PROCEEDINGS OF SCIENCE

QED in external EM fields

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Approximate Truncated Schwinger-Dyson methods have predicted that when massless QED is subjected to strong external magnetic fields, chiral symmetry, which is a good symmetry for massless QED without these external fields, is broken. We use Rational Hybrid Monte Carlo (RHMC) simulations to study lattice QED in a strong constant homogeneous external magnetic field. We use the chiral condensate $\langle\bar{\psi}\psi\rangle$ as an order parameter for chiral symmetry breaking. A non-zero value of this condensate in the limit of zero mass electrons in a strong magnetic field indicates that the magnetic field does break chiral symmetry.

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1. Introduction

We simulate lattice QED in (large) external electromagnetic(EM) fields using methods developed for lattice QCD. For more details see [1].

Our first project is simulating lattice QED in large constant (in space and time) magnetic fields using the RHMC method [2]. We shall assume that the external magnetic field $\boldsymbol{B}$ is oriented in the z (3) direction so that the external vector potential A_{ext} lies in the $x-y$ (1-2) plane. Classically electrons(positrons) in such a field traverse helical orbits around magnetic field lines. In quantum mechanics the motion in the $x-y$ plane is in discrete (transverse) energy levels – Landau levels – while that in the z direction is free.

$$E_n(p_z) = \pm\sqrt{m^2 + 2eBn + p_z^2} \tag{1}$$

where $n = 0, 1, 2, ...$, and the degeneracy of the lowest Landau level ($n = 0$) is half that of the higher levels. When QED is taken into account, for large enough eB, we expect the energy levels to behave similarly, and the lowest Landau level (LLL) to give the dominant contribution to the functional integral.

One of the more interesting predictions of approximate analyses of QED in large B using a truncated Schwinger-Dyson approach, is Magnetic Catalysis of dynamical symmetry breaking, giving a dynamical (non-perturbative) mass to the electron $\propto \sqrt{eB}$ [3–13] and a non-vanishing chiral condensate $\propto (eB)^{3/2}$, [14, 15] when the input electron mass vanishes, associated with a dimensional reduction from $3+1$ to $1+1$ dimensions for charged particles.

For fine-structure constant $\alpha = 1/137$, the Schwinger-Dyson prediction for the dynamical electron mass at our chosen $eB = 2\pi \times 100/36^2 \approx 0.4848...$, $m_{dyn} \approx 2 \times 10^{-35}$ [5]. Since this is far below anything we could measure on the lattice, we choose a stronger electron charge $\alpha = e^2/(4\pi) = 1/5$.

Our simulations show clear evidence that the chiral condensate $\langle\bar{\psi}\psi\rangle$ remains non-zero as $m \to 0$. Hence chiral symmetry is broken dynamically by the magnetic field.

2. Extracting the chiral condensate from lattice simulations

We simulate lattice QED in a strong magnetic field using the RHMC algorithm. A non-compact gauge action is used for the internal electromagnetic fields. We use staggered fermions and a rational approximation to tune to 1 electron flavour. We use a compact interaction between the electromagnetic fields (internal and external) and the fermions to render the action gauge-invariant. As mentioned above we choose $\alpha = 1/5$ to give a measurable signal. Since for free fermions in an external magnetic field, our approach fails when $eB \gtrsim 0.63$ we choose $eB = 2\pi \times 100/36^2 \approx 0.4848...$ on lattices with $N_x = N_y = 36$ or $N_x = N_y = 18$.

Since we are interested in the limit $m \to 0$, we need to perform simulations down to rather small m (We use masses as small as $m = 0.001$). For the smallest masses we will need to measure the chiral condensate on a series of lattice sizes until increasing the lattice size does not increase $\langle\bar{\psi}\psi\rangle$. Because we expect the functional integral will be dominated by the lowest Landau levels (LLLs), then provided N_x and N_y are appreciably greater than the size of an LLL in the $x-y$ plane

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($\sim 1/\sqrt{eB}$), we do not need to increase these dimensions in our simulations. Hence we run our simulations for a series of $N_z = N_t$ values with $N_x = N_y = 36$ or $N_x = N_y = 18$.

