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Monopoles of the Dirac type and color confinement in QCD - Study of the continuum limit -

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Non-Abelian gauge fields having a line-singularity of the Dirac type lead us to violation of the non-Abelian Bianchi identity (VNABI). VNABI as an operator is equivalent to violation of Abelian-like Bianchi identities corresponding to eight Abelian-like conserved magnetic monopole currents of the Dirac type in $SU(3)$ QCD. If these new Abelian-like monopoles exist in the continuum limit, the Abelian dual Meissner effect occurs, so that the linear part of the static potential between a quark-antiquark pair is reproduced fully by those of Abelian and monopole static potentials. Monte-Carlo studies of pure QCD using the Iwasaki gluonic action at various β on 48^4, the perfect Abelian dominance is reproduced fairly well, whereas the perfect monopole dominance seems to be realized for large β when use is made of smooth lattice configurations in the maximally Abelian (MA) gauge. Making use of a block spin transformation with respect to monopoles, the scaling behaviors of the monopole density and the effective monopole action are studied. Both monopole density and the effective monopole action which are usually a two-point function of β and the number of times n of the block spin transformation are found to be a function of $b = na(\beta)$ alone for $n = 1, 2, 3, 4, 6, 8, 12$ at 13 different β on 48^4. The scaling behaviors suggest the existence of the continuum limit, since $a(\beta) \to 0$ when $n \to \infty$ for fixed $b = na(\beta)$.

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1. Introduction

Color confinement in quantum chromodynamics (QCD) is still an important unsolved problem. As a picture of color confinement, it was conjectured that the QCD vacuum is a kind of a magnetic superconducting state caused by condensation of magnetic monopoles and an effect dual to the Meissner effect works to confine color charges [1, 2]. However to find color magnetic monopoles is not straightforward in QCD. If the dual Meissner effect picture is correct, it is absolutely necessary to derive such color-magnetic monopoles from gluon dynamics of QCD.

Motivated by an interesting work by Bonati et al.[3] which found violation of non-Abelian Bianchi identity (VNABI) exists behind the 'tHooft Abelian monopoles[4], the present author found in 2014 [5] an interesting and more fundamental fact that, when original gluon fields have a singularity where partial derivatives are not commutative, the non-Abelian Bianchi identity is broken and VNABI is just equal to the violation of Abelian-like Bianchi identities[6, 7].

2. Abelian-like monopoles of the Dirac type in QCD

Consider a covariant derivative operator $D_\mu = \partial_\mu - igA_\mu$. The Jacobi identities are expressed as $\epsilon_{\mu\nu\rho\sigma}[D_\nu, [D_\rho, D_\sigma]] = 0$. By direct calculations, one gets $[D_\rho, D_\sigma] = -igG_{\rho\sigma} + [\partial_\rho, \partial_\sigma]$, where $G_{\rho\sigma}$ is a non-Abelian field strength. The second commutator term can not be neglected if gauge fields have a line-like singularity. The relation $[D_\nu, G_{\rho\sigma}] = D_\nu G_{\rho\sigma}$ and the Jacobi identities lead us to

$$\begin{aligned} D_\nu G^*_{\mu\nu} &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} D_\nu G_{\rho\sigma} = -\frac{i}{2g}\epsilon_{\mu\nu\rho\sigma}[D_\nu, [\partial_\rho, \partial_\sigma]] \\ &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}[\partial_\rho, \partial_\sigma]A_\nu = \partial_\nu f^*_{\mu\nu}, \end{aligned} \tag{1}$$

where $f_{\mu\nu}$ is defined as $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\lambda^a/2$. Denoting the violation of the non-Abelian Bianchi identities (VNABI) as $\frac{1}{2}J^a_\mu \sigma^a = D_\nu G^*_{\mu\nu}$ and Abelian-like monopole currents without any gauge-fixing as $\frac{1}{2}k^a_\mu \sigma^a = \partial_\nu f^*_{\mu\nu}$, we get $J^a_\mu = k^a_\mu$ $(a = 1 \sim 8)$ from Eq.(1). It is to be noted that the Abelian-like monopole currents come from the singularity of the gauge fields and hence they are much the same as the Dirac monopole in QED[8]. Due to the antisymmetric property of the Abelian-like field strength, we get Abelian-like conservation conditions $\partial_\mu k_\mu = 0$ [9].

