PROCEEDINGS OF SCIENCE

Valery Rubakov and quantum cosmology: origin of the Universe

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This is a set of notes dedicated to the memory of Valery Rubakov who has inspired by his works the concept of the microcanonical density matrix of the Universe, which is likely to resolve the puzzles and inconsistencies of the known no-boundary and tunneling prescriptions for the cosmological wavefunction and suggests the existence of pre-inflationary thermal stage of the cosmological evolution with its UV bounded subplanckian energy scale.

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1. Introduction

This set of notes is dedicated to the memory of Valery Rubakov and inspired by vivid memories of our studentship years at Moscow University where we studied in very remote 1970-es in a group of students specializing in theoretical and statistical physics. At that time we were both under the spell of rapidly growing interest in rising quantum gravity theory – the interest carried by Valery throughout his life, which have materialized in his remarkable scientific achievements in various areas of quantum field theory, particle theory and cosmology. In his diverse scientific activity I would like to dwell on quantum gravitational side of his pioneering works which now lie at the foundation of such notions and phenomena as quantum birth of the Universe, baby universes and gravitational decoherence, macroscopic extra dimensions and cosmological evolution. One of the goals will be to discuss the idea of the density matrix model of quantum initial conditions for the evolution of the Universe, which stemmed from our conversations about the problem of time in quantum cosmology, the role of the cosmological wavefunction and the controversy of its various prescriptions. The motivation for this model comes from the scope of ideas suggesting that the initial state of the Universe should be prescribed not from some ad hoc and freely variable initial conditions like in a generic Cauchy problem, but rather intrinsically fixed by the field theory model of the Universe. The pioneering implementation of these ideas was the prescription of the tunneling and no-boundary cosmological wavefunctions, and I will try to develop the line of thought circumventing the difficulties of these two prescriptions and suggesting the way to resolve them.

2. Schroedinger equation vs Wheeler-DeWitt equations

Quantum cosmology or the quantum theory of the Universe was our major passion in those early years of our studentship, and the hope to understand the meaning of the mysterious and devoid of the notion of time Wheeler-DeWitt equation was one of the main driving forces of our school year efforts. I remember very well how Valery once came up to me and with his usual benevolent and somewhat condescending smile told me that he derived the Schroedinger equation for quantum matter from the system of Wheeler-DeWitt equations for the state of the whole gravity-matter system $|\boldsymbol{\Psi}\,\rangle$ – the statement that can symbolically be written down as

$$\hat{H}_\mu|\boldsymbol{\Psi}\,\rangle = 0 \quad\Longrightarrow\quad i\frac{\partial}{\partial t}|\,\Psi(t)\,\rangle = \hat{H}|\,\Psi(t)\,\rangle, \tag{1}$$

the full set of quantum Hamiltonian and momentum constraints [1] being denoted here as $\hat{H}_\mu = \hat{H}_\perp(\mathbf{x}),\, \hat{H}_i(\mathbf{x})$. With uneasy feeling that I am failing in our scientific competition with Valery I went away, and it took me a good stroll over the campus of Moscow University to figure out how this proof can go on – just the way it is shown in this equation,

$$\hat{H}_\mu = \hat{H}^{\rm grav}_\mu + \hat{H}^{\rm matter}_\mu, \quad |\boldsymbol{\Psi}\,\rangle = \boldsymbol{\Psi}[\,g_{ij},\phi\,] = e^{iS[\,g_{ij}\,]}\,\Psi_{\rm matter}[\,g_{ij},\phi\,]. \tag{2}$$

Here the decomposition of quantum gravitational constraints into purely gravitational and matter contributions allows one to disentangle from the full wavefunction the semiclassical gravitational factor $e^{iS[\,g_{ij}\,]}$ and show that the rest of the function,

$$|\,\Psi(t)\,\rangle = \Psi_{\rm matter}[\,g_{ij}(t),\phi\,], \tag{3}$$

evaluated at the classical curved space background $g_{ij}(t) = g_{ij}(t, \mathrm{x})$ solving the vacuum Einstein equations with some choice of the ADM lapse and shift functions $N^\perp(t, \mathrm{x})$, $N^i(t, \mathrm{x})$, satisfies in virtue of the Wheeler-DeWitt equations the usual Schroedinger equation with the quantum matter Hamiltonian [2] — the effect which is valid, of course, modulo graviton loop corrections,

$$\hat{H}_\mu \,|\,\boldsymbol{\Psi}\,\rangle = 0 \quad \Rightarrow \quad i\frac{\partial}{\partial t}|\,\Psi(t)\,\rangle = \hat{H}\,|\,\Psi(t)\,\rangle + \text{graviton loops} \tag{4}$$

$$\hat{H} = \int d^3x\,\big(N^\perp \hat{H}^{\text{matter}}_\perp + N^i \hat{H}^{\text{matter}}_i\big). \tag{5}$$

