PROCEEDINGS OF SCIENCE

Metastable strings and grand unification

Wilfried Buchmüller[*]

Deutsches Elektronen Synchrotron,
Notkestrasse 85, Hamburg, Germany

E-mail: `wilfried.buchmueller@desy.de`

The structure of the Standard Model (SM) of particle physics points toward grand unified theories (GUTs) where strong and electroweak interactions are unified in a non-Abelian GUT group. The spontaneous breaking of the GUT symmetry to the SM symmetry, together with cosmic inflation, generically leads to metastable topological defects, the most prominent example being cosmic strings. The gravitational-wave background (GWB) produced by a cosmic string network is one of the candidates for an explanation of the GWB recently observed by pulsar timing array (PTA) experiments. We review some properties of the predicted GWB with emphasis on potential implications for GUT model building. The most striking prediction is a GWB in the LIGO-Virgo-KAGRA band that could be discovered in the near future.

International Conference on Particle Physics and Cosmology (ICPPCRubakov2023)
02-07, October 2023
Yerevan, Armenia

---

[*]Speaker

https://pos.sissa.it/

PoS(ICPPCRubakov2023)004

1. Ultraviolet completion of the Standard Model

The structure of the Standard Model of particle physics points toward "grand unification" of strong and electroweak interactions: quarks and leptons form complete multiplets of grand unified (GUT) groups, and the gauge couplings of strong and electroweak interactions unify at a large energy scale (GUT scale), approximately without supersymmetry and more precisely with supersymmetry. The Pati-Salam [1] and SO(10) [2, 3] GUT groups contain $B-L$, the difference between baryon and lepton number, as a local symmetry. The spontaneous breaking of $B-L$ can generate large Majorana masses for right-handed neutrinos, and if Yukawa couplings in the neutrino sector have a similar structure as Yukawa couplings of quarks and charged leptons, the right order of magnitude of the light-neutrino mass scale emerges as consequence of the seesaw mechanism. Supersymmetric GUTs can be derived from higher-dimensional field theories and string theory, providing a UV completion of the Standard Model (for recent reviews, see, e.g. [4, 5]). From gauge coupling unification we know the order of magnitude of the GUT scale, $\Lambda_{\mathrm{GUT}} \simeq 10^{15} \ldots 10^{16}$ GeV. Since no evidence for superparticles has been found at the LHC, the scale of supersymmetry breaking has to lie above a TeV, as for instance in PeV-scale supersymmetry [6, 7].

Supersymmetric GUT models contain all ingredients needed in early universe cosmology: the observed matter-antimatter asymmetry can be generated by sphaleron processes [8] from a lepton asymmetry produced in decays of heavy Majorana neutrinos [9]; the lightest superparticle, gravitino, wino or higgsino, is a candidate for dark matter [10]; inflation can be realized in several ways, in particular as hybrid inflation [11, 12]. A specific example that illustrates the interplay of these mechanisms is the decay of a false $B-L$ vacuum, broken close to the GUT scale and ending in tachyonic preheating [13] where the initial conditions of the hot early universe are generated [14]. The relevant energy scales are displayed in Fig. 1: the effective neutrino mass $\tilde{m}_1$ controls leptogenesis, the gravitino mass $m_{3/2}$ provides a constant slope in the almost flat inflaton potential, v_{B-L} is the scale of $B-L$ breaking, and T_{rh} is the reheating temperature; inflation requires a small Yukawa coupling in the $B-L$ Higgs superpotential, leading to a reheating temperatures $T_{\mathrm{rh}} \simeq 10^{8} \ldots 10^{10}$ GeV, and the observed tensor-to-scalar ratio is obtained for gravitino masses in the range $m_{3/3} \simeq 10$ TeV $\ldots$ 10 PeV.

During the $B-L$ phase transition following hybrid inflation a network of cosmic strings is formed that acts as source of a gravitational wave background (GWB) (for reviews and references, see, e.g. [15, 16]). GUT-scale strings have a string tension in the range $G\mu \simeq 10^{-8} \ldots 10^{-6}$, where G denotes Newton's constant and μ is the energy per unit length of the string. Stable GUT-scale strings are excluded by pulsar timing array (PTA) experiments [17–19], whereas strings below the GUT scale are possible and may even render the GWB a probe of thermal leptogenesis [20]. Moreover, as shown in [21], the PTA bound on the string tension can be avoided for metastable cosmic strings that decay into string segments connecting monopole–antimonopole pairs [22]. In the semiclassical approximation, the decay rate per string unit length is given by [23–25]

$$\Gamma_d \simeq \frac{\mu}{2\pi} \exp\left(-\pi\kappa\right) , \quad \kappa = \frac{m_M^2}{\mu} , \tag{1}$$

where m_M is the monopole mass. Since the decay rate of a string is proportional to its length, the spectrum of radiated GWs is suppressed at small frequencies. This effect is clearly visible in Fig. 2, and a quantitative analysis shows that the PTA bound can be avoided for values $\kappa \lesssim 8$.

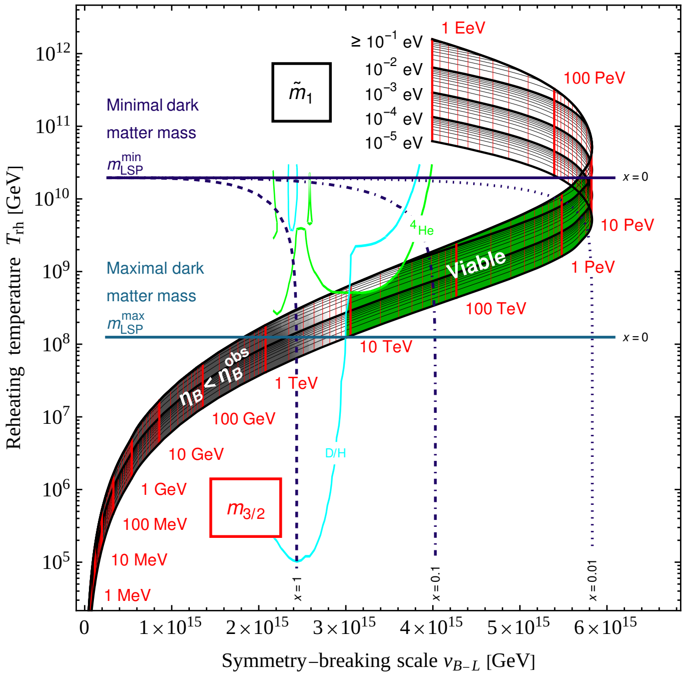

Figure 1: Viable parameter space (green) for hybrid inflation, leptogenesis, neutralino DM, and big bang nucleosynthesis. Hybrid inflation and the dynamics of reheating correlate the parameters v_{B-L}, $T_{\rm rh}$, $m_{3/2}$ and $\widetilde{m}_1$ (black curves). Successful leptogenesis occurs outside the gray-shaded region. Neutralino dark matter is viable in the green region, corresponding to a higgsino (wino) with mass $100 \leq m_{\rm LSP}/{\rm GeV} \leq 1060$ (2680). From [21].