3. Perfect Abelian dominance and monopole dominance

It is very interesting to study the new Abelian-like monopoles in $SU(3)$ QCD. To check if the Dirac-type monopoles are a key quantity of color confinement in the continuum $SU(3)$ QCD, it is necessary to study monopoles numerically in the framework of lattice $SU(3)$ QCD and to study then if the continuum limit exists.

Refer to the reference[6] for the detailed method of extracting Abelian link fields from non-Abelian one and defining monopoles from Abelian link fields in $SU(3)$ QCD,

If the Abelian dual Meissner effect is the color confinement mechanism, the string tension σ_F of the non-Abelian static potential is expected to be equal to the Abelian string tension σ_a with respect to any color, i.e. $\sigma_F = \sigma_a$, since the string tension σ_F is regarded as the average of Abelian string tensions of 8 colors[10]. This is called as perfect Abelian dominance. The linear potential of Abelian static potentials comes from the solenoidal monopole current, so that the Abelian string tension is equal to that from the monopole solenoidal current, i.e. $\sigma_a = \sigma_m$[10]. This is called as perfect monopole dominance.

3.1 $SU(2)$ QCD case

In the framework of simpler $SU(2)$ QCD, interesting numerical results were obtained. Abelian and monopole dominances as well as the Abelian dual Meissner effect are seen clearly without any additional gauge-fixing already in 2009 [11, 12], although at that time, no theoretical explanation was clarified with respect to Abelian-like monopoles without any gauge-fixing. They are now found to be just Abelian-like monopoles proposed in the above paper [5].

3.2 $SU(3)$ QCD case

The perfect Abelian dominance is proved to exist with the help of the multilevel method[13, 14] but without introducing additional smoothing techniques like partial gauge fixings, although lattice sizes studied are not large enough to study the infinite volume limit[15]. Perfect monopole dominance on $24^3 \times 4$ at $\beta = 5.6$ is also shown without any additional gauge fixing but with a million of thermalized configurations.

Lattice size	β	σ_a/σ_F
12^4	5.6	0.87(13)
16^4	5.6	1.05(9)
12^4	5.7	0.91(8)
12^4	5.8	1.01(11)

Table 1: σ_a/σ_F determined by applying the multi-level method in the Wilson action

Types of the potential	σa^2
non-Abelian	0.178(1)
Abelian	0.16(3)
monopole	0.17(2)
photon	-0.0007(1)

Table 2: String tensions from Polyakov-loop correlations in the Wilson action at $\beta = 5.6$ on $24^3 \times 4$

3.3 $SU(3)$ QCD in MA gauge

The above results in $SU(2)$ and $SU(3)$ QCD are exact without any additional assumptions. However they are based on very small lattice volumes at few β points. To study the continuum scaling limit, we have to adopt larger lattice volume at a wide range of coupling constants. However exact Monte-Carlo simulations on large lattices are practically impossible to get meaningful results since the vacuum is full of dirty lattice-artifact monopoles. It is inevitable first to make the lattice vacuum as smooth as possible. The maximally Abelian (MA) gauge is known to reduce lattice-artifact monopoles[16, 17].

We adopt the Iwasaki gauge action[18, 19] on 48^4 lattices at $\beta = 2, 3 \sim 3.5$ in MA gauge. The perfect Abelian dominance is seen at $\beta = 2.9 \sim 3.5$ as shown in Fig.1 and the perfect monopole dominance is observed at $\beta = 2.9 \sim 3.5$ as in Fig.2.

