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Computation of the Kugo-Ojima function from lattice simulations

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In addition to its connection with a standard confinement criterion, the Kugo-Ojima function constitutes an indispensable component in a multitude of applications in the gauge sector of QCD. In the present work we report on preliminary results of an ongoing large-volume lattice simulation of this special function. In particular, the volume-dependence of the data is studied in detail, and a comparison with results obtained from Schwinger-Dyson equations is carried out.

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1. Introduction

The Kugo-Ojima (KO) function, $u(p^2)$, appears naturally in the context of the quantization formalism developed in [1, 2], where a non-trivial connection between confinement and the infrared behaviour of the gluon and ghost propagators in the Landau gauge was put forth. Within the KO formalism, the requirement of having a well-defined BRST charge leads to the confinement criterion associated with the infrared behavior of $u(p^2)$, namely that $u(0) = -1$. It turns out that, in the Landau gauge, the realization of this condition would cause the divergence of the ghost dressing function, $F(p^2)$, at the origin [2], by virtue of the relation $F^{-1}(0) = 1 + u(0)$; for a review, see [3]. However, as was established in a large number of works, the KO confinement condition is not fulfilled on the lattice [4, 5], and generally, in the context of the so-called "decoupling solutions", see, e.g., [6–9] and [10, 11]; in particular, $u(0) \neq -1$, and $F(0) = c$, where c is a finite constant.

The interest in the KO function resurged within the confines of the PT-BFM framework, namely the formalism that emerges from the fusion of the pinch technique (PT) [12] with the background field method (BFM) [13]. The relevance of $u(p^2)$ in this context originates from its coincidence with a central auxiliary function, denoted by $G(p^2)$ in the related literature [14], i.e.,

$$G(p^2) = u(p^2)\,. \tag{1}$$

The function $G(p^2)$ constitutes one of the cornerstones of the aforementioned framework, and is a central element in a multitude of theoretical relations and physical applications derived from it [15]. In particular, $G(p^2)$ is a common component of all formal identities relating the BFM correlation functions with those in the linear covariant gauges. The prime example of such an identity is the relation connecting the background and ordinary gluon propagators, $\widehat{\Delta}(p^2)$ and $\Delta(p^2)$, respectively, namely [15]

$$\Delta(p^2) = \widehat{\Delta}(p^2)[1 + G(p^2)]\,, \tag{2}$$

Thus, $G(p^2)$ constitutes a crucial ingredient in the determination of the effective interaction, $\mathcal{I}(p^2)$,

$$\mathcal{I}(p^2) = \alpha_s p^2 \Delta(p^2)[1 + G(p^2)]^{-2}\,, \tag{3}$$

which is employed in the computation of hadronic observables [16], where $\alpha_s = g^2/4\pi$.

In view of these considerations, in this work we present preliminary results of a new large-volume lattice simulation of the KO function that is currently underway.

2. Lattice procedure and setup

The KO function $u(p^2)$ is defined as the scalar co-factor of the following two-point function of composite operators,

$$\int d^4x e^{ip(x-y)} \langle 0|T\left([(D^{ae}_\mu c^e(x)][f^{bcd}A^d_\nu(y)\bar{c}^c(y)]\right)|0\rangle = \delta^{ab}\left(\delta^{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)u(p^2)\,, \tag{4}$$

where D^{ae}_μ is the covariant derivative in the adjoint representation, and T denotes the standard time-ordering operation.

An explicit lattice definition for $u^{ab}(p^2) = \delta^{ab} u(p^2)$ can be given as follows

$$\mathcal{U}^{ab}_{\mu\nu}(p) = \frac{1}{V} \left\langle \sum_{x,y} \sum_{c,d,e} e^{-ip\cdot(x-y)} D^{ae}_{\mu} \left(M^{-1}\right)^{ec}_{xy} f^{bcd} A^{d}_{\nu}(y) \right\rangle_U , \tag{5}$$

where M^{-1} is the ghost propagator and the scalar function $u(q^2)$ is given by

$$u(p^2) = \frac{1}{(N_d - 1)(N_c^2 - 1)} \sum_{\mu,a} \mathcal{U}^{aa}_{\mu\mu}(p) . \tag{6}$$