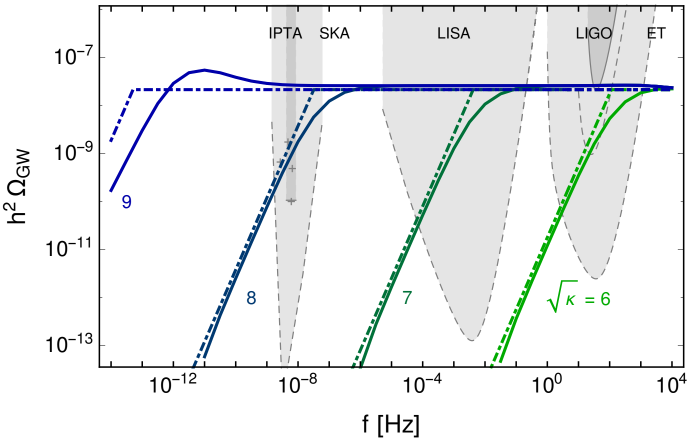

Figure 2: GW spectrum for $G\mu = 2\times10^{-7}$. Different values of $\sqrt{\kappa}$ are indicated in different colors; the blue curve corresponds to a cosmic-string network surviving until today; the number of SM degrees of freedom is fixed to its high-temperature value, $g_* = 106.75$; the dot-dashed lines depict an analytical estimate. The (lighter) gray-shaded areas indicate the sensitivities of (planned) experiments SKA [26], LISA [27], LIGO [28] and ET [29], the crosses within the SKA band indicate constraints by the IPTA [30]. From [21].

2. Gravitational-wave background

Three years ago the NANOGrav collaboration reported evidence for a stochastic common-spectrum process at nanohertz frequencies [31]. The results received considerable attention since the signal could be interpreted as a GWB of astrophysical [32] and/or cosmological origin including stable strings [33–36] as well as metastable strings with $\sqrt{\kappa} \simeq 8$ [37].

Given the exponential dependence of the decay rate on the parameter κ, and considering monopole masses larger than the string scale, metastable strings were generally assumed to be effectively stable [15]. However, one easily verifies that a value $\sqrt{\kappa} \sim 8$ can indeed be obtained in realistic GUTs. Taking as a simple example the electroweak gauge group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$, broken to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{U}(1)_{B-L}$ by an $\mathrm{SU}(2)_R$ triplet at a scale v_u, and further to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ by an $\mathrm{SU}(2)_R$ doublet at scale v_s, with $Y = T^3_R + \frac{1}{2}(B-L)$, one finds [38]

$$\kappa = \frac{m_M^2}{\mu} \sim \frac{4\pi}{g^2 \cos^2\Theta}\left(\frac{m_V}{m_X}\right)^2 , \tag{2}$$

where monopole mass and string tension for isolated systems have been used as as a rough estimate. Here $m_V = g\sqrt{2v_u^2 + v_s^2}/\sqrt{2}$ and $m_X = g v_s/(\sqrt{2}\cos\Theta)$ are the masses of the charged and neutral vector bosons, respectively, and Θ is the mixing angle of the $\mathrm{U}(1)_R$ and $\mathrm{U}(1)_{B-L}$ vector bosons. For an embedding of the electroweak group into a Pati-Salam or SO(10) GUT group one has $\tan\Theta = \sqrt{3/2}$, and with a GUT-scale gauge coupling $g \simeq 1/\sqrt{2}$ one obtains $\sqrt{\kappa} \sim 8$ for $m_V \sim m_X$, i.e., the SU(2) and U(1) breaking scales have to be of the same order of magnitude.

Computing the GW spectrum emitted from a cosmic string network is a complicated problem where many questions are still open, even for stable strings (for a discussion and references, see [39–41]). One first studies the approach of the network to a scaling regime in which the relative contribution to the total energy density of the universe remains constant and computes the number density of non-self-interacting loops per unit string length by means of Nambu-Goto string simulations. Together with a model for the average power radiated off in GWs by each loop, this yields the GW spectrum.

For metastable cosmic strings a detailed study has been carried out in [42], see also [43, 44]. Compared to stable strings, the kinetic equations for the string network are modified to take into account the decay of string loops to segments through the formation of a monopole–antimonopole pair, as well as the formation of segments from longer segments and super-horizon strings. As demonstrated in [42], the GW spectrum generated by string loops alone provides a good approximation to the full spectrum in most of the parameter space. The key change compared to stable cosmic strings is an additional decay term in the kinetic equation for the loop number density accounting for the monopole–antimonopole formation on the loops. Following [24], the number density $\overset{\circ}{n}(\ell,t)$ of loops with length ℓ is matched to the loop number density of stable strings at early times, $t \ll t_s = 1/\Gamma_d^{1/2}$, which is determined by numerical simulations. In the radiation-dominated era one then obtains for the loop number density of metastable strings at $t > 1/\Gamma_d^{1/2}$ [42],

$$\overset{\circ}{n}{}^{\mathrm{rad}}(\ell,t) = \frac{B}{t^{3/2}(\ell + \Gamma G\mu t)^{5/2}}\, e^{-\Gamma_d[\ell(t-t_s)+\frac{1}{2}\Gamma G\mu(t-t_s)^2]}\,\Theta(\alpha t_s - \ell - \Gamma\mu(t-t_s))\,. \tag{3}$$