We simulate at a selection of masses covering the range $0.001 \leq m \leq 0.2$. In order to remove finite size effects, we need to simulate on lattices with $N_z = N_t$ as large as 128 (in particular $18^2 \times 128^2$) at $m = 0.001$. We simulate for a total of at least 1250 trajectories for each choice of parameters.

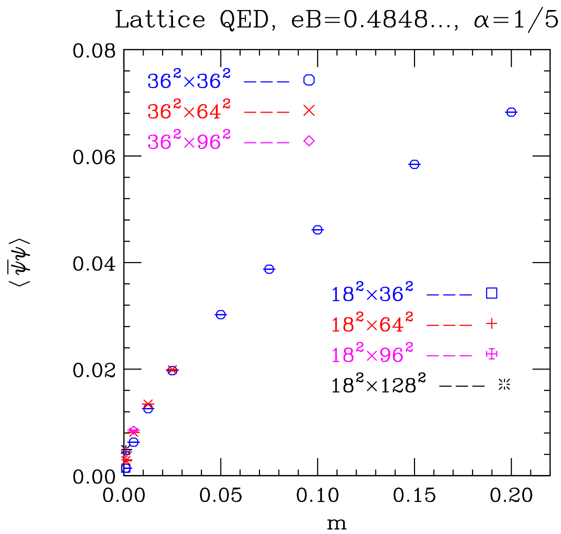

Figure 1: $\langle\bar{\psi}\psi\rangle$ as a function of mass at $eB = 2\pi \times 100/36^2$,showing dependence on lattice size in the z and t directions.

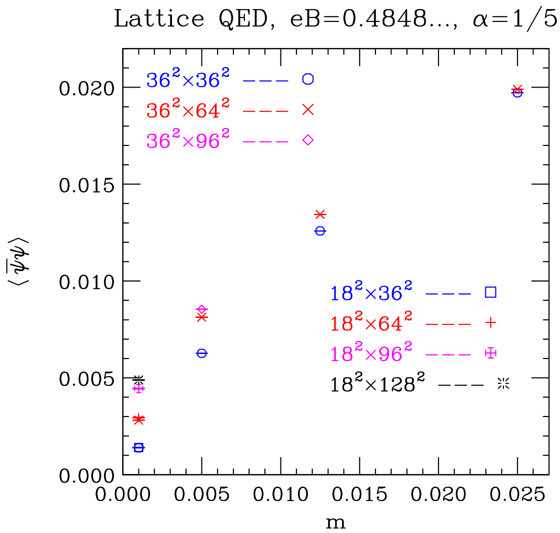

Figure 2: As in figure 1, but on an expanded scale

Figures 1 and 2 show the mass dependence of the chiral condensates $\langle\bar{\psi}\psi\rangle$ for the lattice sizes we use. Note that, for each m value, we consider the measurement for the lattice with the largest $N_z = N_t$ to be our best estimate for the infinite lattice value.

From these plots, it would appear that the chiral condensate remains finite as $m \to 0$, and that the $m = 0$ value ≈ 0.004. Indeed a selected set of fits [1] indicate that

$$0.003 \lesssim \langle\bar{\psi}\psi\rangle \lesssim 0.004 \qquad (2)$$

which is clearly non-zero.

For comparison we have also performed simulations at $\alpha = 1/5$ with $eB = 0$ on a 36^4 lattice for a range of masses $0.001 \leq m \leq 0.2$ which are consistent with $\langle\bar{\psi}\psi\rangle = 0$ in the limit as $m \to 0$. We have repeated the simulations at $m = 0.001$ on a 48^4 lattice, which indicate that the finite size effects are very small. These results are plotted in figures 3 and 4.