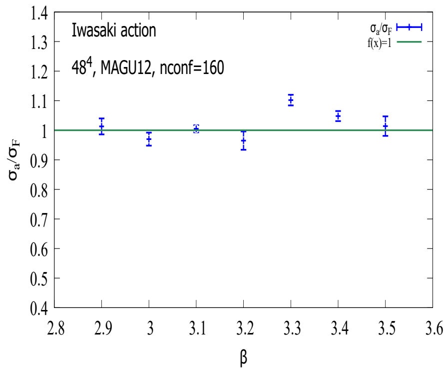

Figure 1: σ_a/σ_F

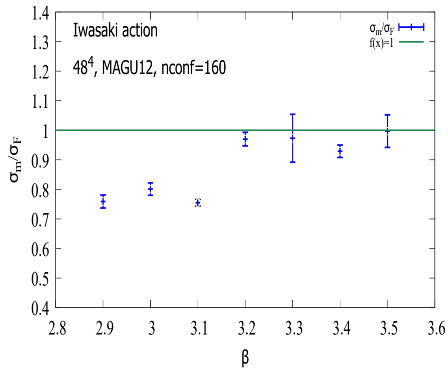

Figure 2: σ_m/σ_F

4. Block spin transformation studies of the monopoles

4.1 The block spin transformation method

The renormalization-group method based on the block spin transformation is known to be a powerful tool for studying the continuum limit and critical phenomena. The idea of the block spin with respect to Abelian monopoles on lattice was first introduced by Ivanenko et al.[22] and applied to the study obtaining an infrared effective monopole action in Ref.[23]. The n blocked monopole has a total magnetic charge inside the n^3 cube and is defined on a blocked reduced lattice with the spacing $b = na$.

The respective magnetic currents for each color are defined as

$$k_\mu^{(n)}(s_n) \quad = \quad \sum_{i,j,l=0}^{n-1} k_\mu(ns_n + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma}), \tag{2}$$

where s_n is a site number on the reduced lattice and the color indices are not shown explicitly. After the block spin transformation, the number of short lattice artifact monopole loops decreases while loops having larger magnetic charges appear. For details, see Ref.[6].

4.2 Monopole density

The first observable is the gauge-invariant monopole density. If the Abelian monopoles exist in the continuum limit, the monopole density must exist non-vanishing in the continuum. In $SU(2)$, this seems to be realized actually [6].

In $SU(3)$ we have eight Abelian-like conserved monopole currents instead of three in $SU(2)$. Since monopoles are three-dimensional objects, the monopole density is defined as follows:

$$\rho = \frac{\sum_{\mu,s_n} \sqrt{\sum_a (k_\mu^a(s_n))^2}}{4\sqrt{8}V_n b^3}, \tag{3}$$

where $V_n = V/n^4$ is the 4 dimensional volume of the reduced lattice, $b = na(\beta)$ is the spacing of the reduced lattice after n-step block spin transformation. In general, the density ρ is a function of two variables β and n, i.e., $\rho = \rho(n, a(\beta))$. When we change β larger for fixed number of blocking step, the monopole density decreases in the case of original unblocked monopole currents. No asymptotic scaling is seen for fixed number of blocking. On the otherhand, we change the number of blocking steps from $n = 1$ to $n = 12$, the monopole density increases monotonously for fixed β.
But it is interesting to show that, if we plot the monopole density versus blocked lattice distance $b = na(\beta)$, we get a universal curve $\rho(n, a(\beta)) \to \rho(b = na(\beta))$ depending on b alone as shown in Fig. 3. There is a beautiful scaling function similarly as observed in $SU(2)$[6] as in Fig. 4, although the latter $SU(2)$ results have smaller errorbars and more appealing. If the same behavior $\rho(b = na(\beta))$ is kept for $n \to \infty$, it correponds to the non-zero monopole density at $a(\beta) \to 0$, i.e., in the continuum limit. Although we have studied the block spin transformation up to $n = 12$, the results

obtained support strongly existence of the continuum limit of the Abelian-like monopoles, since the universal scaling function depending only on b is realized.

In $SU(2)$, we have studied three other smooth gauge fixings as well as MAU1 and no gauge-dependence is seen as expected from the new type of Abelian-like monopoles[5]. On the otherhand in $SU(3)$, we have not yet obtained another reliable gauge-fixed smooth vacuum ensemble except for those in MAU12. Hence to prove existence of the new type of Abelian-like monopoles in $SU(3)$, the scaling behavior in MAU12 alone is not enough. Gauge independence is still to be studied.

