,1)$ for blue and red points and $(1,1,1)$ for purple and orange points. As one can see, the CG matrix elements is free from the linear divergence which makes the ratios at different a significantly different in the GI case.

per time slice. Each gauge fixing takes 600 sweeps and reaches a precision of $< 10^{-7}$. Though imprecise fixing and the presence of Gribov copies [20, 21] can affect gauge-variant correlations, they most likely contribute to the statistical noise with our algorithm [22]. In our simulation, increasing the statistics reduces the overall statistical error, suggesting that the Gribov noise is not important [23]. Besides, lattice studies of the SU(2) gluon propagator in the Landau gauge [24] and CG [25] show that Gribov copies only affect the far infrared region $\lesssim 0.2$ GeV, which implies that they should have negligible impact on the QCD PDF at $2x|\vec{p}| \gg 0.2$ GeV where LaMET is reliable [7]. Using an off-axis momentum $\vec{n} = (n_x, n_y, n_z)$, one can achieve the same $|\vec{n}|$ with less oscillatory modes $n_{x,y,z}$. Compared to $\vec{n} = (0,0,5)$, we observe in $\vec{n} = (3,3,3)$ about 20% increase in the signal-to-noise ratios of both two-point and three-point correlations at $t_s/a \leq 10$. Besides, we also find that 3D rotational symmetry is precisely maintained in the case of CG correlations, whereas it is broken to some extent in the GI case.

Since QCD has been proven renormalizable in CG [26–28] without linear divergence [29], renormalization of the quark correlator is simply multiplicative through the quark wave function renormalization factor. This has been verified for the quark propagator on the lattice [23]. For hadronic matrix elements, the ratio $\tilde{h}(z,0,a)/\tilde{h}(z_0,0,a)$ should have a continuum limit, which we verify with a finer HotQCD ensemble [10, 13, 30] with $a = 0.04$ fm and $L_s^3 \times L_t = 64^3 \times 64$, as shown in Fig. 1.

Then we do renormalization in the hybrid scheme [5],

$$\begin{aligned}\tilde{h}(z,z_s,P^z,\mu) &= N\big[\tilde{h}(z,P^z,a)/\tilde{h}(z,0,a)\big]\theta(z_s-|z|) \\ &\quad + Ne^{(\delta m+m_0)(|z|-z_s)}\big[\tilde{h}(z,P^z,a)/\tilde{h}(z_s,0,a)\big]\theta(|z|-z_s)\,,\end{aligned} \tag{6}$$

where $N = \tilde{h}(0,0,a)/\tilde{h}(0,P^z,a)$, and $z_s = 4a$ and $2\sqrt{3}a$ for on- and off-axis momenta, respectively. For the GI correlation, δm is the same as that in Ref. [30], and $m_0(\mu)$ is fitted with the leading-renormalon resummation (LRR) approach under large-β_0 approximation [6, 7]. A precise determination of δm and m_0 typically requires multiple fine lattice spacings [7, 30, 31]. In contrast, for the CG correlation $\delta m = m_0 = 0$, which does not need extra calculation, thus greatly simplifying the renormalization and eliminating related systematics. Fig. 2 compares the hybrid-scheme CG

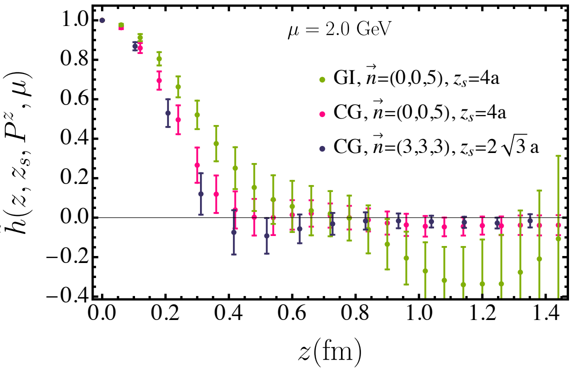

Figure 2: CG and GI correlations in the hybrid scheme at on-axis momentum 2.15 GeV with $\vec{n} = (0, 0, 5)$ and off-axis momentum 2.24 GeV with $\vec{n} = (3, 3, 3)$.

