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Neutron electric dipole moment from isovector quark chromo-electric dipole moment

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We present results from our lattice QCD study of the contribution of the isovector quark cEDM (qcEDM) operator to the neutron EDM (nEDM). The calculation was carried out on four $2+1+1$-flavor highly improved staggered quark (HISQ) ensembles (provided to us by the Multiple Instruction, Multiple Data (MIMD) Lattice Computation (MILC) collaboration [1, 2]) using Wilson-clover quarks to construct correlation functions. We use the nonsinglet axial Ward identity (AWI) including corrections up to $O(a)$ to show how to control the power-divergent mixing of the isovector qcEDM operator with the lower dimensional pseudoscalar operator. Results for the nEDM are presented after conversion to the $\overline{\text{MS}}$ scheme at the leading-log order.

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1. Introduction

The physics beyond the standard model (BSM) of particle physics is needed to explain the observed universe [3]. In particular, such physics needs to violate the symmetry under simultaneous interchange of left with right and particle with its antiparticle (CP) to be able to generate the observed excess of matter [4]. CP-violating ($\not{C}\not{P}$) interactions can impart electric dipole moments (EDMs) to nondegenerate quantum eigenstates and observing the EDM of an elementary particle might be the first indication of such BSM physics.

Beyond the scale of electroweak breaking, the standard model can violate CP through a gluonic topological term and CP-violation (CPV) in the couplings of the leptons to the Higgs. When the weak-symmetry is spontaneously broken, the latter give rise to an irreducible $\not{C}\not{P}$ phase in the quark mass determinant and four-fermion couplings arising from the single irreducible phase in the Cabbibo-Kobayashi-Maskawa quark-mixing (CKM) matrix when the weak gauge bosons are integrated out. Due to the axial anomaly, in the absence of BSM interactions, the CPV due to the topological term and that due to the phase of the quark mass determinant can be rotated into each other, and in our previous work, we studied the nEDM induced by this [5]. The nEDM due to the phase of the CKM matrix are expected to be much smaller than those due to BSM CPV in the strong sector.

The lowest mass-dimension BSM $\not{C}\not{P}$ operators are of dimension six. After weak-symmetry breaking, these give rise (i) to dimension-five EDMs of leptons and both EDMs and chromoelectric dipole moments (cEDMs) of quarks, (ii) to a dimension-six cEDM of the gluon, also called the $\not{C}\not{P}$ Weinberg operator, and (iii) to various $\not{C}\not{P}$ lepton-quark and four-quark four-fermion operators. The nEDM due to the EDMs of the quarks are given by the tensor charge, and, in our previous work, we have also calculated these [6–8]. The corresponding calculations for the qcEDM and Weinberg operators are preliminary [9], and no lattice calculation of the nEDM due to the four-quark operators has been reported yet.

Here we present our recent work [10] on the nEDM due to the isovector qcEDM operator

$$\bar{\psi}\Sigma \cdot \widetilde{G}\tau\psi \,, \tag{1}$$

where ψ denotes the quark-flavor multiplet, $\tilde{G}$ the dual chromoelectric field strength, and τ a diagonal non-singlet flavor matrix. This operator is the $SU(3)$-color analog of the quark EDM (qEDM) and breaks the chiral symmetry and the discrete symmetries under parity and CP, but conserves the charge-conjugation symmetry. The lattice calculations involving this operator can be conveniently carried out using the Schwinger-source trick: since it is a quark-bilinear, it merely modifies the quark propagator:

$$\mathcal{P} = \left[\not{D} + m - \frac{r}{2}D^2 + c_{\rm SW}\Sigma \cdot G\right]^{-1} \to \left[\not{D} + m - \frac{r}{2}D^2 + \Sigma \cdot \left(c_{\rm SW}G + i\,\epsilon\tau\widetilde{G}\right)\right]^{-1} , \tag{2}$$

where $\mathcal{P}$ is the Wilson-clover quark propagator, $\not{D}$ is the lattice-discretized Dirac operator, m is the quark mass assumed to be isoscalar, r is the Wilson parameter, $c_{\rm SW}$ is the clover parameter, ϵ is the strength of the qcEDM operator, and we have implicitly absorbed powers of the lattice spacing a to make all quantities dimensionless. Since the qcEDM operator is dimension-five, insertion of multiple instances of this operator give rise to uncontrolled a^{-1} divergences as we take the

