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Axions and Festina Lente

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I provide a brief review of the Festina lente bound, which provides a lower bound on the mass of charged particles in quasi-de Sitter spacetimes. From this, I go on to explain how from Festina Lente one can obtain an analogous bound for axions on the decay constant and instanton action. I provide a brief overview of some of the phenomenological implications of this bound.

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1. Introduction

In recent years, the Swampland Program has played an important role in the development of string theory and quantum gravity (See [1–4] for reviews). The Swampland program aims constrain low-energy effective theories by demanding that in the UV they can complete into consistent theories of quantum gravity. The constraints which the Swampland Program imposes on low-energy physics are referred to as swampland conjectures. The Swampland Program is also of interest for axion physics as some of the Swampland conjectures constrain axion physics, for instance the axionic weak gravity conjecture (aWGC) [5, 6].

Here, I will review a swampland conjecture known as Festina Lente (FL) [7, 8] in Sec. 2. FL constrains the masses of charged particles in quasi-de Sitter cosmologies. In Sec. 3 I will review how FL can be turned into a bound on axions known as axionic Festina Lente (aFL) [9]. In Sec. 4 I will discuss some phenomenological implications of (a)FL. For simplicity and for the sake of phenomenological interest this note focuses on the case of four external spacetime dimensions.

2. The Festina Lente Bound

One of the most fruitful avenues for exploring swampland conjectures is by considering black holes. One demands that black holes 'behave nicely' with respect to what physical principles one think should hold in a theory of quantum gravity and from this derives constraints on the physics. A famous example of this is one of the arguments for the Weak Gravity Conjecture [5]. One argues that subextremally charged black holes should be able to decay for physical consistency. Demanding that near-extremal black holes charged under a $U(1)$ guage field with coupling g can decay, one finds that there must exist a particle with mass m and charge q obeying

$$m \leq \sqrt{2} g q M_P \,, \tag{1}$$

with M_P the Planck mass, in order for the black hole decay process to be possible.

one can now consider subextremally charged black holes in de Sitter space rather than flat space and demand that these decay back to empty de Sitter space. This is precisely what was done in [7]. The novel feature in de Sitter space the a cosmic horizon which effectively sets a maximum size for subextremal black holes. If one makes a black hole in de Sitter space too massive one obtains a naked singularity because, loosely speaking, the black hole horizon no longer fits inside the cosmic horizon. There is now a new kind of extremal black hole, the (charged) Nariai black hole [10, 11], which is the most massive black hole (at a given charge) one can have in a de Sitter background as shown in Fig. 1. Demanding that charged near-Nariai black holes evaporate to empty de Sitter space requires that *all* charged particles obey the bound[1] [7]

$$m^4 \geq 6(g q M_P H)^2 = 2(g q)^2 V \,, \tag{2}$$

which is known as the Festina Lente (FL) bound. If there exists a particle which is lighter than the FL bound, this will trigger a very rapid discharge for near-Nariai black holes with a large charge, shown

[1]The black hole decay analysis of [7] yields this bound parametrically. The numerical factor is fixed by combining this result with other swampland conjectures [8].

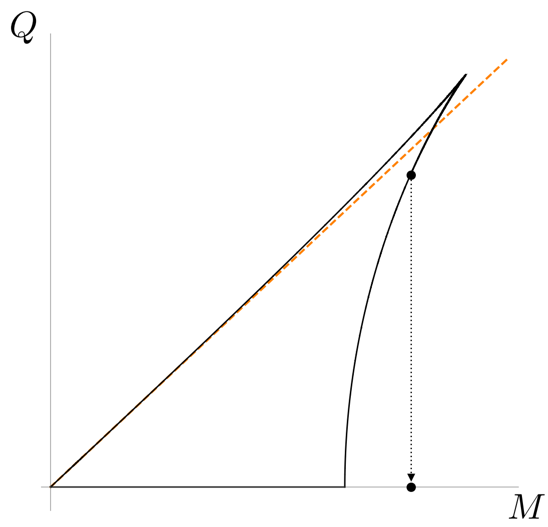

Figure 1: Diagram of possible black hole mass M versus charge Q in de Sitter space. The solid left black line are extremal charged black holes, the solid right black line are charged Nariai black holes and the orange dashed line is $Q = M$. The region inside sharkfin-shaped region given by the solid black lines corresponds to subextremal black holes. The geometries outside the sharkfin shape correspond to singular geometries. The solid dot is a Near-nariai geometry with large charge and the black dashed trajectory qualitatively describes its evaporation in the presence of a light charged particle violating eq. 2. This figure was taken from [7].

in Fig. 1. During this process the black hole rapidly sheds charge without a substantial change in mass. As a result the black hole evolves out of the subextremal regime into the superextremal singular regime, finally ending in a singular big crunch rather than evolving back to empty de Sitter space.