Here, the exponential factor accounts for the decay of the loops at $t > t_s$ through the generation of monopoles, and the Heaviside function ensures that loop formation only occurs at $t < t_s$. $\Gamma \simeq 50$

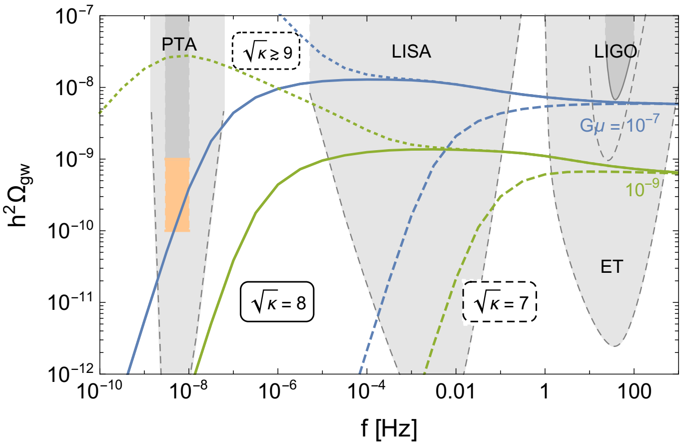

Figure 3: GW spectrum from metastable cosmic strings with a string tension of $G\mu = 10^{-7}$ (blue) and $G\mu = 10^{-9}$ (green), with $g_* = g_*(T)$. Different line styles indicate different string lifetimes, ranging from $\sqrt{\kappa} = 7$ (dashed) over $\sqrt{\kappa} = 8$ (solid) to the limit of (quasi-)stable strings $\sqrt{\kappa} \gtrsim 9$ (dotted). The gray-shaded areas indicate the sensitivity of current (solid) and planned (dashed) GW experiments. The orange region indicates the preferred region of the possible GWB signal observed by PTAs. See [42, 45].

is the power in GWs emitted by a single loop, and the parameters B and α are determined by numerical simulations. Expressions for the loop number densities involving evolution during the matter-dominated era, as well as expressions for the number densities of super-horizon strings and segments, are given in [42].

As for stable cosmic strings, a population of string loops with number density $\mathring{n}(\ell, t)$ yields the spectral energy density in GWs today normalized by the critical energy density [40, 41],

$$\Omega_{\rm gw}(t_0, f) = \frac{16\pi (G\mu)^2}{3H_0^2 f} \sum_k k P_k \int_0^{z_i} \frac{dz'}{H(z')(1+z')^6} \mathring{n}(2k/f', t(z')) \,. \tag{4}$$

Here $f' = 2k/\ell$ indicates the GW frequency, emitted by a loop of length ℓ oscillating in its kth harmonic excitation at time t' of GW emission, and the integration over t' from some initial time t_i to t_0 has been traded for an integration over the redshift z; $H(z)$ is the Hubble parameter, and the argument of the loop number density ensures that we are accounting for all GWs emitted at frequency f' such that after red-shifting, they are observed at frequency f today. $\Omega_{\rm gw}$ depends on two parameters, the string tension $G\mu$ and the decay parameter κ.

Fig. 3 shows the GW spectrum obtained by inserting Eq. (3) and the corresponding expression for the matter-dominated era into Eq. (4). The colored curves indicate predictions for the GW spectrum for different values of κ and μ. The dotted black curves show the limit of stable cosmic strings. Large frequencies correspond to GWs produced at early times, and hence the spectrum produced by stable and metastable strings is identical, featuring a plateau at

$$\Omega_{\rm gw}^{\rm plateau} \simeq \frac{128\pi}{9} B\, \Omega_r \left(\frac{G\mu}{\Gamma}\right)^{1/2} , \tag{5}$$

where $\Omega_r h^2 = 4.15 \cdot 10^{-5}$ is the density parameter of radiation today. At small frequencies, the decay of the metastable cosmic string loops suppresses the GW signal, leading to a drop in the spectrum proportional to f^2.

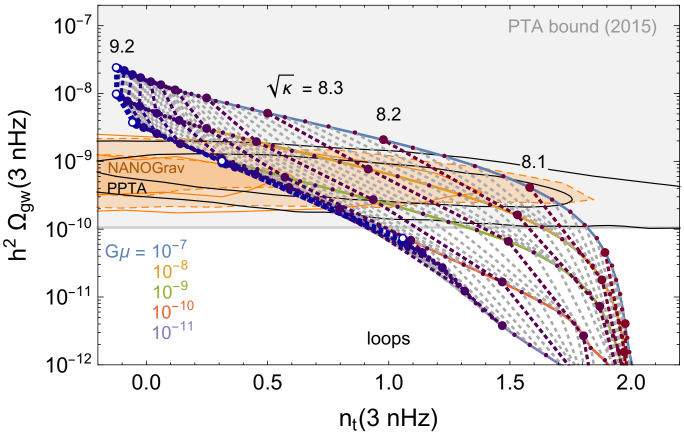

Figure 4: Predictions for the GW spectrum in the frequency range of pulsar timing arrays, for different values of $G\mu$ and $\sqrt{\kappa}$, including contributions from loops decaying during the matter era. The bound from the Parkes Pulsar Timing Array (PPTA) [19] published in 2015 is shown in gray; the 1/2 σ credible regions of NANOGrav [31] and PPTA [46] are indicated in orange and black, respectively. From [42].

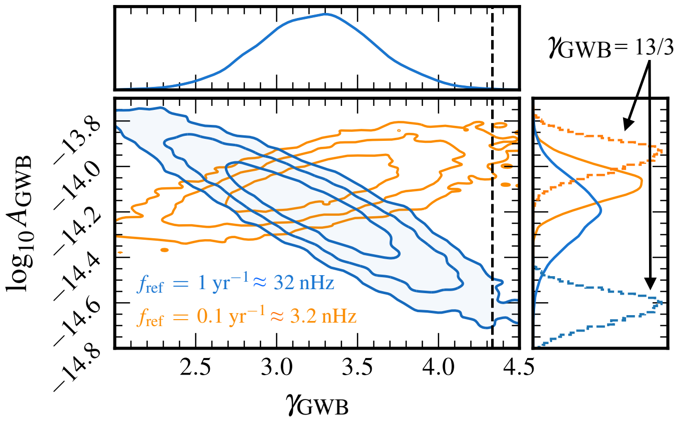

Figure 5: Posterior probability distribution of GWB amplitude and spectral exponent in a Hellings-Downs power-law model, showing 1/2/3 σ credible regions. The value $\gamma_{\rm GWB} = 13/3$ (dashed black line) is included in the 99% credible region. The amplitude is referenced to $f_{\rm ref} = 1{\rm yr}^{-1}$ (blue) and $0.1{\rm yr}^{-1}$ (orange). The dashed blue and orange curves in the $\log_{10} A_{\rm GWB}$ subpanel shows its marginal posterior density for a $\gamma_{\rm GWB} = 13/3$ model, with $f_{\rm ref} = 1{\rm yr}^{-1}$ and $0.1{\rm yr}^{-1}$, respectively. From [47].