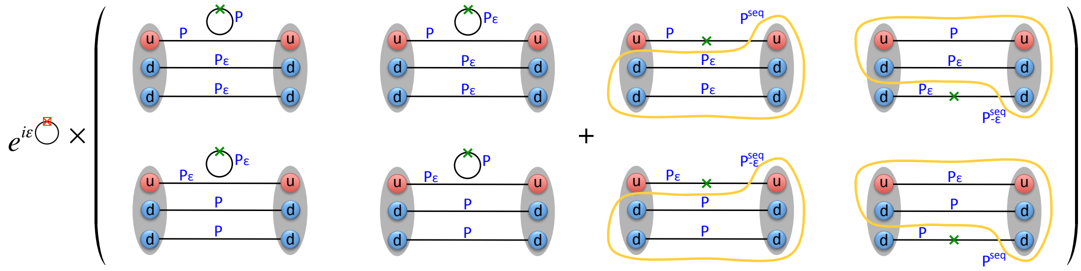


Figure 1: The Wick contractions contributing to the three-point functions

ID	a (fm)	$M_\pi^{\rm sea}$ (MeV)	$M_\pi^{\rm val}$ (MeV)	$L^3 \times T$	$N_{\rm conf}$	ϵ	ϵ_5
$a12m310$	0.1207(11)	305.3(4)	310.2(2.8)	$24^3 \times 64$	1013	0.008	0.0024
$a12m220L$	0.1189(09)	217.0(2)	227.6(1.7)	$40^3 \times 64$	475	0.001	0.0003
$a09m310$	0.0888(08)	312.7(6)	313.0(2.8)	$32^3 \times 96$	447	0.008	0.0024
$a06m310$	0.0582(04)	319.3(5)	319.3(0.5)	$48^3 \times 144$	72	0.009	0.0012

Table 1: The names (ID) and the lattice parameters of the HISQ ensembles from the MILC collaboration [1, 2] used in the calculation. $N_{\rm conf}$ provides the number of configurations analyzed. ϵ is defined in Eq. (2) and ϵ_5 is the corresponding quantity in propagators evaluated with the pseudoscalar operator $\bar\psi\gamma_5\tau\psi$ replacing the qcEDM operator.

continuum limit, which necessitates a correspondingly decreasing value for ϵ. Details on these issues is presented in our longer publication [10].

The nEDM can be calculated from the two-point function of the nucleon and the three-point function of the vector-current, whose Wick contractions we display in Fig. 1. Since we are concerned with only the isovector qcEDM, the disconnected loops from the fermion determinant do not contribute, and in this work we ignore the disconnected loops arising from the isoscalar parts of the electromagnetic current, which are found to be small in other matrix element calculations. As a result, we are left only with connected contributions that we proceed to evaluate.

For this calculation, we use a mixed-action setup of tree-level tadpole-improved clover quarks on HISQ lattices obtained from the MILC collaboration [1, 2]. All the calculations were done with ensembles where the valence and the sea pion masses were roughly equal, i.e., $M_\pi^{\rm sea} \approx M_\pi^{\rm val}$; and the lattice sizes in the temporal and spatial directions, $T \geq L$, were large, $M_\pi L \gtrsim 4$, where finite volume effects are expected to be small. The lattice parameters are shown in Table 1.