Thus, this very well known by now derivation was done long before the paper by T.Banks [3] and others, and it is likely to disprove a somewhat strange, but sometimes pronounced, opinion that the Wheeler-DeWitt equation is the most useless one in theoretical physics. Indeed, this opinion is certainly incorrect because, even if this equation is not directly used in concrete applications, it still fundamentally underlies the results obtained by alternative methods, which are intrinsically equivalent to the Wheeler-DeWitt formalism. To the same extent, the Schroedinger equation is rarely directly usable as a tool of high energy relativistic scattering but it fully underlies scattering phenomenology, and what we see is the fact that the Schroedinger equation itself is a derivative of the Wheeler-DeWitt one.

In addition, this derivation reveals the role of time in timeless formalism of the diffeomorphism invariant quantum gravity. That is, time appears as a parameter of chronology synchronization of quantum matter events as a clock device played by a semiclassical subsystem – semiclassically treated gravitational field. This is a semiclassical implementation of the idea by Bryce DeWitt that the role of the clock is played by a certain variable of the gravitational system [1]. Below we will also see another incarnation of time variable within the timeless formalism of the cosmological density matrix.

3. No-boundary (Hartle-Hawking) vs tunneling wavefunction

No wonder that after this start Valery Rubakov has turned to quantum cosmology – the theory of quantum origin of the Universe – the subject that was going to be very popular in early 1980-es especially because of invention of inflation paradigm successfully resolving the puzzles of standard Big Bang scenario. And here he became one of the pioneers of the particle creation theory within the tunneling prescription for the quantum state of the Universe [4], that was initially suggested by Alex Vilenkin [5, 6]. This prescription was competing with the Hartle-Hawking no-boundary prescription [7, 8] that also underlay the quasi-exponential expansion of the Universe befitting the chaotic inflation scenario. Both prescriptions can be interpreted as a quantum birth of the Universe from nothing or tunneling from classically forbidden, underbarrier, state of the gravitational field described by the Euclidean deSitter (or quasi-deSitter) instanton with the FRW metruc $ds^2 = N^2\,d\tau^2 + a^2\,d\Omega^2_{(3)}$, $a_0(\tau) = \sin(H\tau)/H$, $H = \sqrt{\Lambda_{\text{eff}}/3}$ and the effective cosmological constant generated by the potential of a slowly varying inflaton field φ, $\Lambda_{\text{eff}} = V(\varphi)/M_P^2$.

Because of the hyperbolic nature of the Wheeler-DeWitt equation the semiclassical behavior of these two wavefunctions is characterized by two inverse to one another amplitudes in terms of the exponentiated Euclidean action of the de Sitter instanton,

$$\Psi_{\pm}\big(\varphi,\Phi(\mathbf{x})\big) = \exp\Big(\mp\tfrac{1}{2}S_E(\varphi)\Big)\,\Psi_{\rm matter}\big(\varphi,\Phi(\mathbf{x})\big), \quad S_E(\varphi) \simeq -\frac{24\pi^2 M_{\rm P}^4}{V(\varphi)} < 0. \tag{6}$$

The no-boundary state, as a source of inflation, suffers a major difficulty associated with the negative definiteness of the de Sitter action — it produces insufficient amount of inflation because its amplitude is maximal at the minimum values of the inflaton potential generating small (or even zero) Hubble factor,

$$\boldsymbol{\Psi}_{HH} \sim \exp(-S_E) = \exp\left(12\pi^2\frac{M_P^4}{V(\varphi)}\right) \to \infty, \quad \frac{V(\varphi)}{M_P^2} = \Lambda_{\rm eff} \to 0. \tag{7}$$

This looks counterintuitive because it predicts as infinitely more probable the quantum birth of the Universe of infinitely big size.