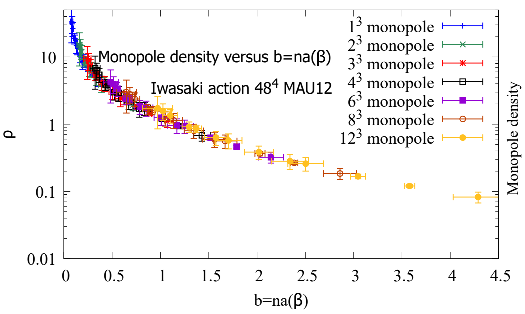

Figure 3: Monopole density versus $b = na(\beta)$

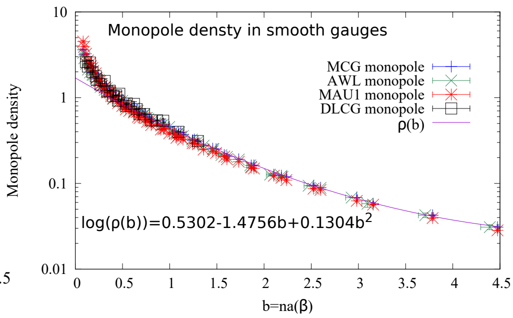

Figure 4: Monopole density behaviors versus $b = na(\beta)$ in SU2

4.3 The renormalization flow studies of the monopole infrared effective action

The effective monopole action is defined as follows:

$$e^{-\mathcal{S}[k]} = \int DU(s,\mu)e^{-S(U)} \times \prod_a \delta(k^a_\mu(s) - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu n^a_{\rho\sigma}(s+\hat{\mu})).$$

Practically, we have to restrict the number of interaction terms of monopoles. It is natural to assume that monopoles which are far apart do not interact strongly and to consider only short-ranged local interactions of monopoles. We adopt most dominant two-point monopole interactions $\mathcal{S}[k] = \sum_i^{10} F(i)\mathcal{S}_i[k]$ as shown in Table3. We determine the set of couplings $F(i)$ from the monopole current ensemble $\left\{k^a_\mu(s)\right\}$ with the aid of an inverse Monte-Carlo method first developed by Swendsen [24] and extended to closed monopole currents by Shiba and Suzuki [23]. The details of the inverse Monte-Carlo method are reviewed in AppendixA of Ref. [21]. Since we now consider vacuum configurations in the smooth MAU12 gauge, only the diagonal components are important. Hence, we consider only the monopole currents having a color 3. As studied in the previous section discussing the monopole density, we perform the block-spin transformation of monopole currents for $n = 1, 2, 3, 4, 6, 8, 12$ on 48^4 at $\beta = 2.3 \sim 3.5$ and try to fix the infrared monopole actions for all blocked monopoles.

Table 3: The quadratic interactions used for the modified Swendsen method.

coupling	distance	*type*			
$F(1)$	(0,0,0,0)	$k_\mu(s)k_\mu(s)$	$F(6)$	(1,1,1,0)	$k_\mu(s)k_\mu(s+\hat{\mu}+\hat{\nu}+\hat{\rho})$
$F(2)$	(1,0,0,0)	$k_\mu(s)k_\mu(s+\hat{\mu})$	$F(7)$	(0,1,1,1)	$k_\mu(s)k_\mu(s+\hat{\nu}+\hat{\rho}+\hat{\sigma})$
$F(3)$	(0,1,0,0)	$k_\mu(s)k_\mu(s+\hat{\nu})$	$F(8)$	(2,0,0,0)	$k_\mu(s)k_\mu(s+2\hat{\mu})$
$F(4)$	(1,1,0,0)	$k_\mu(s)k_\mu(s+\hat{\mu}+\hat{\nu})$	$F(9)$	(1,1,1,1)	$k_\mu(s)k_\mu(s+\hat{\mu}+\hat{\nu}+\hat{\rho}+\hat{\sigma})$
$F(5)$	(0,1,1,0)	$k_\mu(s)k_\mu(s+\hat{\nu}+\hat{\rho})$	$F(10)$	(0,2,0,0)	$k_\mu(s)k_\mu(s+2\hat{\nu})$

Although we restrict ourselves to important two-point monopole current interactions, we get the scaling behaviors also. Namely, all coupling constants which are a two-point function of n and $a(\beta)$ are actually found to be a function of $b = a(\beta)$ alone.