3. Comparison of lattice QED with Schwinger-Dyson results

Our lattice QED simulations at $\alpha = 1/5$, $eB = 0.4848...$, $N_f = 1$ are consistent with a chiral condensate $0.003 \lesssim \langle\bar{\psi}\psi\rangle \lesssim 0.004$ in the limit $m \to 0$. This result is gauge invariant.

The 'best' Schwinger-Dyson estimates of the chiral condensate at $\alpha = 1/5$, $eB = 0.4848...$, $N_f = 1$ give $\langle\bar{\psi}\psi\rangle \approx 1.2\times10^{-4}$ in the massless limit. This is for what is considered to be the optimal

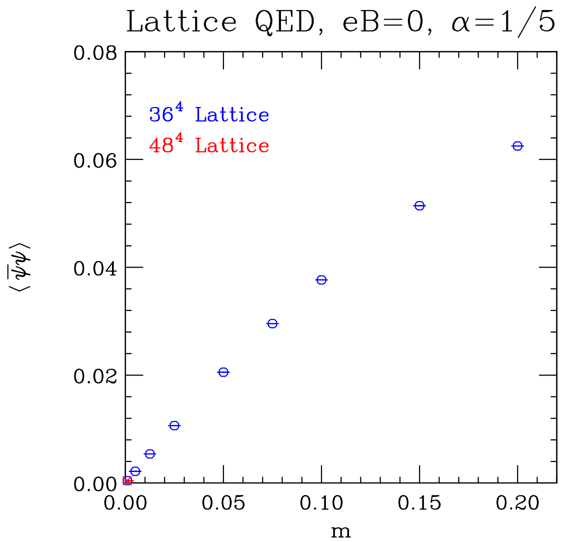

Figure 3: $\langle\bar{\psi}\psi\rangle$ as a function of mass at eB = 0,showing dependence on lattice size on 36^4 and 48^4 lattices.

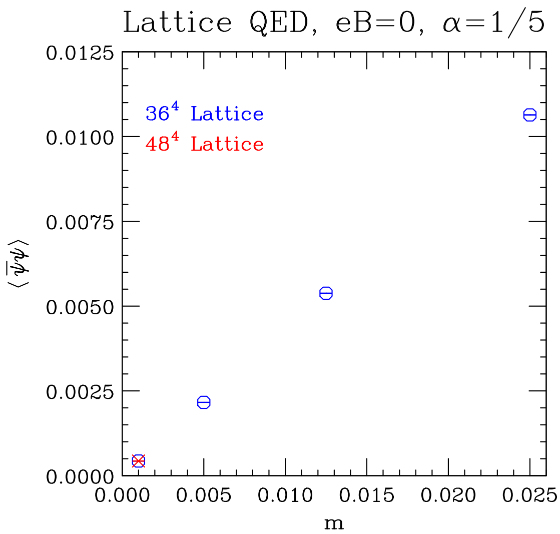

Figure 4: As in figure 3, but on an expanded scale

gauge for this truncation. However, generic covariant gauge choices yield $\langle\bar{\psi}\psi\rangle \approx 2.4\times10^{-3}$ in the limit $m \to 0$, much closer to what lattice simulations predict.

However, the parameters in our simulations are bare parameters, while those in the Schwinger-Dyson analysis are renormalized quantities. Since the renormalized α is smaller than its bare value, this only makes the disagreement worse. This is based on perturbation theory, where treating the external magnetic field perturbatively makes the renormalization constants independent of the external magnetic field. This ignores the fact that the external magnetic field completely changes the physics, which finite order perturbative analyses completely ignore. Our lattice simulations include the external magnetic field in the action which we simulate and so automatically include all non-perturbative effects. In the Schwinger-Dyson analyses, some of this physics is included by hand, starting with using the known propagators for free fermions in the external magnetic field, restricting them to the LLL. For the inverse photon propagator they include only the contribution of a single electron loop in an external magnetic field. The electron propagator in the external magnetic field is then evaluated in the rainbow approximation using these photon propagators in the optimal gauge.