Note that the WGC in the bare-bones form which we discuss here only requires one particle to obey Eq. 1 while all particles should obey FL. This is because the WGC follows from demanding that a particle exists to trigger the BH decay while FL follows from forbidding the existence of a particle that results in singular behaviour in contradiction with cosmic censorship.

3. Festina Lente and Axions

Having obtained the Festina Lente bound for gauge fields, it is natural to ask whether an analogous bound exists for axions. One could attempt to directly derive the analogous bound by considering a physical process involving axions in de Sitter space. Two such processes were considered in [9]: the decay of black holes with axion charge in de Sitter space and euclidean black holes in de Sitter space. Neither of these led to a direct bound on axions. Instead, [9] started from the Festina Lente bound for gauge fields as given and dualized this into a bound on axions. The rough outline[2] of this dualization procedure is that one assumes the four-dimensional theory one started from is the result of the dimensional reduction of a ten-dimensional theory. A Dp-brane in the ten-dimensional theory wrapping a p-cycle Σ_p in the internal dimensions reduces

[2]The detailes of this dualization are disucssed in [9]. In fact, it is the dipole version of FL that one must dualize, not the charged particle version discussed in Sec. 2.

to a particle in the four-dimensional theory. One may then translate the FL and WGC bounds on the charged particle into geometric constraints on the cycle which the Dp-brane is wrapping. Taking the compactification geometry to be a Calabi-Yau threefold, X one finds

$$\frac{1}{2} \lesssim \frac{2V_X|q^{\Sigma^p}|^2}{V_{\Sigma^p}^2} \lesssim \frac{M_P}{H}\,, \tag{3}$$

where the inequality on the right (left) follows from FL (WGC). Here V_X is the CY volume and V_{Σ^p} is the volume of the cycle Σ_p which the brane is wrapping. Further,

$$|q^{\Sigma^p}|^2 \equiv K^{ij} q_i^{\Sigma^p} q_i^{\Sigma^p}\,; \quad q_i^{\Sigma^p} \equiv \int_{\Sigma^p} \omega_i\,; \quad K_{ij} \equiv \int_X \omega_i \wedge *\omega_j\,, \tag{4}$$

where the ω_i form a symplectic basis of the harmonic p-forms of the pth cohomology of X. By now considering a D$p-1$ brane wrapping Σ_p, one obtains from Eq. 4 a constraint on axions and their instantons. One finds for the axion decay constant f and instanton acion S the constraint

$$\frac{1}{\sqrt{2}} \lesssim \frac{M_p}{Sf} \lesssim \sqrt{\frac{M_P}{H}}\,, \tag{5}$$

where the right inequality follows from FL and is called axionic Festina Lente (aFL). The left inequality is the axionic Weak Gravity Conjecture (aWGC) [6].

4. Phenomenological implications

The bounds of Sec. 2 and 3 yield a wealth of phenomenological implications, see e.g. [7–9, 12, 13]. We will review a few implications covered [7–9] in here.

As a basic check of FL, in our world for electromagnetism one has $\sqrt{gM_PH} \sim 10^{-3}$eV. As the electron has a mass of order 0.5 MeV, the standard model indeed satisfies FL for the electromagnetic force [7]. An interesting coincidence (?) is that the scale $\sqrt{M_PH}$ is roughly the neutrino mass difference scale. While neutrinos are not known to be charged under any $U(1)$ gauge field, it could be that the neutrinos are charged under some hidden $U(1)$. If so, FL would provide a reason why neutrinos are not massless. If the hidden gauge coupling is not too small and the neutrino masses are of the same scale as the mass differences, then FL would motivate the mass scale of the neutrinos.

Nonabelian gauge theories carry $U(1)$ Cartan subgroups. By applying the argument of Sec. 2 to this subgroup, FL should also apply to nonabelian gauge theories [8]. The nonabelian gauge fields are themselves charged under the $U(1)$ subgroup. If there gauge fields are massless and interacting at long range, they violate FL. To avoid this, the gauge fields must be either Higgsed or confined, with the Higgsing or confinement scale at shorter lengths than the Hubble scale to avoid long-range interactions. Without long-range interaction the BH analysis of Sec. 2 no longer goes through and one sidesteps the FL bound. All nonabelian gauge fields in the SM are indeed confined or Higgsed at energies above the Hubble scale.

By considering the geometric version of the FL bound, Eq. 4, one can constrain string compactifications and in this way constrain e.g. inflation. For blow-up inflation [14] where inflation is driven by a rolling small four-cycle $\tau_{\rm inf}$ the geometry is constrained as

$$\frac{H}{M_P} \lesssim \frac{\tau_{\rm inf}^{3/2}}{V_X} \lesssim 4\,, \tag{6}$$

where H is the Hubble scale during inflation. This leads to a preference for low-scale inflation [9] and constrains the slow-roll parameter $\varepsilon^* < 3 \cdot 10^6/V_X^2$ for a CY with a typical volume $V_X \sim 10^6$ this leads to a bound on the tensor-to-scalar ratio of $r < 5 \cdot 10^{-5}$.

