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New Insights into the Properties of Matter at High Baryon Density

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New insights into the properties of matter at high baryon density and low temperature arise from the Quarkyonic hypothesis. The general features of Quarkyonic matter are discussed as well as the application of its theory to the description of neutron stars. An exactly solvable model, with two phases and a hard equation of state at high densities is presented. This theory is a free theory of nucleons, composed of quarks, with the Fermi exclusion principle applied for both nucleon and quark densities. It is exactly solvable for a specific choice of probability to find a quark inside a baryon as a function of quark momentum. The solution shows explicitly the formation of a shell of baryons surrounding a filled Fermi sea of quarks in the high density phase. The low density phase is a free Fermi gas. The theory is explicitly dual between quarks and nucleons.

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1. Introduction

The observed properties of neutron stars provide a good determination of the equation of state of nuclear matter. For a recent review of the status of such determination and a comprehensive set of references, see Ref. [1], and in particular the seminal theoretical analysis of Refs. [2],[3]. A conclusion of such a determination is that the equation of state is stiff at densities a few times that of nuclear matter with the sound velocity squared of the order of and possibly exceeding $\frac{1}{3}c^2$[4], [5], [6]. This is a truly remarkable observation since at nuclear matter density, the degrees of freedom are non-relativistic nucleons, with binding energies and kinetic energies small compared to the nucleon mass. Apparently if one squeezes nuclear matter increasing the density by a few times, the system rapidly becomes relativistic. This observation hints that the transition may be from nucleons to quarks. However, this is unlikely to be a first or second order phase transition since at such transition the sound velocity drops to zero and the equation of state becomes soft.

The subject of these lectures will be to show how a rapid transition to a stiff equation of state might occur if matter is Quarkyonic. Such Quarkyonic matter was proposed to explain a remarkable fact observed in the large N_c limit of QCD.[7] At large N_c, de-confinement occurs at a quark chemical potential (or Fermi energy) that is parametrically large compared to the QCD scale, $\mu_{quark} \sim \sqrt{N_c}\Lambda_{QCD}$. On the other hand, we might naively expect that the effects of interactions will be small when $\mu_{quark} >> \Lambda_{QCD}$. The resolution of this paradox is that at the Fermi surface, there are strong interactions since Fermi surface interactions are not cutoff by Debye screening. The Fermi surface may be thought of as confined baryons with confined mesonic excitations. On the other hand, deep inside the Fermi surface, interactions are weak and the degrees of freedom may be thought of as those of quarks. Since the bulk properties of the system should be largely determined by the bulk of particles which are in the Fermi sea, the system becomes relativistic at a scale $\mu_{quark} \sim \Lambda_{QCD}$, and this corresponds to a density scale not large compared to that of nuclear matter. There may or may not be a true phase transition to Quarkyonic matter, but the transition to this matter is distinct from the de-confinement transition. I shall later show that such a transition can occur at lowish density, and that it allows for a rapid stiffening in the equation of state of high density matter.

The outline of this lecture is the following: In the second section, I shall discuss the phenomenological implications of neutron star observations. In particular, I argue that matter may become approximately scale invariant at a density scale a few times that of nuclear matter. In the third section, I discuss the properties of Quarkyonic matter and argue that a rapid transition to approximately scale invariant matter might be possible if matter is Quarkyonic. In the fourth section, I present an exactly solvable model that is Quarkyonic. This model has a rapid transition to a hard equation of state, is explicitly dual between quarks and nucleons, and shows an accumulation of nucleons on a Fermi surface that surrounds a filled Fermi sphere of quarks. The fifth section concerns future areas research.