In order to study the KO function on the lattice, we rely on Eq. (5). However, for practical reasons, it is convenient to compute $\mathcal{U}^{ab}_{\mu\nu}(p)$ using a point source y_0 in the inversion of the lattice Faddeev-Popov operator

$$\mathcal{U}^{ab}_{\mu\nu}(p) = \left\langle \sum_{x} \sum_{c,d,e} e^{-ip\cdot(x-y_0)} D^{ae}_{\mu} \left(M^{-1}\right)^{ec}_{xy_0} f^{bcd} A^{d}_{\nu}(y_0) \right\rangle_U . \tag{7}$$

The computation of the KO function on the lattice is performed following the procedure:

1. prepare the source, using a suitable lattice definition for $f_{abc} A^c_\mu$:

$$f_{abc} A^c_\mu(x) = -\frac{1}{2} \mathrm{Tr} \left[\left\{ \left(U^\dagger_{x,-\mu} + U_{x,\mu} \right) - \left(U^\dagger_{x,-\mu} + U_{x,\mu} \right)^\dagger \right\} [t^a, t^b] \right] ;$$

2. solve the linear system of equations to get the ghost propagator, taking care of zero modes

$$MY = M\phi_{b,\nu} \quad ; \quad M\psi_{b,\nu} = Y \quad ;$$

3. apply the covariant derivative, which can be written on the lattice as [5]

$$\left(D_\mu[U]\right)^{ab}_{xy} = 2\,\mathrm{Re}\,\mathrm{Tr}\left[t^b t^a U_{x,\mu}\right] \delta_{x+\hat{\mu},y} - 2\,\mathrm{Re}\,\mathrm{Tr}\left[t^a t^b U_{x,\mu}\right] \delta_{x,y} ;$$

4. apply a Fast Fourier Transform (FFT) and include the correction due to the location of the point source.

In this work we consider quenched lattice ensembles generated with the Wilson gauge action, with $\beta = 6.0$ ($a \sim 0.1$fm) for the lattice volumes 32^4, 48^4, 64^4, and 80^4, whose physical volumes go from $(3\text{ fm})^4$ to $(8\text{ fm})^4$. The results we show use 100 configurations for each of the lattice ensembles, except for the largest volume, where the number of gauge configurations is 50. For the smallest lattice volume, we consider an average over several point sources; for the other volumes, only one point source is considered. Computer simulations have been performed with the help of Chroma [17] and PFFT [18] libraries. The interested reader may find further details on the work reported here in [19].

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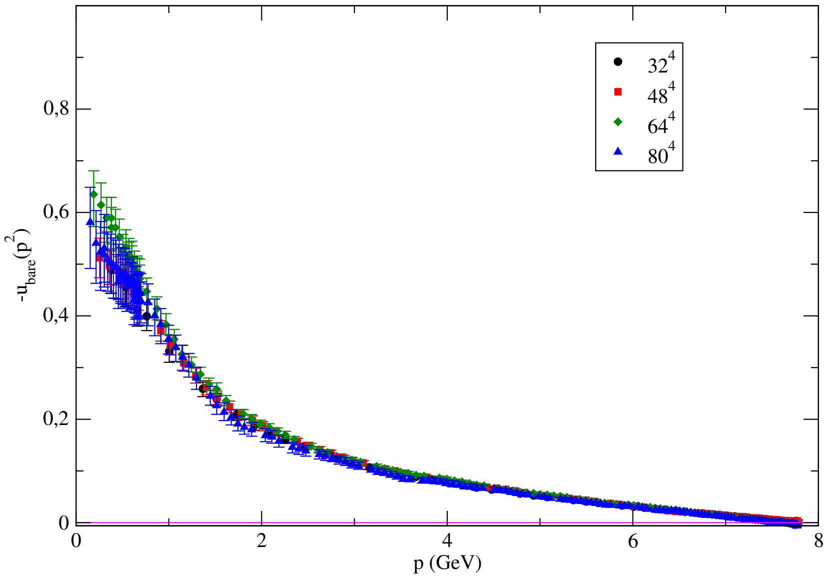
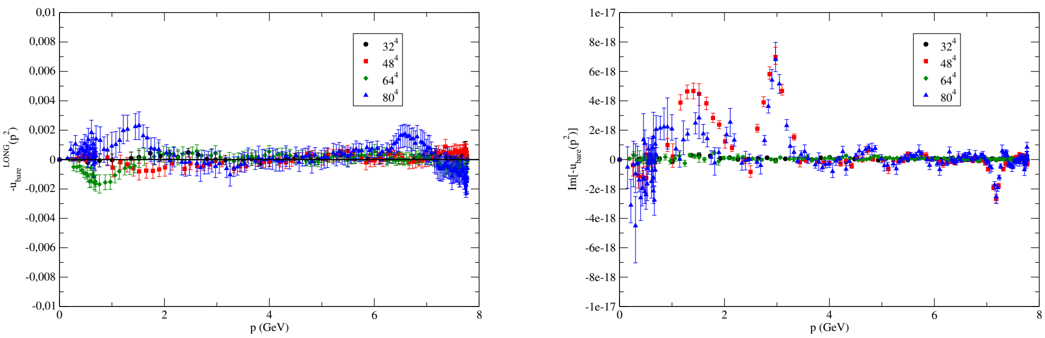