In the PTA frequency band a power law model is used to describe the data,

$$\Omega_{\rm gw} = \frac{2\pi^2 f_{\rm PTA}^2 A_{\rm GWB}^2}{3H_0^2}\left(\frac{f}{f_{\rm PTA}}\right)^{n_t} , \qquad (6)$$

where $f_{\rm PTA}$ is a suitably chosen reference frequency. For specified values of $G\mu$ and κ, the amplitude $h^2\Omega_{\rm gw}$ and the spectral index n_t are predicted. Lines of constant $G\mu$ and κ, respectively, are displayed in Fig. 4 and compared with the NANOGrav data [31] (orange), with the solid contours showing the 1σ and 2σ regions. The black solid lines show the preferred region reported by PPTA [46] when performing a similar analysis. The 2σ regions are consistent with the parameter range $10^{-11} \lesssim G\mu \lesssim 10^{-7}$ and $\sqrt{\kappa} \gtrsim 8$. Note that the PTA data are essentially a measurement of the amplitude with the spectral index still subject to a large uncertainty and largely uncorrelated with the amplitude.

Pulsar timing array observations have entered a new phase with evidence for Hellings–Downs angular correlation, the smoking-gun signal of a GWB, which has recently been reported by PTA collaborations across the world [47–50]. The result of the NANOGraph collaboration for amplitude and spectral index is shown for two reference frequencies in Fig. 5, where instead of $h^2\Omega_{\mathrm{gw}}$ and n_t the variables A_{GWB} and $\gamma_{\mathrm{GWB}} = 5 - n_t$ are used. The observed amplitude is consistent with the result reported in [31], $10^{-10} \lesssim h^2\Omega_{\mathrm{gw}} \lesssim 10^{-9}$ at the PTA peak sensitivity $f_{\mathrm{PTA}} = 3$ nHz. The preferred spectral index is now restricted to $0 \lesssim n_t \lesssim 3$, with the PPTA data set [49] preferring slightly smaller values and the EPTA 10.5 year data set [48] preferring larger values.

Beyond the astrophysical interpretation of the GWB in terms of inspiraling supermassive black-hole binaries [32, 51, 52] there are several viable cosmological interpretations including cosmic strings (for an overview, see, e.g. [52–54]). As consequence of the restricted range of the spectral index, metastable strings are now favoured over stable strings [54]. Cosmic superstrings [55–57], with a GW spectrum estimated by a rescaled GW spectrum of stable strings, provide an even better fit to the data [54]. Upcoming data will improve our understanding of the spectral index, the isotropy and the presence of resolvable individual sources in this GWB, which will all help to distinguish an astrophysical from a cosmological origin, as well as different cosmic string interpretations.

For the physical interpretation of the PTA results it is useful to express the decay parameter κ in terms of the U(1) and SU(2) symmetry breaking scales v_s and v_u, respectively. The string tension is given by $\mu \simeq 2\pi v_s^2$, and a monopole with flux $2\pi n/g$ has mass $m_M \simeq 2\pi n v_u/g$. For a monopole–string–antimonopole configuration, the magnetic fluxes of the string and the (anti)monopole have to match (for a discussion, see, e.g. [58]). The string solution with lowest energy has winding number $n = 1$. For the considered Pati-Salam embedding the symmetry breaking doublet has U(1) charge 1/2 and carries magnetic flux $4\pi/g$. This can be matched by a $n = 2$ monopole with mass $m_M \simeq 4\pi v_u/g$, which yields for the decay parameter[1][45]

$$\kappa = \frac{m_M^2}{\mu} \sim \frac{8\pi}{g^2}\frac{v_u^2}{v_s^2}\,. \tag{7}$$

With $g^2 \sim 1/2$ at the unification scale, one finds again that $\sqrt{\kappa} \sim 8$ can be achieved if the two symmetry breaking scales v_u and v_s have comparable size. Note that this value of κ favours supersymmetric GUTs. Smaller gauge couplings in non-supersymmetric GUTs, like $g^2 \sim 1/4$ at the GUT scale, would require $v_u < v_s$ and may render the cosmic strings unstable.

The decay rate (1) is derived for infinitely thin strings with point-like monopoles at the end. Clearly, this approximation is very questionable in the case $v_u \sim v_s$. Moreover, the effect of scalar fields has been neglected in the expressions (2) and (7) for the decay parameter, which for comparable symmetry breaking scales can be rough estimates at best. It is not even clear whether in this parameter range metastable strings exist at all. We know that for $v_u \to 0$ unstable dumbbells form instead of metastable strings [38]. Hence, at some critical value of the ratio v_u/v_s the domain of metastabily should turn into a domain of instability. The difficult problem of computing the decay

---

[1] From Eq. (2) one obtains $\kappa = (4\pi/g^2)(2v_u^2 + v_s^2)/v_s^2$. For $v_u \gg v_s$, this agrees with Eq. (7), a consequence of the fact that in Eq. (2) the mass of a 't Hooft-Polyakov monopole was used which has charge $n = 2$. The difference between the expressions (2) and (7) in the regime $v_u \sim v_s$ illustrates the theoretical uncertainties of the two estimates.