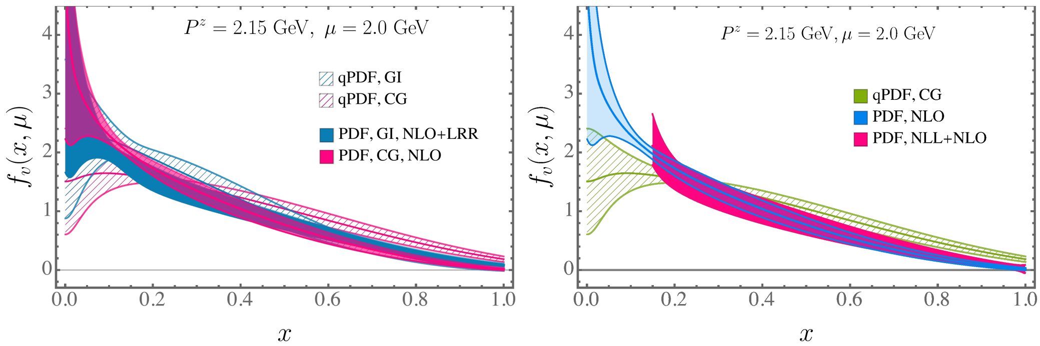

Figure 3: Left panel: comparison of the CG and GI qPDFs before and after matching at $P^z = 2.15$ GeV and $\mu = 2.0$ GeV. Right panel: comparison of NLO and NLL+NLO matching corrections. The error band only includes the statistical uncertainty.

and GI correlations. Both fall close to zero at large z, but the errors in the GI case are significantly larger due to the exponential enhancement by the subtraction of δm. Next, we Fourier transform the correlations to obtain the qPDFs. The discrete data are interpolated with a cubic polynomial, whose uncertainty is small compared to the other systematics. For the GI correlation, we extrapolate to $z = \infty$ with a physically motivated model $e^{-m|z|}/\tilde{\lambda}^d$ [30], which mainly affects the small-x region. Meanwhile, thanks to the simple renormalization, the extrapolation has much less impact on the CG qPDF as both the central value and error of the correlation remain small at large z.

Now we match the qPDFs to the PDF. Fig. 3 compares the CG and GI qPDFs before and after matching at NLO accuracy. Finally, we conclude the analysis of CG qPDFs by resumming the small-x logarithms through PDF evolution [32, 33]. Fig. 4 shows the results at on-axis and off-axis momenta $|\vec{p}| = 2.15$ and 2.24 GeV, respectively, which are compared to the recent global fits by `xFitter20` [34] and JAM21NLO [35]. The error has included scale variation, which is estimated by setting $\mu = 2\kappa x|\vec{p}|$ with $\kappa = \sqrt{2}, 1, 1/\sqrt{2}$ in the matching and evolving the results to $\mu = 2.0$ GeV at next-to-leading-logarithmic (NLL) order, whose effect is also demonstrated in Fig. 3. For $x > 0.2$, the lattice results agree with the global fits within errors.

In summary, we have proposed a new method to calculate the PDF from CG correlations within the LaMET framework. With an exploratory lattice calculation, we show that the CG correlation is

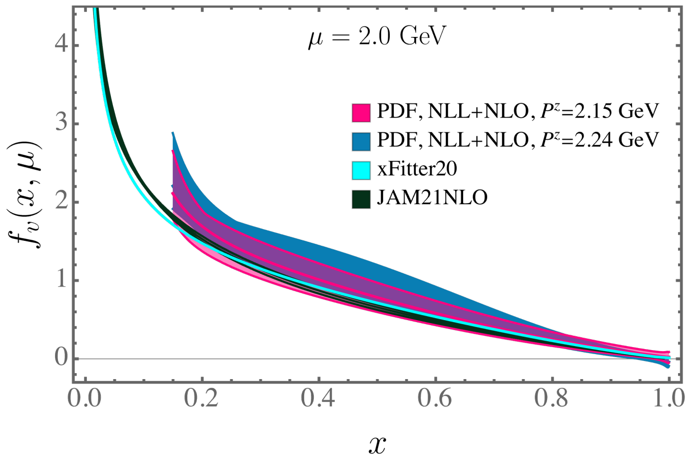

Figure 4: PDFs from the CG method at $|\vec{p}| = 2.15$ and 2.24 GeV with NLO matching and NLL evolution, compared to `xFitter20` [34] and JAM21NLO [35] fits. The lattice error bands include statistical and scale variation uncertainties.

free from linear divergence and renormalon, which greatly simplifies the renormalization, and that it yields consistent results with the GI method. It also enables access to larger off-axis momenta under 3D rotational symmetry and enhances long-range precision, both contributing to more efficient analysis. The CG method can be applied to broader parton physics like generalized parton distributions and transverse-momentum distributions (TMDs) [36], which are more computationally demanding than the PDFs.

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