Our operator creating the nucleon in the standard basis [5, 11, 12] is given by N_α below. The lattice calculations are actually carried out with $N_0 \equiv N_{\alpha=0}$, from which the rotation phase α_N is determined,

$$N_\alpha = e^{-i\alpha_N}\epsilon^{abc}\left[\psi_d^{aT}(\gamma_0\gamma_2)\gamma_5\frac{1\pm\gamma_4}{2}\psi_u^b\right]\psi_d^c \tag{3}$$

$$\alpha_N = \mathrm{Lim}_{\tau\to\pm\infty}\frac{\Im\,\mathrm{Tr}\,\gamma_5(1\pm\gamma_4)\langle N_0(0)\bar N_0(\tau)\rangle}{\Re\,\mathrm{Tr}(1\pm\gamma_4)\langle N_0(0)\bar N_0(\tau)\rangle} \approx -\frac{r\epsilon}{8ma}\frac{a^2\langle\Omega|\bar\psi\Sigma\cdot G\psi|\Omega\rangle}{\langle\Omega|\bar\psi\psi|\Omega\rangle}\,. \tag{4}$$

where the last expression is a leading order chiral perturbation theory result [10]. We show an

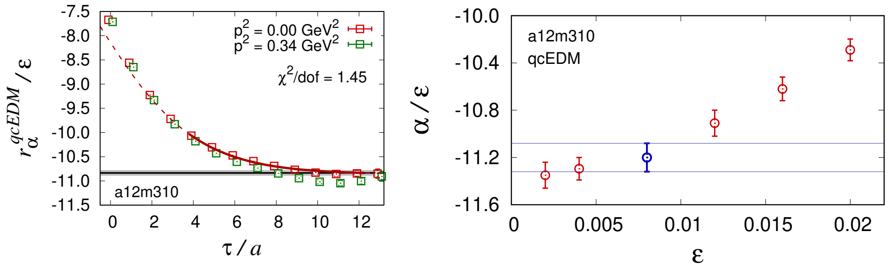


Figure 2: Determination of the $\not{C}\not{P}$ phase α_N at various momenta, and checking its linearity.

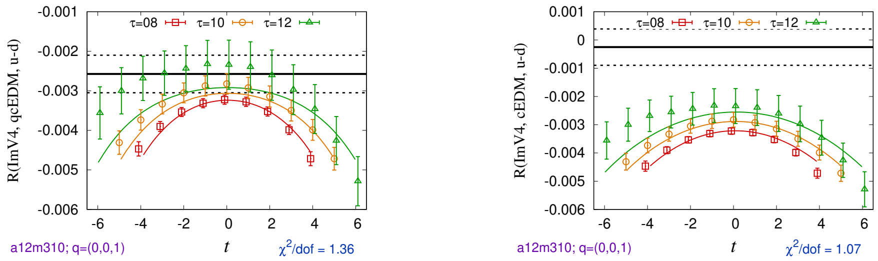


Figure 3: Effect of excited states on the nucleon matrix elements. The two fits show the same data, but the left hand one assumes a spectrum of excited states obtained from a best fit to the two-point data, whereas the right one assumes a light $N\pi$ contribution saturating the excited state contribution.

example of the quality of the data determining α_N in Fig. 2, and check that it is linear in ϵ and momentum-independent. We then use the determined α_N to obtain the form factors $F_{1,2,3}$

$$\langle N_\alpha(p')|J)\mu^{\rm EM}|N_\alpha(p)\rangle = \bar{u}(p')\Big[\gamma_\mu F_1 + \Sigma_{\mu\nu}\frac{q^\nu}{2M_N}(F_2 - iF_3\gamma_5)\Big]u(p)\,, \tag{5}$$

which decomposition holds when there are no excited state contributions and the theory conserves charge-conjugation.

2. Excited State Contribution

The interpolating operators used couple not only to the nucleon state, but other states allowed by the symmetries of the theory. Especially with CPV, among these are light multiparticle states like the $N\pi$ state whose correlations are volume suppressed. This volume suppression often makes it difficult to see them in two-point correlators. But since the vector current has a strong coupling to the two pion channel, it is, in principle, possible that the contribution of this state is relatively enhanced in the three-point functions. Unfortunately, as we found for other matrix elements [5, 8, 13], a direct check of this is difficult since the χ^2-surface has strongly flat directions, and goodness-of-fit tests do not choose between the alternatives. As shown in the example Fig. 3, this leads to large uncertainties in the final determination of the matrix elements, and, correspondingly, on the predictions for the nEDM.