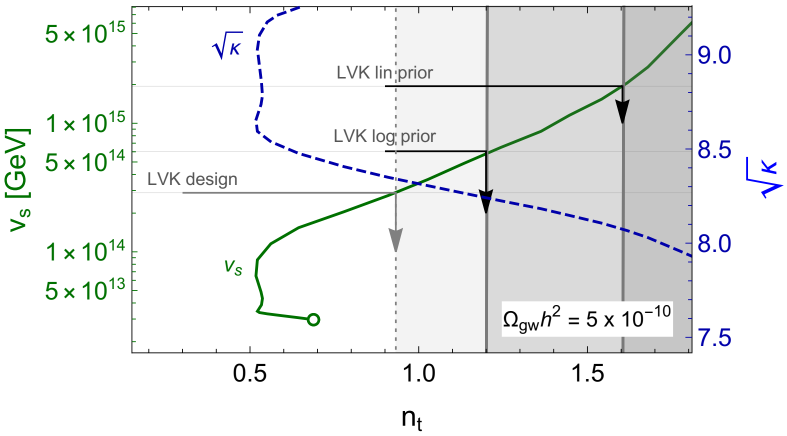

Figure 6: $U(1)$ symmetry breaking scale v_s (solid green, left axis) and decay parameter $\sqrt{\kappa}$ (dashed blue, right axis) as functions of the spectral index n_t of the GWB, assuming an amplitude $h^2\Omega_{\rm gw} \simeq 5 \cdot 10^{-10}$ at $f_{\rm PTA} = 3$ nHz. The grey region indicates values $G\mu > 1.5 \cdot 10^{-8}$ which is disfavoured by LIGO-Virgo–KAGRA (LVK) for a logarithmic (linear) prior on the amplitude of the GWB signal in the LVK band [61]; the light grey region corresponds to the design sensitivity. From [45].

rate of metastable strings beyond the thin-defect approximation has recently been addressed in [59], building on earlier work in [60]. Starting vom the "unwinding process" discussed in [60], a bounce action describing the decay of metastable strings has been obtained and numerically analyzed in the regime $v_u \sim v_s$. The results show that for scalar masses of the same order of magnitude there is indeed a regime of metastability with $\sqrt{\kappa} \sim 8$, as required by the PTA data [59].

Using Eq. (7) and $\mu \simeq 2\pi v_s^2$, the two observables $\Omega_{\rm gw}$ and n_t allow to determine the two symmetry breaking scales, v_u and v_s. As shown in Fig. 4, the amplitude $h^2\Omega_{\rm gw}$ is rather well determined at the reference frequency $f_{\rm PTA} = 3$ nHz, whereas the spectral index n_t is much more uncertain. This allows a simplified analysis: Fixing the amplitude of the GW spectrum at 3 nHz to the representative value $h^2\Omega_{\rm gw} = 5 \cdot 10^{-10}$, the analysis in [42] can be used to determine $G\mu$ and $\sqrt{\kappa}$ as functions of n_t (see Fig. 4), which can be translated to functions v_u and v_s of n_t. The results for $\sqrt{\kappa}$ and v_s are shown in Fig. 6. As the string lifetime and hence κ is increased, the string tension $G\mu$ needs to be reduced to maintain the same GWB amplitude at 3 nHz. An increase in κ comes with a decrease in n_t at $f_{\rm PTA}$, until with a further increase of κ the f^2 part of the spectrum enters the PTA band and the spectral index starts increasing again. In the limit of stable strings, $\sqrt{\kappa} \to \infty$, one finds $G\mu \simeq 4 \cdot 10^{-11}$ and $n_t \simeq 0.7$. For small κ (large v_s), the GWB will exceed the bound $\Omega_{\rm gw} \leq 5.8 \cdot 10^{-9}$ set by the LIGO–Virgo–KAGRA (LVK) collaboration in the 100 Hz range for a logarithmic prior [61]. As shown in Fig. 6, this leads to an upper bound on the spectral index, $n_t \lesssim 1.2$. From Fig. 5 one reads off the 2σ upper bound $\gamma_{\rm GWB} \lesssim 4.1$, corresponding to the 2σ lower bound on the spectral index $n_t \gtrsim 0.9$, which coincides with the LVK design sensitivity for setting an upper bound on n_t.

3. GUT model building

Metastable strings are a characteristic prediction of GUTs which lead, via several steps of spontaneous symmetry breaking, to the Standard Model gauge group $\mathrm{G}_{SM} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times$

$U(1)_Y$. Strings with tensions above the electroweak scale result from the breaking of a U(1) group that commutes with G_{SM}. Similarly, monopoles arise from the breaking of a non-Abelian gauge group that leaves a U(1) subgroup unbroken. If this U(1) overlaps with the U(1) symmetry responsible for string formation, the strings become metastable by quantum tunneling. Hence, gauge groups containing the SM which can give rise to metastable strings must have at least rank 5. A typical sequence of SM embeddings is

$$G_{SM} \subset SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \subset G_{PS} \subset SO(10) \subset E(6) \subset \ldots\,, \tag{8}$$

where $G_{PS} = SU(4) \times SU(2)_L \times SU(2)_R$ denotes the Pati–Salam group.

If a symmetry group G is broken to a subgroup H, the quotient $\mathcal{M} = G/H$ corresponds to the manifold of degenerate vacuum states. The types of defects that may be formed in the symmetry breaking are governed by the topology of $\mathcal{M}$, which is encoded in the homotopy groups $\pi_n(\mathcal{M})$ (for a review and referenes, see, e.g. [15]. Topologically stable strings can form if the first homotopy group is nontrivial, $\pi_1(\mathcal{M}) \neq I$, and topologically stable magnetic monopoles can arise if the second homotopy group is nontrivial, $\pi_2(\mathcal{M}) \neq I$. In two-step symmetry breakings G → H → K, where the homotopy group G/K is trivial, but the homotopy groups of the individual steps, G/H and H/K, are nontrivial, metastable defects can form.

The vacuum manifold crucially depends on the chosen Higgs representation. A simple example is the breaking of SO(10) to the Standard Model group via SU(5) [62]. The breaking chain

$$SO(10) \overset{\mathbf{45}}{\to} SU(5) \times U(1) \overset{\mathbf{45}\oplus\mathbf{126}}{\to} G_{SM} \times \mathbb{Z}_2 \tag{9}$$

yields stable monopoles and, in the second step, also stable strings. On the contrary, for the closely related symmetry breaking with a **16**-plet,

$$SO(10) \overset{\mathbf{45}}{\to} SU(5) \times U(1) \overset{\mathbf{45}\oplus\mathbf{16}}{\to} G_{SM}\,, \tag{10}$$

the homotopy group of $\mathcal{M} = SO(10)/G_{SM}$ is trivial, $\pi_1(\mathcal{M}) = I$, and there are no topologically stable strings. However, cosmologically interesting metastable strings can now form.