2. Scale Invaiance and High Density Matter

The mass and radius of a neutron star are determined by equating the outward pressure against the inward force of gravity, using the Tolman-Oppenheimer-Volkov equation of hydrostatic equilibrium. One property of an equation of state is the sound velocity

$$v_s^2 = \frac{dP}{d\epsilon} \tag{1}$$

where P is the pressure and ϵ is the energy density. Another quantity is the trace of the stress energy tensor

$$T^\mu_\mu = \frac{1}{3}\epsilon - P \tag{2}$$

which can be characterized by

$$\Delta = \frac{1}{3} - \frac{P}{\epsilon} \tag{3}$$

These two dimensionless quantities, Δ and v_s^2 characterize the hardness or softness of the equation of state. $\Delta \sim 1/3$ and $v_s^2 \sim 0$ are characteristic of soft non-relativistic matter, while $\Delta \sim 0$ and $v_s^2 \sim \frac{1}{3}$ are for hard, relativistic matter.

The relationship to scale invariance is easily seen since for a scale invariant theory at fixed baryon number N with volume V, the number density is $n = N/V$, and the energy density is $\epsilon = \kappa n^{4/3}$, where κ is a constant. The pressure is

$$P = -\frac{dE}{dV} = -\frac{d}{dV}\frac{\kappa N^{4/3}}{V^{1/3}} = \frac{1}{3}\epsilon \tag{4}$$

so that

$$v_s^2 = \frac{1}{3} \tag{5}$$

and

$$\Delta = 0 \tag{6}$$

In QCD, there is a scale associated with the distance of confinement, $1/\Lambda_{QCD}$. With the beta function of QCD defined by $\beta(g) = dg/dln(\Lambda_{QCD})$,where g is the QCD interaction strength, deviations from scale invariance are characterized by the trace anomaly

$$T^\mu_\mu = -\frac{\beta(g)}{g}(E^2 - B^2) + m_q(1 + \gamma_q)\overline{\psi}\psi \tag{7}$$

In this equation, E and B are the color electric and color magnetic fields, m_q are quark masses, γ_q is an anomalous dimension of Fermion operators, and ψ is a Fermion field. The β function is negative for QCD, and usually the effect of quark masses may be ignored, except for pion physics. For single particle states

$$< p \mid T^\mu_\mu \mid p >\sim p^2 = m^2 \geq 0 \tag{8}$$

Ignoring the last term, this equation means that $E^2 > B^2$ inside massive hadrons, which is consistent with our intuition for bound states made of massive quarks, where electric fields should be larger than magnetic fields.

For gasses composed of hadrons we might therefore expect that $< T^\mu_\mu >$ is positive, and approaches zero from above as temperatures and densities increase. This might be violated if there were condensates or systems dominated by pions where the quark mass term might be important.

One of the remarkable achievements of neutron star studies is that one can extract to a fair approximation the equation of state of nuclear matter from neutron properties[1], [2]. In Fig. 1,

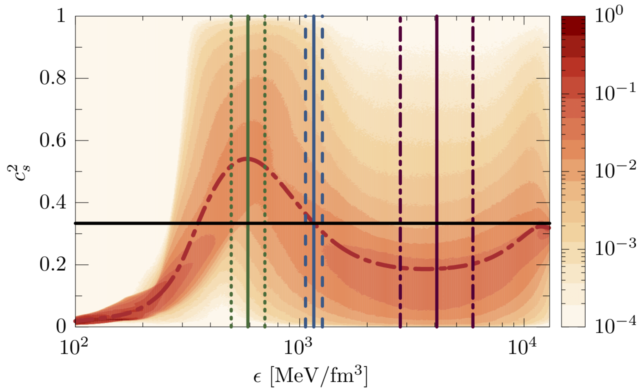

Figure 1: The sound velocity as a function of energy density from Ref. [9] The green lines indicate where a maximum of the sound velocity is found and the blue lines are typical maximum central densities of neutron stars. The inference above this maximum density comes from requiring that fits to data smoothly connect to QCD computations at the highest densities.