Ensemble	$\tilde{F}_3^{P_3}/\tilde{F}_3^C$					$\frac{K}{2am+AK}$
	$Q^2 = 1$	$Q^2 = 2$	$Q^2 = 3$	$Q^2 = 4$	$Q^2 = 5$	
a12m310	0.879(17)	0.863(14)	0.867(18)	0.844(23)	0.864(13)	0.694(48)
a12m220L	0.81(10)	0.769(77)	0.869(75)	0.98(18)	0.94(11)	0.7807(70)
a09m310	1.063(35)	1.042(40)	1.078(45)	1.006(58)	1.039(44)	0.740(61)
a06m310						0.859(64)

Table 2: Verification of the expected relation between CPV due to the pseudoscalar and qcEDM insertions

3. Mixing

Under renormalization, the isovector qcEDM operator, $C \equiv C^{(3)}$, has a power divergent mixing with the pseudoscalar operator $P_3 \equiv P^{(3)} \equiv \bar{\psi}\gamma_5\tau\psi$ even when the regularization preserves chiral symmetry. When chiral symmetry is broken, there is additional divergent mixing with the topological term, which is, however, prohibited for the isovector qcEDM operator due to the unbroken isospin symmetry in our calculation.

In the continuum, the isovector pseudoscalar operator can be rotated away by the nonanomalous nonsinglet chiral symmetry, and has no effect. The lattice situation is more subtle due to the explicit breaking of chiral symmetry. In fact, the AWI for Wilson-like fermions is

$$Z_A(m)\left[\partial_\mu A_3^\mu + iac_A\partial^2 P_3 + 2imP_3\right] = iaZ_A(m)K\tilde{C}_3 + O(a^2) \tag{6}$$

where we have restored the explicit powers of the lattice spacing a, A_3 is the axial current, $\tilde{C}_3 \equiv C - a^{-2}AP_3$ is defined to be an operator free of power divergence, m is the quark mass and the term involving K appears because tree-level tadpole-improved c_{SW} does not remove all $O(a)$ effects in the theory even after introducing the improvement constant [14, 15] c_A into the axial current.[1] Note that there is an $O(a^2)$ ambiguity in the definition of the coefficient A that affects only the interpolating operators and not the physical matrix elements, since in the continuum limit a term proportional to P_3 in the action can be rotated away. In our work, we determine by demanding that the vacuum-to-pion matrix element of the operator C_3 is zero at each lattice spacing—since the corresponding matrix element of the interpolating operator P_3 is nonzero, this guarantees the absence of remaining power divergences. This is most conveniently done by taking the ratio of the 2-point correlators of C and P_3 with a pion interpolating operator π at long Euclidean times (see Fig. 4 left).

From Eq. (6), we immediately find that

$$\frac{2am}{K}\frac{P_3}{a} \sim \frac{2am}{2am+K}aC_3 + O(a^2) \tag{7}$$

is power-divergence free, and this quantity gives, up to logarithmic renormalization, the qcEDM operator in the continuum. The operator equality of the two sides in Eq. (7) then allows us to determine the constant K and the quark mass m (see Fig. 4 middle) and hence the effects of the

[1]For later convenience, we have defined c_A, m and K with the axial renormalization constant $Z_A(m)$ factored out.

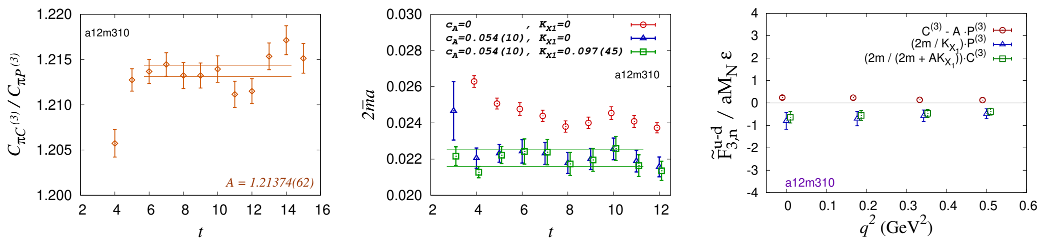


Figure 4: Example of determination of (left) the constant A that subtracts the power-divergence of the qcEDM operator C, (middle) the quark mass m (labeled $\bar{m}$ here) and the constants c_A and K (labeled K_{X1} here) needed for the improved Ward identity in Eq. (6), and (right) the $\not{C}\not{P}$ form-factor $\tilde{F}_3$ using three separate lattice operator combinations.