Realistic GUTs require large Higgs representations in order to break the GUT gauge group down to the SM, which complicates the analysis of symmetry breaking patterns. Moreover, non-supersymmetric models are sensitive to large radiative corrections and hence suffer from a severe naturalness problem. This lead to even more complicated models: supersymmetric GUTs with even more complicated Higgs sectors. As a possible alternative, one can consider higher-dimensional theories such as orbifold GUTs or string models (for a review, see, e.g. [4, 5]). In these constructions, the GUT gauge group is first partially broken in a geometric way, by the compactification of extra dimensions, and only the remnant subgroup remaining after this first step is further reduced to the SM group via conventional spontaneous symmetry breaking.

The simplest group leading to potentially realistic metastable strings is $SU(3)_C \times SU(2)_L \times SU(2)_R$, the electroweak subgroup of the Pati–Salam and SO(10) GUT groups. A Higgs triplet U, breaking $SU(2)_R$ to $U(1)_R$ with $\langle U^a \rangle = v_u \delta_{a3}/\sqrt{2}$, is contained in the adjoint representation, and Higgs doublets S and S_c, breaking $SU(2)_R \times U(1)_R$ to $U(1)_Y$ with $\langle S_i \rangle = \langle S_{ci} \rangle = v_s \delta_{i1}/\sqrt{2}$, are contained in the Pati–Salam multiplets $(\mathbf{4}, 1, \mathbf{2})$ and $(\bar{\mathbf{4}}, 1, \bar{\mathbf{2}})$, and the SO(10) multiplets **16** and $\overline{\mathbf{16}}$, respectively. Embedding the doublets S, S_c in **16**-, $\overline{\mathbf{16}}$-plets Φ, Φ^c of SO(10) implies that heavy Majorana neutrino masses must be generated by the nonrenormalizable operator

$$\mathcal{L}_n = \frac{1}{M_*} h_{ij}\, S^T L_i^c S^T L_j^c \subset \frac{1}{M_*} h_{ij}\, \Phi^c \psi_i \Phi^c \psi_j \,. \tag{11}$$

Here, the fields $L_i^c = (n_i^c, e_i^c)^T$, $i = 1, .., 3$, denote the $SU(2)_R$ doublets of right-handed neutral and charged leptons that are contained in the SO(10) **16** representations ψ_i of matter, and h_{ij} are Yukawa couplings. Alternatively, one can break SO(10) with **126**-, $\overline{\mathbf{126}}$-plets $\tilde{\Phi}$, $\tilde{\Phi}^c$ containing the SU(5) singlets $\tilde{S}$, $\tilde{S}_c$. Heavy neutrino masses are now generated by the renormalizable couplings[2]

$$\mathcal{L}_n = h_{ij}\tilde{S} L_i^c L_j^c \subset h_{ij}\tilde{\Phi}\psi_i\psi_j \,. \tag{12}$$

The VEVs of $\tilde{S}$, $\tilde{S}_c$ leave a $\mathbb{Z}_2$ discrete symmetry unbroken, which leads to topologically stable strings. Various aspects of metastable strings in SO(10) models have been discussed in [21, 38, 45, 63–67]. A viable alternative are flipped-SU(5) models [68, 69].

Metastable strings that can account for the observed PTA GWB require a two-step GUT symmetry breaking whose scales v_u and v_s satisfy the conditions

$$v_u \sim v_s \sim \text{few} \times 10^{14}\ \text{GeV} \ll v_{\text{GUT}} \sim 10^{16}\ \text{GeV}\,. \tag{13}$$

The inequality could be explained if the symmetry breaking scales were related to the compactification scale of a 5-dimensional SO(10) GUT rather than parameters of a 4-dimensional supersymmetric GUT. In such higher-dimensional theories also topological defects can occur, which may shed some light on the connection between the two symmetry breaking scales (for a discussion and references, see, e.g. [4, 70, 71]). On the observational side, it is intriguing that the PTA lower bound on the spectral index n_t coincides with the upper bound on n_t corresponding to the LVK design sensitivity (see Fig. 6). This suggests that a GWB may soon be observed by the LIGO-Virgo–KAGRA collaboration.

Acknowledgments

This article is based on work with V. Domcke, H. Murayama and K. Schmitz whom I thank for an eventful collaboration. Special thanks also to E. Dudas for discussions on metastable strings in grand unified models. I am grateful to the organizing committee for arranging this international conference dedicated to Valery Rubakov in Yerevan in difficult times.

---

[2] This was assumed in [14]. Note that string compactifications prefer symmetry breaking with **16**- and $\overline{\mathbf{16}}$-plets which, contrary to **126**- and $\overline{\mathbf{126}}$-plets, are contained in the adjoint representation of E_8 [4, 5].

References

[1] J. C. Pati and A. Salam, *Unified Lepton-Hadron Symmetry and a Gauge Theory of the Basic Interactions*, *Phys. Rev. D* **8** (1973) 1240.

[2] H. Georgi, *The State of the Art—Gauge Theories*, *AIP Conf. Proc.* **23** (1975) 575.

[3] H. Fritzsch and P. Minkowski, *Unified Interactions of Leptons and Hadrons*, *Annals Phys.* **93** (1975) 193.

[4] S. Raby, *Supersymmetric Grand Unified Theories: From Quarks to Strings via SUSY GUTs*, vol. 939. Springer, 2017, 10.1007/978-3-319-55255-2.

[5] A. Hebecker, *Naturalness, String Landscape and Multiverse: A Modern Introduction with Exercises*, vol. 979 of *Lecture Notes in Physics*. 3, 2021, 10.1007/978-3-030-65151-0.

[6] J. D. Wells, *PeV-scale supersymmetry*, *Phys. Rev. D* **71** (2005) 015013 [hep-ph/0411041].

[7] P. N. Bhattiprolu and J. D. Wells, *Precision unification and the scale of supersymmetry*, 2309.12954.