The tunneling wavefunction is free from this difficulty, $\boldsymbol{\Psi}_T \sim \exp(+S_E)$, but this prescription similarly to the no-boundary one is devoid of reasonable justification from principles of unitary quantum field theory. Even though the no-boundary wavefunction can be considered as an offspring of Euclidean quantum gravity as a special vacuum-type state given by the Euclidean path integral, its status within canonical quantization subject to unitarity requirements remains questionable. Among other limitations of both prescriptions is also the fact that they are concrete pure quantum states – in fact, vacuum states of all particle field modes on top of the (quasi)de Sitter background. This a priori restricts a physical setup by excluding a vast set of possible excited states and impure density matrix states of the Universe. So allow me at this point to proceed to the main question of my notes — how one can go over to the physical setup with the density matrix replacing this distinguished pure state.

4. Cosmological initial conditions: microcanonical density matrix of the Universe

The attempt to do this encounters the problem of constructing the set of physical states $|\boldsymbol{\Psi}\rangle$ along with the set of their weights w_Ψ participating in the construction of the density matrix,

$$\hat{\boldsymbol{\rho}} = \sum_{\text{all } |\boldsymbol{\Psi}\rangle} w_\Psi |\boldsymbol{\Psi}\rangle\langle\boldsymbol{\Psi}|. \tag{8}$$

This problem looks unmanageable without additional assumptions, but the simplest possible assumption — universal microcanonical equipartition of all physical states with $w_\Psi = 1$ — allows one to write down the density matrix in a closed form provided one has a complete set of equations which determine a full set of $|\boldsymbol{\Psi}\rangle$. These are, of course, the above mentioned Wheeler-DeWitt equations $\hat{H}_\mu|\boldsymbol{\Psi}\rangle = 0$ [1], μ being the label enumerating the full set of Hamiltonian and diffeomorphism constraints, which includes also a continuous range of spatial coordinates, $\mu = \big(\perp\mathbf{x},\, i\mathbf{x}\big)$. The density matrix becomes a formal operator projector on the subspace of these states — an operator delta functions,

$$\hat{\boldsymbol{\rho}} = \frac{1}{Z}\prod_\mu \delta(\hat{H}_\mu), \quad Z = {\rm Tr}\prod_\mu \delta(\hat{H}_\mu) \tag{9}$$

the factor Z being a partition function which provides the normalization $\mathrm{tr}\hat{\boldsymbol{\rho}} = 1$ [9].

What kind of motivation can be put forward in favor of this construction? A simplest analogy is an unconstrained system with a conserved Hamiltonian $\hat{H}$ is the microcanonical density matrix at a fixed energy E, $\hat{\boldsymbol{\rho}} = \frac{1}{Z}\delta(\hat{H} - E)$. This analogy would not, however, directly work in quantum gravity, because spatially closed cosmology does not have the notion of global conserved energy and other freely specifiable constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints H_μ, all having a particular value — zero. Hence, comes the above prescription. Another conceptual argument of a more philosophical or ontological nature (after all we are talking about the Universe as a whole, bearing our existence) is that this is an ultimate equipartition in the full set of states of the theory — “Sum over Everything”. Creation of the Universe from *Everything* is conceptually more appealing than creation from *Nothing*. This is because the democracy of microcanonical equipartition better fits the principle of Occam razor, preferring to drop redundant assumptions, than the selection of a concrete state.

Getting back from philosophical grounds to the rules of mathematical physics, let us note the following important feature of this formal projection operation. The detailed construction of the delta function of *noncommuting* operators $\hat{H}_\mu$ (which form an open algebra of first class constraints) leads to the representation of this projector in terms of the Faddeev-Popov or Batalin-Fradkin-Vilkovisky path integral of quantum gravity [9, 10] and makes it tractable within perturbation theory. This representation equally applies to the matrix element of the delta function of constraints as the integral over *paths* – the histories $g_{\mu\nu}(t, \mathbf{x}), \Phi(t, \mathbf{x})$ parameterized by some parameter t and interpolating between the arguments $\varphi_\pm = (g^\pm_{ij}(\mathbf{x}), \Phi_\pm(\mathbf{x}))$ of this two-point element,

$$\boldsymbol{\rho}(\varphi_+, \varphi_-) = \frac{1}{Z}\int D[\, g_{\mu\nu}, \Phi\,]\; e^{iS[\, g_{\mu\nu}, \Phi\,]}\Big|_{g_{ij}(t_\pm) = g^\pm_{ij},\;\; \Phi(t_\pm) = \Phi_\pm} \tag{10}$$