continuum qcEDM operator either from the insertion of the lattice pseudoscalar operator P_3, or from that of the lattice qcEDM operator C. In Table 2, we display the ratio of[2] $\tilde{F}_3$ determined from the insertion of either of these two operators along with the value expected from Eq. (7). We notice that the expected relation is satisfied to about 10–20%, which is not surprising since the difference between them, $O(a^2)/am$, though vanishing in the continuum, could be large for the small quark masses in our calculation.

In addition to determining it from either C, or P_3, we could also determine it directly from C_3. This, however, involves subtraction of the large power-divergent piece explicitly, and, as shown in Fig. 4 right, shows a result that differs significantly from the other two determinations. Without further understanding of the $O(a^2)$ errors, it is currently not clear which determination should be preferred.

4. Renormalization

After the subtraction of power-law divergences, in our isospin-conserving theory, only logarithmic divergences need to be considered that mix the various operators of dimension-five. In pure quantum chromodynamics (QCD), the only mixing is with the qEDM operator, but this is small, $O(\alpha_{\rm EM}) \sim 1\%$. Our calculation of nEDM, however, needs the addition of the electromagnetic interaction $J^{\rm EM} \cdot A$ to the action. With this addition, we find that $\int d^4x \tilde{C}_3 J^{\rm EM}_\mu A^\mu$ has mixing with qEDM at $O(\alpha_s)$.

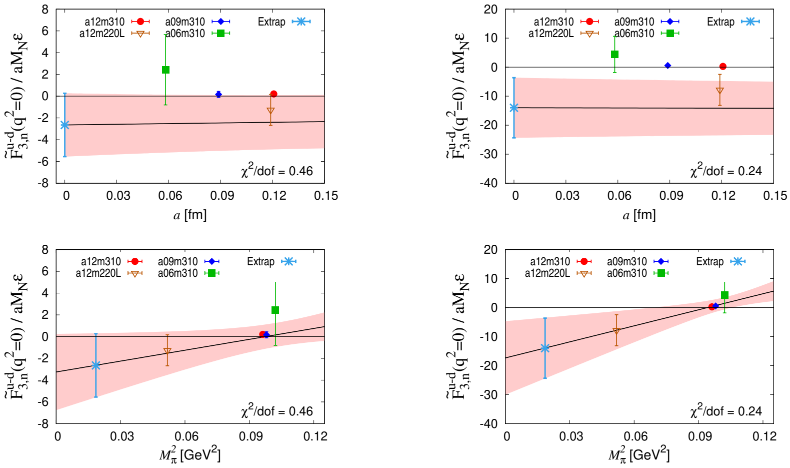
At leading logarithm (i.e., tree-level matching, one-loop running), we can use this to combine our present results for qcEDM with our previous analysis of the qEDM operator:

$$F_3(\vec{O}_{\overline{\rm MS}}) = U \begin{pmatrix} \left(\frac{\alpha_s(\mu)}{\alpha_s(a^{-1})}\right)^{-\gamma_{11}/\beta_0} & 0 \\ 0 & \left(\frac{\alpha_s(\mu)}{\alpha_s(a^{-1})}\right)^{-\gamma_{22}/\beta_0} \end{pmatrix} U^{-1}\, F_3(\vec{O}_L(a)) \tag{8a}$$

$$U = \begin{pmatrix} 1 & -\frac{\gamma_{12}}{\gamma_{11}-\gamma_{22}} \\ 0 & 1 \end{pmatrix} \qquad \vec{O} = \begin{pmatrix} \text{qcEDM} \\ \text{qEDM} \end{pmatrix}, \tag{8b}$$

where the one-loop β-function and all the one-loop anomalous dimensions γ are known [10].