These results are all on 48^4 lattice for various coupling constants of the Iwasaki gauge action, adopting MAU12 gauge for reducing the lattice-artifact monopoles. It is absolutely necessary to show gauge independence to prove the new type of Abelian monopoles coming from the violation of non-Abelian Bianchi identity at least as done in $SU(2)$[11, 12] without adopting any additional gauge fixing. But such studies in $SU(3)$ seem at present stage almost impracticable except for the previous study on a small lattice[15]. Hence it is desirable to study in smooth gauges other than MAU12 as done in $SU(2)$ case[6, 21].

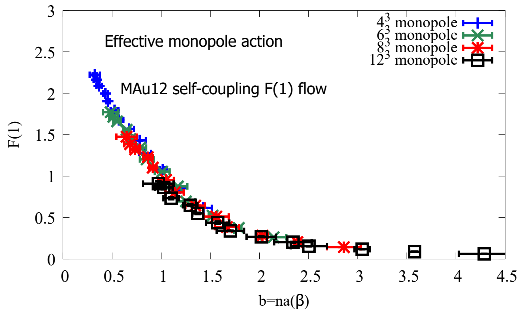

Figure 5: The self-coupling constant $F(1)$ versus $b = na(\beta)$

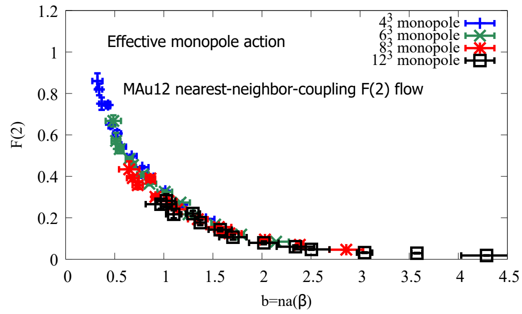

Figure 6: The nearest-neighbor coupling constant $F(2)$ versus $b = na(\beta)$.

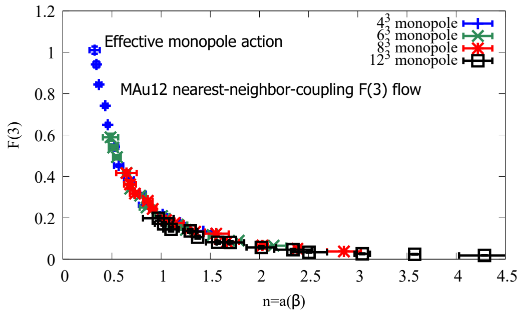

Figure 7: Another nearest-neighbor coupling constant $F(3)$ versus $b = na(\beta)$

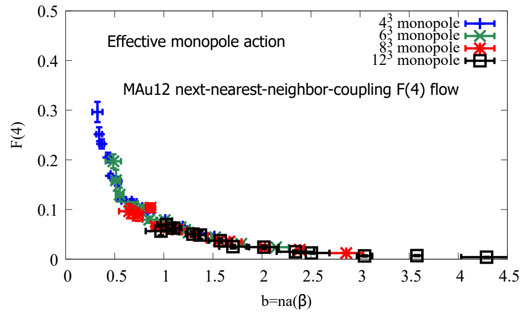

Figure 8: Next to the nearest-neighbor coupling constant $F(4)$ versus $b = na(\beta)$.

5. Summary

In this note, the scaling behaviors of new Abelian-like monopoles in pure $SU(3)$ QCD are studied adopting the Iwasaki improved gauge action for wide range of β and the number of blocking transformations from $n = 1, 2, 3, 4, 6, 8, 12$. To reduce lattice-artifact monopoles, we adopt here the maximally Abelian gauge and $U1 \times U1$ Landau gauge.

(1) The perfect Abelian dominance and the perfect monopole dominance are seen fairly well with respect to Abelian and monopole string tensions. The asymptoic scaling behaviors are observed fairly well in these cases.