(Φ denoting a generic set of matter fields in addition to the spatial metric g_{ij}, and among the components of the 4-metric $g_{\mu\nu}$ the lapse and shift functions N^μ are not fixed at $t_\pm$ but rather integrated over, see [9–11]). Correspondingly the partition function Z, which follows from the normalization of the density matrix and obviously reduces to the path integration over periodic histories, arises as

$$\mathrm{tr}\,\hat{\boldsymbol{\rho}} = \int d\varphi\, \boldsymbol{\rho}(\varphi_+, \varphi_-)\,\big|_{\varphi_\pm = \varphi} = 1 \quad\rightarrow\quad Z = \int\limits_{\text{periodic}} D[\, g_{\mu\nu}, \Phi\,]\; e^{iS[\, g_{\mu\nu}, \Phi\,]}. \tag{11}$$

Note that the parameter t in histories $g_{\mu\nu}(t, \mathbf{x}), \Phi(t, \mathbf{x})$, which looks exactly like a physical time variable in the Lorentzian gravitational action $S[\, g_{\mu\nu}, \Phi\,]$ and which is originally missing in the projector (9), arose as an operator ordering parameter that allows one to account for non-Abelian nature of quantum constraints. In the canonical (phase space) version of this path integral its role was to extend to the operator level with non-commuting $\hat{H}_\mu$ the c-number delta function $\int dN\, \exp(-iN^\mu H_\mu) = \prod_\mu \delta(H_\mu)$ (necessitating the transition from single-time integration to the path-integral one [10, 11]). This is another incarnation of the time variable in the timeless Wheeler-DeWitt formalism, mentioned in Sect.2.

In contrast to the Hartle-Hawking prescription formulated exclusively in Euclidean spacetime this density matrix expression is built within unitary Lorentzian quantum gravity formalism [11].

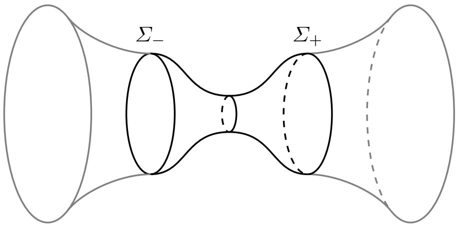

Figure 1: Instanton picture representing the density matrix. Gray lines depict the Lorentzian Universe nucleating from the instanton at the minimal surfaces Σ_- and Σ_+.

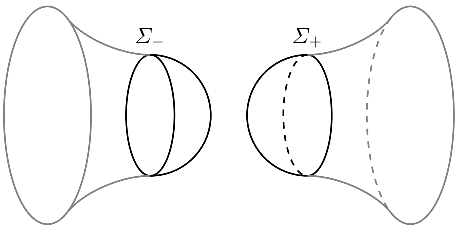

Figure 2: Density matrix of the pure Hartle-Hawking state represented by the union of two no-boundary instantons.

Euclidean quantum gravity, however, arises in this picture at the semiclssical level as a mathematical tool of perturbative loop expansion. The dominant semiclassical contribution to the partition function Z should come from the saddle points — periodic solutions of classical equations of motion. The practice of cosmological applications shows, however, that such solutions do not exist in spacetime with the Lorentzian signature, but they can be constructed in Euclidean spacetime. The deformation of the integration contour into the complex plane of both dynamical variables and their time argument then suggests that these Euclidean configurations with their Euclidean action $S_E[\,g_{\mu\nu},\Phi\,]$ can be taken as a ground for a dominant contribution of the semiclassical expansion of the partition function,

$$Z = \int\limits_{\text{periodic}} D[\,g_{\mu\nu},\Phi\,]\; e^{-S_E[\,g_{\mu\nu},\Phi\,]} \tag{12}$$

For spatially closed cosmology with spherical S^3 spatial slices the graphical image of the density matrix (or at least its diagonal element) looks like a nucleation of the Lorentzian spacetime branches, depicted by dashed lines, from the Euclidean cosmological instanton of tubular topology $R^1\times S^3$, $R^1 = [\tau_-, \tau_+]$, with the finite range of the Euclidean time τ between two minimal boundary surfaces (turning points of the Euclidean solution) Σ_- and Σ_+ (see Fig.1). At these boundaries this instanton is analytically continued to the Lorentzian spacetime.