Figure 1: Bare KO function for all lattice volumes. A horizontal line at $u_{bare}(p^2) = 0$ is drawn to guide the eye.

3. Results

The bare $u(p^2)$ for all lattice volumes is reported in Fig. 1. The lattice data for the various volumes is compatible within errors, suggesting that the lattice volume effects are small or negligible.

According to the definition in Eq. (4), the KO function is transverse. We have tested the transversality of the lattice KO function computing its longitudinal projection that can be seen in Fig. 2(a). Indeed, the numerical results show that the lattice version is, indeed, orthogonal and that it can have a very small imaginary part that seems to decrease with an increase of the lattice volume.



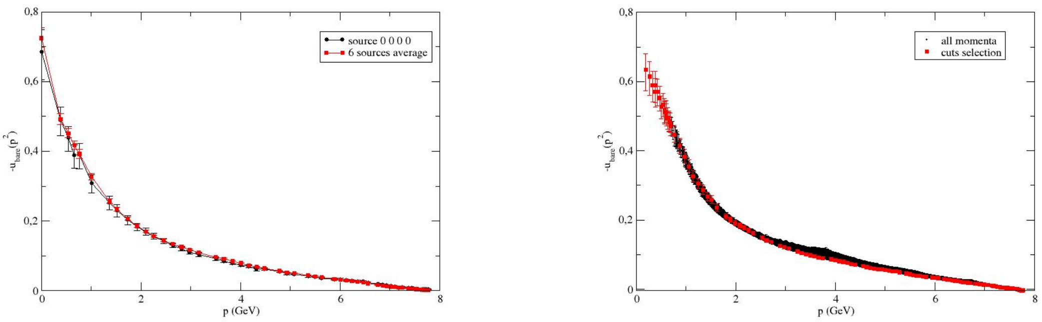
(a) Longitudinal component.

(b) Imaginary part.

Figure 2: Longitudinal component and imaginary part of the Kugo-Ojima function.

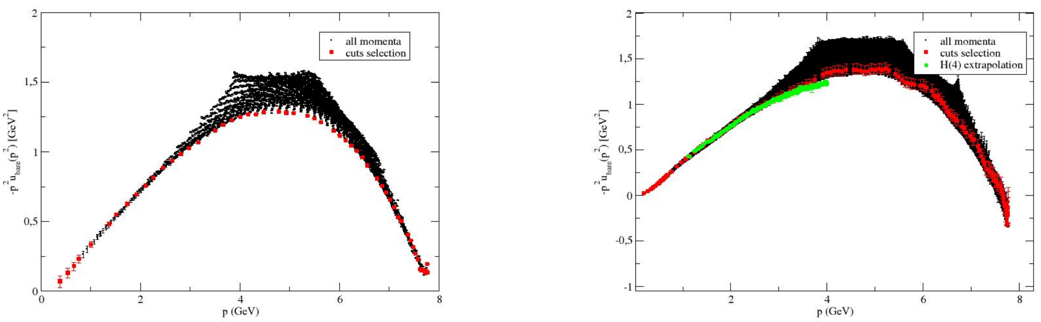
For the smallest lattice volume, where we combined several point sources in the inversion of the Faddeev-Popov matrix, the effect of using several sources is described in Fig. 3(a). The curves compare the average over several point sources with the result computed with a single point source at the origin of the lattice. The results show that averaging over several point sources reduces the fluctuations in $u(p^2)$.