(2) The block-spin transformation studies with respect to Abelian monopoles are done. The behaviors of the monopole densities $\rho(n, a(\beta))$ of the blocked monopole currents show the beautiful scaling behavior: $\rho(n, a(\beta)) = \rho(b = na(\beta))$, i.e. ρ is a function of $b = na(\beta)$ alone. The scaling behaviors are seen here for $n = 1, 2, 3, 4, 6, 8, 12$. If on larger lattices, similar scaling behaviors are seen for $n \to \infty$, it means $a(\beta) \to 0$, the continuum limit. It is stressed that, although we adopt MAU12 gauge, the scaling behavior of the monopole density is seen with respect to $SU(3)$ invariant combination summing over all color components.

(3) Adopting the inverse Monte Carlo method, we determine the coupling constant flow of the effective monopole action under the blocking transformation. Although we restrict ourselves to important two-point monopole current interactions, we get the scaling behaviors also. Namely, all coupling constants which usually a two-point function of n and $a(\beta)$ are actually found to be a function of $b = a(\beta)$ alone.

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References

[1] G. 't Hooft, in *Proceedings of the EPS International*, edited by A. Zichichi, p. 1225, 1976.

[2] S. Mandelstam, Phys. Rept. **23**, 245 (1976).

[3] C. Bonati, A. Di Giacomo, L. Lepori and F. Pucci, Phys. Rev. **D81**, 085022 (2010).

[4] G. 't Hooft, Nucl. Phys. **B190**, 455 (1981).

[5] T. Suzuki, A new scheme for color confinement due to violation of the non-Abelian Bianchi identities, hep-lat: arXiv:1402.1294 (2014)

[6] T. Suzuki, K. Ishiguro and V. Bornyakov, Phys. Rev. **D97**, 034501 (2018); Phys. Rev. **D97**, 099905(E) (2018).

[7] T.Suzuki, Monopoles of the Dirac type and color confinement in QCD - $SU(3)$ invariant picture of color confinement -, hep-lat: arXiv:2204.11514.

[8] P. Dirac, Proc. Roy. Soc. (London) A 133, 60 (1931).

[9] J. Arafune, P.G.O. Freund and C.J. Goebel, J.Math.Phys. **16**, 433 (1975).

[10] Y. Koma, M. Koma, E.-M. Ilgenfritz, T. Suzuki and M.I. Polikarpov, Phys.Rev. D68 (2003) 094018, arXiv: hep-lat/0302006.

[11] T. Suzuki, K. Ishiguro, Y. Koma and T. Sekido, Phys. Rev. **D77**, 034502 (2008).

[12] T. Suzuki, M. Hasegawa, K. Ishiguro, Y. Koma and T. Sekido, Phys. Rev. **D80**, 054504 (2009).

[13] M. Lüscher and P. Weisz, JHEP **0109**, 010 (2001), hep-lat/0108014.

[14] M. Lüscher and P. Weisz, JHEP **0207**, 049 (2002), hep-lat/0207003.

[15] K.Ishiguro, A.Hiraguchi and T. Suzuki,Phys Rev. **D106** .014515 (2022), arXiv:2207.04436.

[16] A. S. Kronfeld, M. L. Laursen, G. Schierholz, and U. J. Wiese, Phys. Lett. **B198**, 516 (1987).

[17] A. S. Kronfeld, G. Schierholz, and U. J. Wiese, Nucl. Phys. **B293**, 461 (1987).

[18] Y. Iwasaki, Nucl. Phys. B258 (1985) 141; Univ. of Tsukuba report UTHEP-118 (1983), unpublished.

[19] S. Takeda et al. (CP-PACS Collaboration), Phys. Rev. D 70, 074510 (2004).

[20] T. Suzuki, Phys. Rev **D107**, 094503 (2023).

[21] T. Suzuki, Phys. Rev **D97**, 034509 (2018).

[22] T.L. Ivanenko, A. V. Pochinsky and M.I. Polikarpov, Phys. Lett. **B302**, 458 (1993).

[23] H. Shiba and T. Suzuki, Phys. Lett. **B351**, 519 (1995).

[24] R.H. Swendsen,Phys. Rev. Lett. **52**,1165 (1984).