This picture includes as a particular case the factorized density matrix of the pure no-boundary state, when the Euclidean bridge gets pinched to form the union of two disjoint 4-dimensional discs, each representing the no-boundary wavefunction (see Fig.2). Tracing the relevant two-point kernels leads to the corresponding partition functions associated respectively with the donut topology $S^1\times S^3$ and the S^4 topology, which arise as the result of identification and gluing of these minimal surfaces

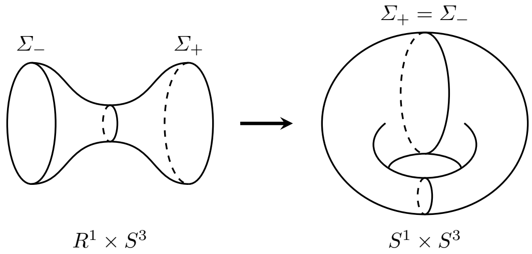

Figure 3: Origin of the partition function instanton from the density matrix instanton by the procedure of gluing the boundaries Σ_+ and Σ_- — tracing the density matrix.

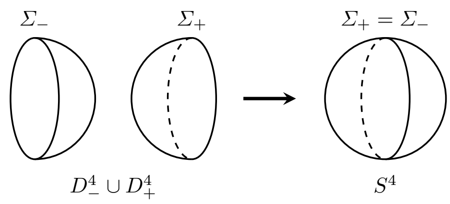

Figure 4: Origin of the S^4 partition function instanton from the density matrix instanton of the pure Hartle-Hawking state.

(see Fig.3 and Fig.4). Compactification of the Euclidean time τ to the circle clearly indicates a thermal nature in the first case [12].

5. Inflationary model driven by the trace anomaly of Weyl invariant fields — CFT driven cosmology

Productive application of this model is the Einstein theory with the cosmological term dominated by a large number of Weyl invariant matter fields Φ,

$$S_E[\,g_{\mu\nu},\Phi\,] = -\frac{M_P^2}{2}\int d^4x\, g^{1/2}\,(R-2\Lambda) + S_{CFT}[\,g_{\mu\nu},\Phi\,] \tag{13}$$

(M_P denotes here reduced Planck mass). Integrating Φ out in the approximation omitting graviton loops one arrives at the gravitational effective action with the conformal field theory (CFT) part $\Gamma_{CFT}[\,g_{\mu\nu}]$ which can be recovered from the known trace anomaly,

$$S_{\rm eff}[g_{\mu\nu}] = -\frac{M_P^2}{2}\int d^4x\, g^{1/2}(R-2\Lambda) + \Gamma_{CFT}[g_{\mu\nu}], \tag{14}$$

$$\tag{15}$$

$$e^{-\Gamma_{CFT}[\,g_{\mu\nu}]} = \int D\Phi\, e^{-S_{CFT}[\,g_{\mu\nu},\Phi\,]}. \tag{16}$$

On the conformally flat and periodic in time Euclidean Friedmann background with the metric,

$$g_{\mu\nu}dx^\mu dx^\nu = N^2(\tau)\,d\tau^2 + a^2(\tau)\,d^2\Omega^{(3)}, \tag{17}$$

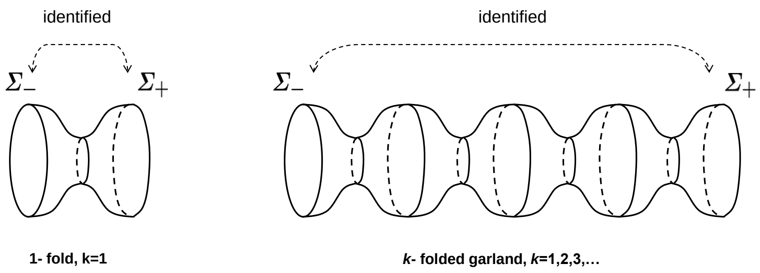

Figure 5: Garland instantons with $k = 1$ and $k = 1, 2, 3, 4, \ldots$ folds.