The lattice data in the previous figures corresponds to the momenta that survives the cylindrical and conical cuts [20]. In Fig. 3(b) we compare the outcome of these cuts with all lattice data. Furthermore, in Fig. 4(a) the product $p^2u(p^2)$, which allows to better understand the breaking of rotational invariance on the lattice calculation of $u(p^2)$, is plotted. Fig. 4(b) includes an H(4) extrapolation of the data that seems to provide good results for momenta below 3 GeV.



(a) Effect of number of sources, 32^4 lattice. (b) Momentum cuts, 64^4 lattice.

Figure 3: Other issues in the lattice computation of the KO function.



(a) 32^4 lattice. (b) 64^4 lattice, H(4) extrapolation.

Figure 4: Plots of $p^2u(p^2)$.

It is important to compare the results of this simulation with those obtained from the Schwinger-Dyson equation (SDE) that governs the evolution of the function $G(p^2)$ [21], and, therefore, by virtue of Eq. (1), of $u(p^2)$. To that end, we renormalize the KO function for the simulation with the 64^4 lattice by matching the lattice data with the outcome of an SDE calculation; the renormalization scale μ was chosen to be $\mu = 4.3$ GeV. As can be seen in Fig. 5, the renormalized lattice data and the SDE curve are in good agreement, especially in the region of momenta between $2 - 6$ GeV.

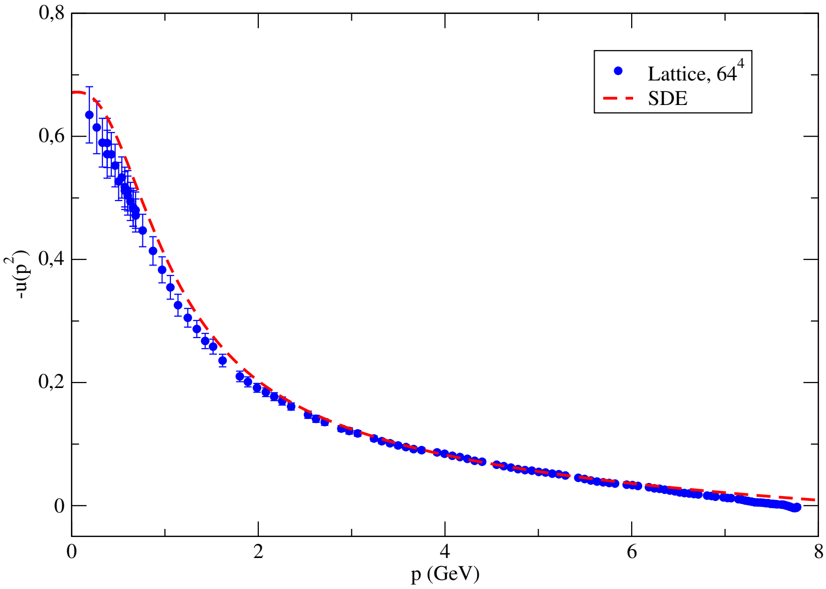


Figure 5: Renormalized KO function and comparison with SDE results.

4. Conclusions and Outlook

In the present work we have reported recent results on the evaluation of the KO function on the lattice, for several lattice volumes. The results obtained are in agreement with those of earlier lattice studies [5] using smaller volumes. Moreover, they show good coincidence with the SDE results of [21].

We are currently increasing the statistics of our lattice ensembles, in order to reduce the errors in the deep infrared region; the final results will be reported elsewhere soon. These results, in conjunction with existing lattice data for the gluon propagator, offer the possibility of deriving the effective interaction $\mathcal{I}(p^2)$ of Eq. (3) using exclusively ingredients from lattice QCD.

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