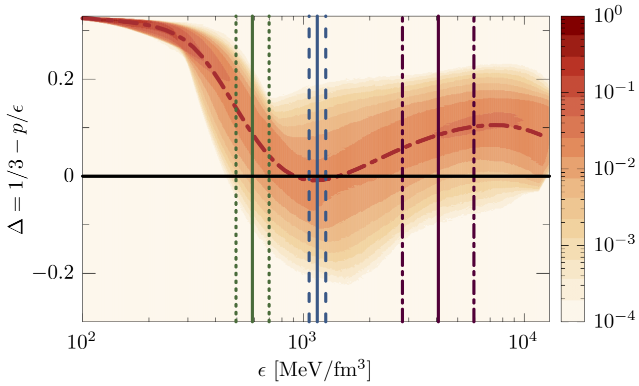

Figure 2: The scaled trace anomaly Δ as a function of energy density from Ref. [9]

the sound velocity extracted from Bayesian analysis of data is shown from Ref. [8],[9] and it appears that the the equation of state is close to the conformal limit at high density. as shown in Fig. 2. This is a surprise since the energy density is not at so high compared to the QCD scale. Generally, the sound velocity squared exceeds $1/3$ above some density and there is some evidence for a maximum at a density a few times that of nuclear matter. The scaled trace of the stress energy tensor approaches zero from above, characteristic of a gas of massive particles

There is a relation between the sound velocity and the trace anomaly since

$$v_s^2 = -\epsilon \frac{d}{d\epsilon}\Delta - \Delta + \frac{1}{3} \tag{9}$$

If the scaled trace approaches zero monotonically, this equation means that a large sound velocity accelerates this approach, and a large sound velocity is signal for a precocious approach to the conformal limit.

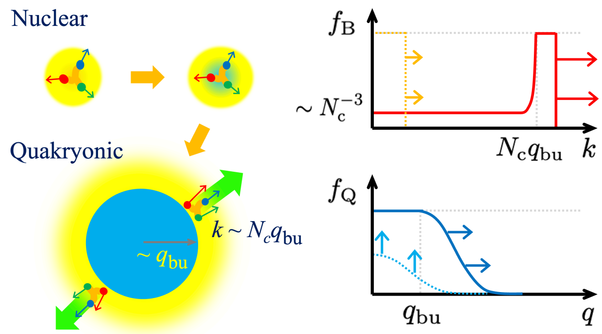

Figure 3: The shell structure of Quarkyonic matter from Ref. [10]

3. Properties of Quarkyonic Matter

The observation that the sound velocity changes rapidly to a relativistic value as the baryon number density changes by of order one, has strong consequences. The sound velocity can be expressed in terms of the baryon density and baryon chemical potential as

$$\frac{n_B}{\mu_B dn_B/d\mu_B} = v_s^2 \tag{10}$$

means that

$$\frac{\delta\mu_B}{\mu_B} \sim v_s^2 \frac{\delta n_B}{n_B} \tag{11}$$

If the sound velocity is of order one, an order one change in the baryon density generates a change in the baryon chemical potential which is of the order the baryon chemical potential. Initially, the baryon chemical potential for a non-relativistic system is of order the nucleon mass, with a small correction due to nucleon binding and kinetic energies,

$$\mu_B = M + \mu_B^{kinetic} \tag{12}$$

but after a change of order one, the kinetic energies are relativistic. If the distribution of particles were uniform in momentum space, we would expect, $k_F \sim M$, and the density $n_B \sim k_F^3 \sim M^3$. If we count factors of the number of colors, the number density would naively be required to jump by of order $N_c^3 n_B$. The jump is only of order n_B, and means that the baryons must appear in some geometrically restricted region of momentum phase space.