the CFT part of the effective action $S_{\rm eff}[\,g_{\mu\nu}\,] = S_{\rm eff}[\,a, N\,]$ turns out to be given by the contributions of Weyl anomaly,

$$g_{\mu\nu}\frac{\delta\Gamma_{CFT}}{\delta g_{\mu\nu}} = \frac{1}{64\pi^2}g^{1/2}\left(\beta E + \alpha\Box R + \gamma C^2_{\mu\nu\alpha\beta}\right), \tag{18}$$

critically depending on the overall coefficient β of the Gauss-Bonnet term with $E = R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2$ (and two other less important coefficients of $\Box R$ and Weyl squared term), Casimir energy and thermal radiation energy of CFT particles [12]. These particles are distributed over the spectrum of their comoving energies ω (eigenvalues of the spatial Laplacian on the 3-sphere of a unit radius) according to Bose-Fermi statistics at the effective (or comoving) temperature which is in its turn determined by the time circumference of the instanton in units of the conformal time

$$\eta = \int_{S^1}\frac{d\tau N}{a}. \tag{19}$$

This directly leads to the effective Friedmann equation $\delta S_{\rm eff}[\,a, N\,]/\delta N(\tau) = 0$ with the total energy density ε, composed of the contributions of the cosmological constant and this thermal radiation, and having the effective Planck mass squared $M^2_{\rm eff}(\varepsilon)$ – the function of ε reflecting the back reaction of quantum matter on instanton geometry,

$$\frac{1}{a^2} - \frac{\dot a^2}{a^2} = \frac{\varepsilon}{3M^2_\pm(\varepsilon)}, \quad \varepsilon = M_P^2\Lambda + \frac{1}{2\pi^2a^4}\sum_\omega\frac{\omega}{e^{\eta\,\omega} - 1}, \tag{20}$$

$$M^2_\pm(\varepsilon) = \frac{M_P^2}{2}\left(1 \pm \sqrt{1 - \tfrac{\beta}{6\pi^2M_P^4}\,\varepsilon}\right). \tag{21}$$

Altogether this signifies the existence of a thermal stage preceding the inflation.

The solutions of this equation consist of the family of garland-type instantons representing multiple periodic oscillations of the cosmological scale factor between minimal and maximal values (see Fig.5) and also the vacuum (zero temperature) S^4-instanton of the Hartle-Hawking type. The latter, however, does not contribute at all because of its infinite positive action – flipping the sign of this action being the joint effect of Weyl anomaly and Casimir energy [12]. This completely resolves the paradox of infinitely more probable quantum birth of infinitely big universes with $\Lambda = 0$ inherent to the prescription of the no-boundary wavefunction.

Garland-type thermal instantons exist in the finite range of values of the cosmological constant below the Planckian bound determined by the overall coefficient β of the topologucal Gauss-Bonnet term in the trace anomaly [12],

$$\Lambda_{\rm min} < \Lambda < \Lambda_{\rm max} = \frac{12\pi^2}{\beta}\, M_P^2. \tag{22}$$

Thus, this parameter is indeed critically important in this theory.

6. New type of hill-top inflation

The garland instantons serve as initial conditions for inflation, and we called this scenario "Some Like It Hot" (SLIH) [12]. In contrast to a known inflation paradigm, which replaced Big Bang with the initial vacuum state, SLIH scenario recovers a new incarnation of Hot Big Bang — effectively it incorporates a thermal state at the onset of the cosmological evolution. Realistic inflation with finite duration can be obtained from the above model by replacing the cosmological constant with the composite operator — inflaton potential in the regime of the slow roll,

$$\Lambda \to \frac{\rho_\phi}{M_P^2}, \quad \rho_\phi = V(\phi) - \frac{\dot\phi^2}{2} \simeq V(\phi). \tag{23}$$

Then, the Lorentzian inflationary Universe starts evolving with intial conditions set by the instanton, that is by its analytic continuation to the Lorentzian time across the turning point (its minimal boundary surface). Qualitatively the further evolution looks like a quick dilution of primordial radiation density, decay of a composite Λ, exit from inflation and particle creation of conformally non-invariant matter with its eventual thermalization.

Quite remarkably, this scenario also solves another problem with the no-boundary state — inflationary evolution starts from the maximum of the inflaton potential rather than from maximally probable minimum [13]. Integrating over the period the Euclidean equation of motion for the inflaton one finds that the extremum of the inflaton potential necessarily belongs to the interior of this period,

$$\frac{d}{d\tau}a^3\dot\phi = a^3\,\frac{\partial V}{\partial\phi} \quad\Rightarrow\quad \oint d\tau\, a^3\,\frac{\partial V}{\partial\phi} = 0$$

(for a smooth $V(\phi)$ the point of $\partial V(\phi)/\partial\phi = 0$ being inside this period), which means the realization of the two alternative types of inflaton oscillations in the vicinity of either minimum or maximum of the potential. But the minimum is impossible, because these oscillations in Euclidean time should take place in the underbarrier regime, that is for values of the potential above the level of the approximately conserved oscillator energy. This points out to the only case of underbarrier oscillations in the vicinity of the potential *maximum*. The picture of the start of the Lorentzian evolution looks as a nucleation of the Lorentzian Universe at the turning point of the oscillatory Euclidean solution followed by slow roll sliding down the slope of the potential in the overbarrier regime (see Fig.6).