[8] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, *On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe*, *Phys. Lett.* **155B** (1985) 36.

[9] M. Fukugita and T. Yanagida, *Baryogenesis Without Grand Unification*, *Phys. Lett.* **B174** (1986) 45.

[10] J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive and M. Srednicki, *Supersymmetric Relics from the Big Bang*, *Nucl. Phys.* **B238** (1984) 453.

[11] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, *False vacuum inflation with Einstein gravity*, *Phys. Rev.* **D49** (1994) 6410 [astro-ph/9401011].

[12] G. R. Dvali, Q. Shafi and R. K. Schaefer, *Large scale structure and supersymmetric inflation without fine tuning*, *Phys. Rev. Lett.* **73** (1994) 1886 [hep-ph/9406319].

[13] G. N. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. D. Linde and I. Tkachev, *Dynamics of symmetry breaking and tachyonic preheating*, *Phys. Rev. Lett.* **87** (2001) 011601 [hep-ph/0012142].

[14] W. Buchmuller, V. Domcke and K. Schmitz, *Spontaneous B-L Breaking as the Origin of the Hot Early Universe*, *Nucl. Phys.* **B862** (2012) 587 [1202.6679].

[15] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects*. Cambridge University Press, 2000.

[16] M. Hindmarsh, *Signals of Inflationary Models with Cosmic Strings*, *Prog. Theor. Phys. Suppl.* **190** (2011) 197 [1106.0391].

[17] NANOGRAV collaboration, *The NANOGrav 11-year Data Set: Pulsar-timing Constraints On The Stochastic Gravitational-wave Background*, Astrophys. J. **859** (2018) 47 [`1801.02617`].

[18] M. Kerr et al., *The Parkes Pulsar Timing Array Project: Second data release*, Publ. Astron. Soc. Austral. **37** (2020) e020 [`2003.09780`].

[19] R. M. Shannon et al., *Gravitational waves from binary supermassive black holes missing in pulsar observations*, Science **349** (2015) 1522 [`1509.07320`].

[20] J. A. Dror, T. Hiramatsu, K. Kohri, H. Murayama and G. White, *Testing the Seesaw Mechanism and Leptogenesis with Gravitational Waves*, Phys. Rev. Lett. **124** (2020) 041804 [`1908.03227`].

[21] W. Buchmuller, V. Domcke, H. Murayama and K. Schmitz, *Probing the scale of grand unification with gravitational waves*, Phys. Lett. **B** (2020) 135764 [`1912.03695`].

[22] A. Vilenkin, *Cosmological evolution of monopoles connected by strings*, Nucl. Phys. **B196** (1982) 240.

[23] J. Preskill and A. Vilenkin, *Decay of metastable topological defects*, Phys. Rev. **D47** (1993) 2324 [`hep-ph/9209210`].

[24] L. Leblond, B. Shlaer and X. Siemens, *Gravitational Waves from Broken Cosmic Strings: The Bursts and the Beads*, Phys. Rev. **D79** (2009) 123519 [`0903.4686`].

[25] A. Monin and M. B. Voloshin, *The Spontaneous breaking of a metastable string*, Phys. Rev. **D78** (2008) 065048 [`0808.1693`].

[26] R. Smits, M. Kramer, B. Stappers, D. R. Lorimer, J. Cordes and A. Faulkner, *Pulsar searches and timing with the square kilometre array*, Astron. Astrophys. **493** (2009) 1161 [`0811.0211`].

[27] LISA collaboration, *Laser Interferometer Space Antenna*, `1702.00786`.

[28] LIGO SCIENTIFIC, VIRGO collaboration, *Search for the isotropic stochastic background using data from Advanced LIGO's second observing run*, Phys. Rev. **D100** (2019) 061101 [`1903.02886`].

[29] EINSTEIN TELESCOPE collaboration*http://www.et-gw.eu/index.php/etsensitivities* .

[30] J. P. W. Verbiest et al., *The International Pulsar Timing Array: First Data Release*, Mon. Not. Roy. Astron. Soc. **458** (2016) 1267 [`1602.03640`].

[31] NANOGRAV collaboration, *The NANOGrav 12.5 yr Data Set: Search for an Isotropic Stochastic Gravitational-wave Background*, Astrophys. J. Lett. **905** (2020) L34 [`2009.04496`].

[32] H. Middleton, A. Sesana, S. Chen, A. Vecchio, W. Del Pozzo and P. A. Rosado, *Massive black hole binary systems and the NANOGrav 12.5 yr results*, *Mon. Not. Roy. Astron. Soc.* **502** (2021) L99 [`2011.01246`].

[33] J. Ellis and M. Lewicki, *Cosmic String Interpretation of NANOGrav Pulsar Timing Data*, *Phys. Rev. Lett.* **126** (2021) 041304 [`2009.06555`].

[34] S. Blasi, V. Brdar and K. Schmitz, *Has NANOGrav found first evidence for cosmic strings?*, *Phys. Rev. Lett.* **126** (2021) 041305 [`2009.06607`].

[35] R. Samanta and S. Datta, *Gravitational wave complementarity and impact of NANOGrav data on gravitational leptogenesis*, *JHEP* **05** (2021) 211 [`2009.13452`].

[36] L. Bian, R.-G. Cai, J. Liu, X.-Y. Yang and R. Zhou, *Evidence for different gravitational-wave sources in the NANOGrav dataset*, *Phys. Rev. D* **103** (2021) L081301 [`2009.13893`].

[37] W. Buchmuller, V. Domcke and K. Schmitz, *From NANOGrav to LIGO with metastable cosmic strings*, *Phys. Lett.* **B811** (2020) 135914 [`2009.10649`].

[38] W. Buchmuller, *Metastable strings and dumbbells in supersymmetric hybrid inflation*, *JHEP* **04** (2021) 168 [`2102.08923`].

[39] C. J. A. P. Martins and E. P. S. Shellard, *Extending the velocity dependent one scale string evolution model*, *Phys. Rev. D* **65** (2002) 043514 [`hep-ph/0003298`].

[40] J. J. Blanco-Pillado and K. D. Olum, *Stochastic gravitational wave background from smoothed cosmic string loops*, *Phys. Rev.* **D96** (2017) 104046 [`1709.02693`].