This is solved in Quarkyonic matter. Nucleons form a shell surrounding of filled Fermi sea of quarks, as shown in Fig. 3, as computed explicitly in Ref. [10]. The nucleonic shell has a thickness that can become thinner as the baryon density increases, so that the density of baryons associated with the momentum space shell does not rapidly grow, in spite of the fact that the typical nucleon momentum can grow. The typical Fermi momentum of the quarks may be as high as Λ_{QCD}, and the baryon density in the quarks is of order Λ_{QCD}^3 which is not so high, in spite of the fact that the nucleon momentum scale is $N_c \Lambda_{QCD} \sim M_N$. Making a shell of relativistic nucleons surrounding a filled Ferm sea of quarks can therefore be accomplished at a density $n_B \sim \Lambda^3_{QCD} << M_N^3$

I shall later construct a theory which explicitly has this property, and moreover is dual in its description of nucleon and quarks. Duality is a most interesting fundamental property of QCD meaning that one should be able to think about the degree of freedom of quark and nucleons simultaneously because nucleons are composed of quarks, and thinking in terms of quarks or nucleons simply involve an arbitrary choice of basis states[11],[12].

It is useful to consider explicitly the relationship between quark and nucleon densities and to make explicit the N_c dependences. The relationship between constituent quark masses and nucleon masses is

$$m_q = m_N / N_c \tag{13}$$

and chemical potentials

$$\mu_q = \mu_N / N_c \tag{14}$$

For Fermi momenta,

$$k_q^2 = \mu_q^2 - m_q^2 = k_N^2 / N_c \tag{15}$$

For an ideal gas of 2 flavors of quarks

$$n_B^N = \frac{2}{3\pi^2} k_N^3 \tag{16}$$

and

$$n_B^q = \frac{2}{3\pi^2} k_q^3 \tag{17}$$

The baryon number can be kept from growing too rapidly by adjusting the thickness of the shell. The baryon number density of the quarks is naturally $n_B \sim \Lambda^3_{QCD}$. Note that for a gas of nucleons with Fermi momentum Λ_{QCD} the energy density is also of this order, so there need not be parametric changes in densities when one makes a transition between a gas of nucleons and Quarkyonic matter.

If we think about a filled Fermi sea of quark in terms of nucleons, then the corresponding nucleon Fermi momentum for this filled sea of quarks is $k_N \sim N_c k_q$. On the other hand, the phase space density of quarks is $f_q \sim 1$, for a fully occupied Fermi sea, but in order that one gets the same baryon number $n_B \sim f_q k_q^3 = f_B k_N^3$, the occupation number density of nucleons must be $f_N \sim 1/N_c^3$. This is shown in Fig 3.

Energy densities for the nucleons are $\epsilon_B \sim k_F n_B \sim N_c \Lambda^3_{QCD}$ and $\epsilon_Q \sim \Lambda_{QCD} n_q \sim \Lambda_{QCD} N_c n_B^q$ (since the baryon number per quark is of order $1/N_c$), so that a change in the energy density from a nucleon gas to Quarkyonic matter can also be smooth.

The pressure is different however. For an ordinary gas of nucleons at density n_B the pressure is of order $p \sim k_F^5/M_N \sim \epsilon_B / N_c^2$ where as the pressure density of a quark gas at this density is of order $P \sim \epsilon_B$. This means that the equation of state becomes much stiffer when one makes a transition between a non-relativistic system of nucleons and a Quarkyonic system, even though the energy density and baryon number density change by of order one.

This is precisely what is needed to describe neutron stars.

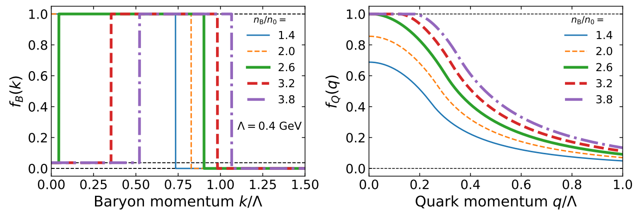

Figure 4: From Ref. [10], the momentum distribution of nucleons (left) and quarks (right) as one increases the baryon number density

4. An Explicit, Exactly Solvable Model for Quarkyonic Matter

Here I construct the solvable Idylliq model of Quarkyonic matter[10]. Idylliq is an acronym for ideal Quarkyonic matter. As such, I consider a noninteracting gas of nucleons. I assume that these nucleons are composed of quarks, and one can compute the phase density of quarks, f_q in terms of the phase space density of nucleons f_N. I will require that the phase space densities satisfy $1 \geq f_q, f_N \geq 0$.