There is a special mechanism forming the needed hill shape of the inflaton potential, which is possible in the Higgs and Starobinsky R^2-inflation models both characterized by a nonminimal

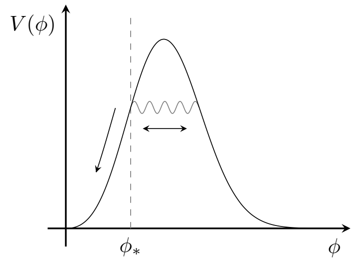

Figure 6: Nucleation of the Lorentzian Universe at the turning point ϕ_* of the oscillatory Euclidean solution followed by a slow roll down the slope of the potential.

coupling of the inflaton (respectively Higgs and scalaron fields) to curvature [14, 15]. This mechanism is based on the quantization of the theory in the Jordan frame followed by the transition to the Einstein frame [13, 16]. Classically these models in the Einstein frame feature a plateau like potential of the inflaton φ at $\varphi \to \infty$ because the Jordan frame action expanded in powers of curvature and gradients,

$$\varGamma[\,g_{\mu\nu},\varphi\,] = \int d^4x\, g^{1/2}\Big(V(\varphi) - U(\varphi)\, R(g_{\mu\nu}) + \frac{1}{2}\, G(\varphi)\,(\nabla\varphi)^2 + \dots\Big), \tag{24}$$

with some coefficient functions $V(\varphi)$, $U(\varphi)$ and $G(\varphi)$ generates the Einstein frame potential $V_{\rm E-frame}(\varphi) \sim M_P^2 V(\varphi)/4U^2(\varphi)$ tending to a constant since $V(\varphi) \sim \varphi^4$ and $U(\varphi) \sim \varphi^2$ at large φ. At the quantum level the Jordan frame potential and other coefficients of this expansion (24) of the effective action acquire logarithmic corrections starting in the one-loop order with

$$V_{\rm loop}(\varphi) \sim \varphi^4 \ln\frac{\varphi^2}{\mu^2}, \quad U_{\rm loop}(\varphi) \sim \varphi^2 \ln\frac{\varphi^2}{\mu^2}, \tag{25}$$

so that the graph of $V_{\rm E-frame}(\varphi) \sim (\ln\frac{\varphi^2}{\mu^2})^{-1} \to 0$ bends down to zero at $\varphi \to \infty$ and thus gets the hill-like shape of Fig.5. A similar mechanism works in all higher l-loop orders where the leading logarithmic behavior gets replaced by $(\ln\frac{\varphi^2}{\mu^2})^l$. One-loop renormalization group improvement of this mechanism confirms this property of the effective potential [17].

7. Hierarchy problem and justification of semiclassical expansion

This model can serve as a source of quantum initial conditions for the Starobinsky R^2-inflation and Higgs inflation theory [13, 16], in which the effective H^2 is generated respectively by the scalaron and Higgs field. In particular, one can obtain the observable value of the CMB amplitude and spectral tilt $n_s \simeq 0.965$, whereas the needed inflation scale in these models $H \sim 10^{-6} M_P$ determines the overall parameter $\beta \sim 10^{13}$ [13, 16]. Gigantic value of β needed to solve this hierarchy problem comprises the most serious difficulty of this scenario. The hope is that this difficulty can be circumvented by means of a hidden sector of numerous conformal fields [12, 18]. A high value of β cannot be attained by a contribution of low spin conformal fields $\beta = (1/180)\big(\mathcal{N}_0 + 11\mathcal{N}_{1/2} + 62\mathcal{N}_1\big)$, unless the numbers $\mathcal{N}_s$ of fields of spin s are tremendously high. On the contrary, this bound on β can be reached with a relatively low tower of higher spin fields, because a partial contribution of spin s to β grows as s^6 [19]. The solution of hierarchy problem thus becomes a playground of $1/\mathcal{N}$-expansion theory for large number $\mathcal{N}$ of conformal species.