[2] $\tilde{F}_3 \equiv F_3 + O(Q^2)$ is a quantity defined [5] and used instead of F_3 because of its better statistical signal.



Standard $N\pi$-fit

Figure 5: The chiral-continuum extrapolation of the $\not{C}\not{P}$ form-factor extracted using two choices for the excited state contamination.

5. Extrapolation

The chiral extrapolation of our results needs caution. CP-transformations and chiral rotations do not commute, and the standard CP transformation is to be chosen among a one-parameter family of chirally rotated operators. Since chiral symmetry is spontaneously broken, the physical CP-violation is the one that leaves the vacuum of the broken theory invariant. The direction of the chiral symmetry breaking is, however, determined by the chiral breaking terms in the theory [16]. The analysis of the physical CP-violation in the theory is therefore easiest if we perform a chiral rotation to align the vacuum condensate along the 'standard' direction where all the pseudoscalar condensates are zero. In this aligned vacuum, the pions are created by isovector pseudoscalar operators and single pion 'tadpoles' are absent. Starting from a Lagrangian

$$\mathcal{L} = \mathcal{L}^{\text{chiral- \& CP-conserving}} + m\bar{\psi}\psi + d^{(3)}\bar{\psi}\Sigma\cdot\tilde{G}\tau^3\psi\,, \tag{9}$$

the appropriate chiral rotation leads to

$$\mathcal{L}^{\text{CPV}} = \frac{md_3}{\sqrt{m^2+\bar{r}^2 d_3^2}}\bar{\psi}(\Sigma\cdot G - \bar{r})\gamma_5\psi\,, \tag{10}$$

where

$$2\bar{r} \equiv \frac{\langle\Omega|\bar{\psi}\Sigma\cdot G\psi|\Omega\rangle}{\langle\Omega|\bar{\psi}\psi|\Omega\rangle}\,. \tag{11}$$

This vanishes at $m = 0$, which is consistent with our previous discussion [17] that when the only chiral violation in a theory is from a single apparently $\not{C}\not{P}$ operator, the vacuum aligns to maintain the CP symmetry.

There is, however, a subtlety in this argument when the $\not{C}\not{P}$ operator is a power-divergent operator as in our calculation. In this situation, the operator is ill-defined since the power divergence needs to be controlled. This involves the subtraction of a lower-dimensional operator, and the residual is ill-defined up to finite terms. One can, in this case, define these finite pieces to set $\bar{r} \equiv 0$, and this is the choice we have made with our 'subtracted qcEDM' operator. It is easy to see that when $\bar{r} = 0$, the operator and its chiral rotations do not tilt the vacuum manifold, *i.e.,* the chiral degeneracy of the vacuum is not lifted by this operator. In this case, in the absence of any other chiral violation, *e.g.,* at the chiral limit in the continuum, chiral dynamics alone does not choose between CP-conserving and $\not{C}\not{P}$ condensates. The CPV in the chiral-continuum limit of a theory with a standard mass term, with or without the standard Wilson and clover terms, however, chooses the standard orientation of the chiral condensate independent of the mass. As a result, the CPV in this setup persists in the chiral limit. Thus, in our fits shown in Fig. 5, we do not enforce the vanishing of the nEDM in the chiral limit.

6. Conclusions

In this work we studied the power-divergence of the isovector qcEDM which is present even with good chiral symmetry. We noticed that the power-divergent mixing is with P_3 which implements chiral rotation, but no CP-violation in the continuum. Since lattice artifacts in this relation are enhanced by $1/ma$, it is important to demonstrate control. We find that this leads to a large uncertainty when using perturbative $O(a)$-improved Wilson fermions, even though the identities following from chiral rotation agree with chiral perturbation theory (χPT) at about 10%. Finally, we note that control over excited state contamination (ESC) still need to be demonstrated.

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