This is a very simple theory, but we will see that it is a non-trivial theory with two different phases. The low density phase is an ideal degenerate Fermi gas of nucleons. At some density of the order of the scale set by the typical momentum of a quark inside a nucleon, there is a transition to a high density phase. This high density phase has a hard equation of state, and is Quarkyonic as shown in Fig. 3. For a particular choice of probability density, this theory is analytically solvable. The theory has an explicit duality between quarks and nucleons.

Explicitly, the duality relationship between quarks and nucleons is

$$f_q(\vec{k}) = \int d^3p \; K(\vec{k} - \vec{p}/N_c) f_N(\vec{p}) \tag{18}$$

where K is a probability distribution for a quark inside the nucleon

$$\int d^3k \; K(\vec{k}) = 1 \tag{19}$$

At low baryon number densities, the constraint that the occupation number density of quarks is small is easy to satisfy. This is because the nucleons are localized in low momentum states, but the distribution of quarks is spread out. This is shown in Fig. 4. As the baryon number density increases, more and more nucleon states pile up the quark density at zero momentum. At some density, the quark occupation phase space density becomes one and beyond this density, the free nucleon Fermi gas solution is no longer valid, as shown in Fig. 4. This critical density occurs when

$$1 = \int d^3p \; K[p/N_c] f_N(p) \tag{20}$$

The form of solution we will find at higher densities is not so hard to guess. If nucleons are concentrated on a Fermi surface, and the Fermi momentum increases, this will not so much affect the density of quarks at low momentum. The quarks on the other hand will begin to fully occupy their phase space, $f_q \sim 1$. This is the form of the solution in Fig. 3.

To proceed further, I take an explicit form for $K(p)$, that allows for an exact analytic solution,

$$K(\vec{k}) = \frac{1}{4\pi\Lambda^2}\frac{e^{-|\vec{k}|}}{|\vec{k}|} \tag{21}$$

This is the Greens function for

$$\{-\nabla_k^2 + \frac{1}{\Lambda^2}\}K(k) = \frac{1}{\Lambda^2}\delta^{(3)}(\vec{k}) \tag{22}$$

Applying this differential operator to the expression for the quark phase space density gives

$$\{-\nabla_k^2 + \frac{1}{\Lambda^2}\}f_q(k) = \frac{N_c^3}{\Lambda^2}f_N(N_c k) \tag{23}$$

This allows us to determine explicitly the quark distribution in terms of a nucleon distribution. For a distribution of nucleons corresponding

$$f_N(p) = \theta(p_1 - p)\theta(p - p_2) + \frac{1}{N_c^3}\theta(p_2 - p) \tag{24}$$

the corresponding quark distribution function is much like a Fermi Dirac distribution, of height one at low momentum, and an exponentially falling tail at the Fermi surface, corresponding to a homogeneous solution of the above equation. The quark Fermi surface is at momentum $k_q = p_2/N_c$.

The energy density of a free gas of nucleons can be re-expressed explicitly in terms of a linear theory of quarks by using the relationship between quark and nucleon densities. The energy density is

$$\epsilon = \int d^3p\,\sqrt{p^2 + M_N^2}f_N(p) = N_c\int d^3k\,E_q(k)f_q(k) \tag{25}$$

where

$$E_q(k) = \sqrt{k^2 + m_q^2} - \frac{m_q^2\Lambda^2}{(k^2 + m_q^2)^{3/2}} \tag{26}$$

In Ref. [10], it is argued that the solution outlined above for the distributions of nucleons and quarks is the minimum energy solution at fixed baryon number. The density at which this solution turns on is when Eqn. 20 is satisfied,

$$k_F \sim \frac{\Lambda}{N_c^{1/2}} \tag{27}$$

The extra factor of $\sqrt{N_c}$ arises from the assumed $1/\mid k\mid$ singularity of the probability distribution and might not be present in more generic less singular models. However, this has the feature that the transition from ordinary nuclear matter to Quarkyonic matter happens at a very low density compared to the natural QCD scale, and might explain why the transition to Quarkyonic matter occurs at such a low density, very close to that of nuclear matter.