Due to the derived above UV bound on the inflation scale, large β and large $\mathcal{N}$ justify the use of semiclassical expansion and omission of graviton loops provided the reduced gravitational cutoff $\Lambda_{\rm grav} \sim M_P/\sqrt{\mathcal{N}}$ [20–23], above which effective field theory stops working, remains much higher than the inflation scale. Fortunately, due to a peculiar property that the number of polarizations of higher spin conformal particles $\mathcal{N} \sim s^2$ grows with spin much slower than $\beta \sim s^6$, this difficulty is possible to circumvent. If the hidden sector is built of higher spin conformal fields (CHS) [18], then the known gravitational cutoff $\Lambda_{\rm grav}$ turns out to be several orders of magnitude higher than the inflation scale. This justifies the omission of the graviton loop contribution and the use of the above conformal anomaly method. Moreover, with the number of these CHS species providing the anticipated subplanckian inflation scale one can predict as a signature of the thermal epoch preceding inflation the contribution to the spectral index of the primordial power spectrum in the third decimal order $\Delta n_s^{\rm thermal} \sim -0.001$ [24] which might be soon within the observational reach of modern precision cosmology. This means that a potential resolution of the hierarchy problem in the CFT scenario via CHS simultaneously would make measurable the thermal contribution to the CMB red tilt. This contribution will be complementary to the most fundamental observational evidence for inflation theory – red tilt of the primordial CMB spectrum caused by the deviation of the slow roll evolution from the exact de Sitter scenario [28].

Final comment concerns the cosmological density parameter. One might have noticed that the model of CFT driven cosmology with microcanonical initial conditions works only for a closed model with $k = +1$, because only this case of a positive spatial curvature guarantees existence of two turning points in the solution of the Euclidean Friedmann equation. This sounds disturbing because inflation is usually assumed to be considered in spatially flat Universe, and its flatness is considered as one of the advantages of the inflation scenario, matching very well with observations. However, as it is recently observed in the exhaustive treatment of the Planck 2018 CMB temperature and polarization data [25, 26], these datasets are now preferring a positive curvature at more than the 99% confidence level with a mean $\Omega_K \simeq -0.04$. Though this preference of closed Universe is associated with discordances known as Hubble tension problem [27], robust observational evidence in favor of a positive spatial curvature might serve a strong motivation for the suggested model of quantum initial conditions.

8. Conclusions

Finishing this set of notes let me emphasize that these results were inspired due to a very thought provoking interaction with Valery Rubakov. It started with our discussions of the problem of time in cosmology, semiclassical derivation of conventional quantum field theory from the Wheeler-DeWitt formalism, etc. In the following years he got preoccupied with other field-theoretical problems, but our joint pessimistic conclusions on fruitfulness of the no-boundary and tunneling prescriptions for the cosmological wavefunction finally materialized in the form of this concept of the density matrix of the Universe. As we see, the model of microcanonical density matrix in cosmology resolves the difficulties of these prescriptions. When unified with the hypothesis of the Weyl invariant nature of quantum matter dominating the early Universe, it leads to the quasi-thermal stage preceding

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inflation and UV bounded range of its energy scale. This CFT driven cosmology suggests a new type of hill-top inflation selecting the maxima of the inflaton potential as the onset the inflation scenario. It also incorporates the the mechanism of formation of such a potential characteristic of the Higgs inflation and Starobinsky R^2-gravity. Conformal higher spin fields dominating the early quantum Universe might suggest the solution of the hierarchy problem – the origin of the Universe in the subplanckian domain and the justification of semiclassical expansion within an anticipated 1/N-expansion. Finally it is likely to predict thermally corrected CMB spectrum with a potentially observable signature of the pre-inflationary thermal epoch.

During the last years Valery Rubakov was preoccupied with the problem of cosmological initial conditions avoiding singularities in context of modern exotic models of classical evolution, like Horndeski, Galileon, genesis, etc. Euclidean approach to the same problem initiated by Stephen Hawking and developed by Valery within the concept of baby universes – that is avoiding singularities by jumping into a quantum classically forbidden domain — is what he, perhaps unwillingly, inspired in the form of the above microcanonical density matrix of the Universe, which suggests this fascinating picture of pre-inflationary hot cosmology.

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