[41] P. Auclair et al., *Probing the gravitational wave background from cosmic strings with LISA*, *JCAP* **2004** (2020) 034 [`1909.00819`].

[42] W. Buchmuller, V. Domcke and K. Schmitz, *Stochastic gravitational-wave background from metastable cosmic strings*, *JCAP* **12** (2021) 006 [`2107.04578`].

[43] Y. Gouttenoire, G. Servant and P. Simakachorn, *Beyond the Standard Models with Cosmic Strings*, *JCAP* **07** (2020) 032 [`1912.02569`].

[44] D. I. Dunsky, A. Ghoshal, H. Murayama, Y. Sakakihara and G. White, *GUTs, hybrid topological defects, and gravitational waves*, *Phys. Rev. D* **106** (2022) 075030 [`2111.08750`].

[45] W. Buchmuller, V. Domcke and K. Schmitz, *Metastable cosmic strings*, *JCAP* **11** (2023) 020 [`2307.04691`].

[46] B. Goncharov et al., *On the Evidence for a Common-spectrum Process in the Search for the Nanohertz Gravitational-wave Background with the Parkes Pulsar Timing Array*, *Astrophys. J. Lett.* **917** (2021) L19 [`2107.12112`].

[47] NANOGrav collaboration, *The NANOGrav 15-year Data Set: Evidence for a Gravitational-Wave Background*, *Astrophys. J. Lett.* **951** (2023) [2306.16213].

[48] J. Antoniadis et al., *The second data release from the European Pulsar Timing Array III. Search for gravitational wave signals*, 2306.16214.

[49] D. J. Reardon et al., *Search for an isotropic gravitational-wave background with the Parkes Pulsar Timing Array*, *Astrophys. J. Lett.* **951** (2023) [2306.16215].

[50] H. Xu et al., *Searching for the nano-Hertz stochastic gravitational wave background with the Chinese Pulsar Timing Array Data Release I*, 2306.16216.

[51] NANOGrav collaboration, *The NANOGrav 15-year Data Set: Constraints on Supermassive Black Hole Binaries from the Gravitational Wave Background*, 2306.16220.

[52] J. Antoniadis et al., *The second data release from the European Pulsar Timing Array: V. Implications for massive black holes, dark matter and the early Universe*, 2306.16227.

[53] E. Madge, E. Morgante, C. P. Ibáñez, N. Ramberg and S. Schenk, *Primordial gravitational waves in the nano-Hertz regime and PTA data – towards solving the GW inverse problem*, 2306.14856.

[54] NANOGrav collaboration, *The NANOGrav 15-year Data Set: Search for Signals from New Physics*, *Astrophys. J. Lett.* **951** (2023) [2306.16219].

[55] G. Dvali and A. Vilenkin, *Formation and evolution of cosmic D strings*, *JCAP* **03** (2004) 010 [hep-th/0312007].

[56] E. J. Copeland, R. C. Myers and J. Polchinski, *Cosmic F and D strings*, *JHEP* **06** (2004) 013 [hep-th/0312067].

[57] J. Ellis, M. Lewicki, C. Lin and V. Vaskonen, *Cosmic superstrings revisited in light of NANOGrav 15-year data*, *Phys. Rev. D* **108** (2023) 103511 [2306.17147].

[58] T. W. B. Kibble and T. Vachaspati, *Monopoles on strings*, *J. Phys.* **G42** (2015) 094002 [1506.02022].

[59] A. Chitose, M. Ibe, Y. Nakayama, S. Shirai and K. Watanabe, *Revisiting Metastable Cosmic String Breaking*, 2312.15662.

[60] M. Shifman and A. Yung, *Metastable strings in Abelian Higgs models embedded in nonAbelian theories: Calculating the decay rate*, *Phys. Rev. D* **66** (2002) 045012 [hep-th/0205025].

[61] KAGRA, Virgo, LIGO Scientific collaboration, *Upper limits on the isotropic gravitational-wave background from Advanced LIGO and Advanced Virgo's third observing run*, *Phys. Rev. D* **104** (2021) 022004 [2101.12130].

[62] T. W. B. Kibble, G. Lazarides and Q. Shafi, *Strings in SO(10)*, *Phys. Lett.* **113B** (1982) 237.

[63] S. Antusch, K. Hinze, S. Saad and J. Steiner, *Singling out SO(10) GUT models using recent PTA results*, *Phys. Rev. D* **108** (2023) 095053 [`2307.04595`].

[64] B. Fu, S. F. King, L. Marsili, S. Pascoli, J. Turner and Y.-L. Zhou, *Testing Realistic $SO(10)$ SUSY GUTs with Proton Decay and Gravitational Waves*, `2308.05799`.

[65] A. Afzal, M. Mehmood, M. U. Rehman and Q. Shafi, *Supersymmetric hybrid inflation and metastable cosmic strings in* $SU(4)_c \times SU(2)_L \times U(1)_R$, `2308.11410`.

[66] W. Ahmed, T. A. Chowdhury, S. Nasri and S. Saad, *Gravitational waves from metastable cosmic strings in the Pati-Salam model in light of new pulsar timing array data*, *Phys. Rev. D* **109** (2024) 015008 [`2308.13248`].

[67] W. Ahmed, M. U. Rehman and U. Zubair, *Probing Stochastic Gravitational Wave Background from* $SU(5) \times U(1)_\chi$ *Strings in Light of NANOGrav 15-Year Data*, `2308.09125`.

[68] G. Lazarides, R. Maji, A. Moursy and Q. Shafi, *Inflation, superheavy metastable strings and gravitational waves in non-supersymmetric flipped SU(5)*, `2308.07094`.

[69] S. F. King, G. K. Leontaris and Y.-L. Zhou, *Flipped SU(5): unification, proton decay, fermion masses and gravitational waves*, `2311.11857`.

[70] R. Dermisek, S. Raby and S. Nandi, *Magnetic monopoles with Wilson loops on a 5-D orbifold*, *Nucl. Phys. B* **641** (2002) 327 [`hep-th/0205122`].

[71] H. D. Kim and S. Raby, *Unification in 5D SO(10)*, *JHEP* **01** (2003) 056 [`hep-ph/0212348`].