Any of these truncations of perturbation theory could lead to incorrect results. For example, for $B >> B_{critical} = m^2/e$ the contributions of multiloop diagrams can increase with the number of loops, and the loop expansion of QED will break down [16–18]. Since one is interested in the limit $m \to 0$ this can be important. Then we have the possibility that for standard QED with only one electron flavour, because the external magnetic field contributes to the axial anomaly, chiral symmetry is broken explicitly by the external magnetic field. This contrasts with multiflavour QED where flavour chiral symmetry can only be broken spontaneously. However, the Schwinger-Dyson analysis would appear to indicate that the mechanism of chiral symmetry breaking is the same in both cases. Hence if the external magnetic field breaks chiral symmetry explicitly, the Schwinger-

Dyson analysis is missing this effect, which would be analogous to the $U(1)_{axial}$ breaking in QCD due to instanton contributions.

The lattice simulations include the external magnetic field in their action, and all gauge configurations make positive contributions, so importance sampling is valid. This contrasts with perturbation theory where not all contributions have the same sign and there can be cancellations. The main sources of errors are lattice size and lattice spacing. Both these are amenable to systematic improvement. Since we use (rooted) staggered fermions, the main source of large errors is the flavour symmetry violation. We know this from the community's experience with lattice QCD. This is a discretization error which can be reduced by decreasing the lattice spacing. There are better fermion actions, but they require considerably more computer resources.

The approximations used to justify the truncations used to make the Schwinger-Dyson approach tractable can only be justified for small α. It is quite possible that $\alpha = 1/5$ is large enough that these approximations have broken down. Although the lattice and Schwinger-Dyson estimates of the chiral condensate differ by between 1 and 2 orders of magnitude at $\alpha = 1/5$ this should be compared with the difference between the value of the dynamical electron masses at $\alpha = 1/137$ and at $\alpha = 1/5$, which is more than 30 orders of magnitude.

4. Discussions and Conclusions

Our simulations of Lattice QED in a constant magnetic field using the RHMC method show evidence of a non-zero chiral condensate in the $m \to 0$ limit at a relatively strong coupling ($\alpha = 1/5$). However, the chiral condensate appears to be about 1.5 orders of magnitude larger than the best estimate using a truncated Schwinger-Dyson approach. The lattice QED simulations are performed at bare (lattice) fine structure constant $1/5$, while the Schwinger-Dyson calculations are performed at renormalized fine structure constant $1/5$. Taking this into account makes the difference between the two approaches even larger.

The lattice simulations admit systematic improvements to understand the source of any disagreements between lattice and continuum methods. We are currently running simulations at a smaller magnetic field to understand how the lattice spacing errors are affecting our lattice simulations, and to check if the chiral condensates are proportional to $(eB)^{3/2}$ as predicted.

Our stored configurations allow for further measurements of properties of QED in an external magnetic field, beyond those that are measured during the simulations. We will measure the effect that QED in an external magnetic field has on the coulomb field of a point charge using (large) Wilson loops on stored configurations. This is expected to be a combination of partial screening and distortion of this electric field [19–22], and should be visible even at physical charge $\alpha \approx 1/137$. Other quantities we plan to calculate on stored configurations include the electron and photon propagators, and the fermion effective action (proportional to $\log(D + m)$) in a small constant external electric field.

QED in an external electric field is of interest because of the Sauter-Schwinger effect (production of electron-positron pairs from the vacuum). We will check lattice calculations against known results in the absence of QED. We are now investigating how we might simulate the Sauter-Schwinger effect, including QED, on the lattice. This is a much more difficult problem, since the action with an external electric field becomes complex. A first attempt will be to simulate with external magnetic and electric fields where this external field configuration comes from boosting a system with only an external magnetic field.

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