Let us now consider constraints following from the axionic version of FL. Axions can serve as a source of kinetic mixing between different $U(1)$ gauge fields. one can start with a theory with a visible gauge field F and a hidden gauge field G with a coupling to an axion a. Consider a Lagrangian containing terms

$$\mathcal{L} \supset -\frac{1}{4} F \wedge \star F - \frac{1}{4} G \wedge \star G + \frac{\chi}{2} F \wedge \star G - \frac{a}{SfN} G \wedge G\,. \tag{7}$$

A Lagrangian with such an axion coupling arises for instance when the gauge field G is the DBI gauge theory of a stack of N D7-branes wrapping an internal four-cycle. One may diagonalize this action into the form

$$\begin{aligned}\mathcal{L} \supset &-\frac{1}{4}\left(1-\chi^2\right) F \wedge \star F - \frac{1}{4}\hat{G} \wedge \star \hat{G} \\ &- \frac{a}{NSf}\left(\hat{G} \wedge \hat{G} + \chi \hat{G} \wedge F + \chi F \wedge \hat{G} + \chi^2 F \wedge F\right)\end{aligned} \tag{8}$$

Defining g to be the coupling between the axion a and the visible photon F, one can now show from aFL that the kinetic mixing is bounded from below as [9]

$$\chi^2 \gtrsim Ng\sqrt{M_P H} \gtrsim g\sqrt{M_P H}\,, \tag{9}$$

where last inequality follows $N \geq 1$ as the number of branes is quantized. While this derivation relies on the specific set-up of a stack of D7 brane, the inequality $\chi^2 \gtrsim g\sqrt{M_P H}$ is independent of set-up specific quantities suggesting that it may hold generically.

One may consider inflation driven by axion monodromy [15, 16]. the potential for the axion ϕ is given by

$$V(\phi) = \mu^{4-p}\phi^p + \Lambda^4 \cos\left(\frac{\phi}{f}\right) \qquad , \tag{10}$$

with $p \leq 2$. One can now turn the bound of Eq. 5 into bounds on inflationary observables [9]. For example, axion monodromy inflation has resonant non-gaussianities $f_{\rm res}$ [17] whose value is constrained to lie within an interval as shown in Fig. 2.

So far, we have focused on the case of a single gauge field (axion) and a single charged particle (instanton) coupled to it. our analysis can be generalized in a natural way to the multifield case, in which case the constraints become stronger. We refer to [8, 9] for a detailed discussion of multifield constraints. To give one example of a constraint, the combination of aFL and aWGC bound becomes increasingly difficult to satisfy the more axions one has. One may argue [9] that for N axions it is generically required that $N < M_P/H$ in order to satisfy both aFL and aWGC. For present-day values of H this is a very weak constraint. The bound is significantly stronger during high-scale inflation and string compactifications with many cycles, and hence many axions, may run into issues with this bound during high-scale inflation. This again suggests a preference for low-scale inflation.

In this section I have briefly reviewed a few phenomenological implications of (a)FL. Subjects where FL provides phenomenological constraints but which I have not discussed include for instance

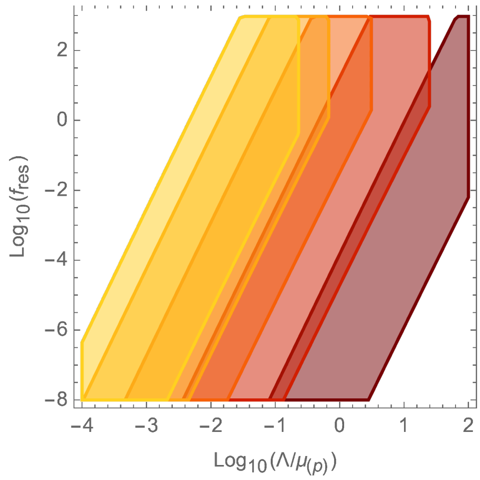

Figure 2: Permitted values of f_{res} based on FL and WG conjectures, monotonicity of the inflationary potential, and experimental constraints. This figure is taking from [9] to which I refer for further details of its derivation.

dark sector constraints [12] and constraints on how supersymmetry-breaking is mediated [13]. I hope that the lightning overview in this section has provided some feeling for the strength and breath of constraints provided by Festina Lente. Several of the constraints discussed were during inflation. As H is larger during inflation, (a)FL is more strongly constraining during the inflationary era than during our current epoch.

The main argument for Festine Lente which we have sketched is based on black hole evaporation. Let me give further pieces of evidence that FL is a legitimate universal principle of quantum gravity. First, [8] studied controlled string theory set-ups and showed that these obey the FL bound. Second, the standard model obeys FL despite the nontrivial predictions made by FL. It would be interesting to establish FL more rigorously and find possible generalizations of FL (or to find a counterargument) in further work. In the context of axion physics, it would be interesting to apply aFL to a broader range of models where axions appear and see what constraints aFL puts on each model.

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