The transition to Quarkyonic matter in the simple Idylliq model is too rapid, and an explicit computation of the sound velocity shows that it has a singularity at the transition [10]. This must be an artifact of ignoring the nucleon interactions, which leads to sharp surfaces where the nucleon number is discontinuous. The region where the discontinuity in the sound speed occurs is very narrow in terms of the baryon number density, and becomes zero in the large N_c limit, and so should be easy to smear out. A proper theory must remove this singularity, and this is one of defects of the Idylliq model. In Fig. 5, a computation of the sound velocity is shown[10].

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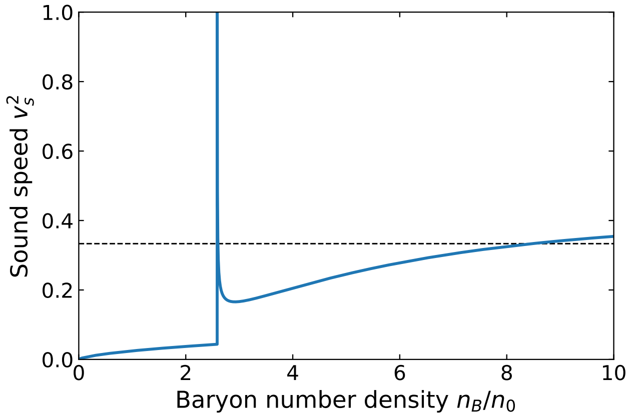

Figure 5: From Ref. [10], the sound velocity as a function of density.

5. Conclusions

The generic features of Quarkyonic matter allow for a rapid transition from a soft nuclear matter equation of state to a hard Quarkyonic equation of state. The Idylliq model provides an example of an explicitly solvable theory which has this property.

There are many issues theoretical and phenomenological which need to be addressed. Clearly there are many possible models that might be constructed using different distribution functions for quarks inside of nucleons. In the explicit model above, a singularity in k exists and one must ask if this is an artifact of the model or is it possible to justify in some limit of QCD. Moreover, the model probability distribution chosen is valid only for non-relativistic systems. The sound velocity has an unphysical singularity. How is this singularity tamed?

The Idylliq model is a free theory of nucleons. What are the properties of this theory when one includes interactions of pions for low densities? Is this a sensible theory?

How does one generalize this theory to include the effects of finite temperature? For describing the dynamics of neutron star collisions, one needs a theory at low but finite temperature and high baryon density.

The equation of state dependence on isospin, and including the effect of hyperons is non-trivial due to the highly occupied phase space of quarks. Do hyperons strongly affect the equation of state computed in their absence? Is the rapid rise in the sound velocity found in neutron stars also characteristic of zero isospin matter?

Can one determine the sound velocities dependence on density in nuclear matter at low temperature from comparision of computation to experiment for heavy ion collsions[13],[14]?[15].

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I wish to gratefully thank the many people who made this memorial meeting for Valery Rubakov possible. Valery was a friend both professional and personal throughout my career in physics. He was an honorable, decent, and honest human being, who combined his personal charisma with his insight and talent for theoretical physics to accomplish extraordinary results for science and for scientific institutions. I remember him vividly from the first time we got to know one another in Armenia in the 1980's, when with Mitya Dyakonov we hiked in the mountains, and later through many hikes and fishing trips in the Rocky Mountains with Valery, Peter Tinyakov and Misha Shaposhnikov. His ability to honestly address truly hard problems in theoretical physics with new ideas was inspirational for me.

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