PROCEEDINGS OF SCIENCE

Inflation without singularity

Kagirov Rinat[a,b,*]

[a] *Institute for Nuclear Research of the Russian Academy of Sciences, 60th October Anniversary Prospect 7a, Moscow 117312, Russia*

[b] *Faculty of Physics, Lomonosov Moscow State University, 1-2 Leninskiye Gory, 119991 Moscow, Russia*

E-mail: kagirov.rr19@physics.msu.ru

The study explored the possibility of constructing cosmological models of inflation without initial singularity. The main focus was on the model of a closed universe, which has a constant radius in the asymptotic past and undergoes inflation in the future, leading to a universe dominated by a massless scalar field ('kination'). This evolution is an analog of the Genesis model with an inflationary exit but in a closed universe. The key feature of the examined model is the absence of the Null Energy Condition (NEC), allowing the use of the simplest quadratic subclass of the Horndeski theory. This subclass essentially represents a scalar field with an unusual kinetic term and General Relativity. By employing the Horndeski theory framework for a closed universe, it was demonstrated that the model remains stable at all times, meaning there are no ghosts or gradient instabilities. Furthermore, the spectrum of scalar perturbations was numerically determined, which is crucial as it corresponds to observable data.

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1. Introduction

Today in gravitation in general and in the general theory of relativity, in particular, there are still many unsolved problems related, for example, to cosmology. A strong motivation for the development of the main number of the newest alternatives to GR are the astronomical observations of recent years, which led to the necessity to introduce into astrophysics and cosmology, built on the General Theory of Relativity, such models as "inflation", "dark matter" and "dark energy". The most general scalar-tensor theory of gravitation, which does not lead to third derivatives in the equations of motion and includes both GR and many other modifications, is the Horndeski theory, in which it is possible to construct various cosmological models without an initial singularity, such as Genesis or rebound.

The inflationary model of the universe solves many of the problems arising in the hot universe model, in particular, due to the extremely high expansion rate in the inflationary stage, the problem of large-scale homogeneity and isotropy of the universe is solved, but the inflationary model has an initial singularity.

The purpose of this paper is to create cosmological models without an initial singularity. Extended theories of gravity can offer a whole class of models that satisfy the properties we need:

1) the absence of tachyons, spirits and other instabilities;

2) restrictions on the propagation rates of tensor and scalar perturbations;

3) isotropic energy-dominance condition;

4) geodesic completeness;

5) flat spectrum of scalar perturbations;

6) absence of initial singularity.

In the framework of the Horndeski theories for a flat universe without an initial singularity there are models in which geodesic incompleteness can be circumvented by adding additional terms to the Lagrangian. On the other hand, there are models of the Genesis type, where initially the energy is zero and then it increases, thus violating the isotropic energy dominance condition (NEC). In a flat universe, when the NEC is satisfied, there is a geodesic incompleteness associated with the singularity, which is called Penrose's theorem [1] . Thus, within the standard GR with a scalar field in the flat universe one has to sacrifice either geodesic completeness or NEC.

In the proposed model, considering almost flat closed universe in the future GR with a scalar field, it turns out to get rid of the mentioned problems. In this case the violation of NEC does not occur due to the fact that the energy-momentum tensor includes also curvature – now the energy passes from curvature to the cosmological constant Λ [8]. That is why for the solution of the initial singularity problem we can consider a closed universe.

2. Horndesky theories in closed universe

By now it is well known that various interesting cosmological solutions can be constructed in Horndeski theories, in particular, the possibility of constructing stable solutions in the flat universe has already been shown in [2–6] and [9–12]. We can consider apparatus developed by many authors for stability analysis in the flat universe to the case of the closed universe [7].

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As mentioned earlier in the introduction, in order to achieve the objectives, it is necessary to consider a closed universe with metric:

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j$$

where γ_{ij} is the metric of the maximally symmetric hypersurface, which can be written as $\gamma_{ij}dx^i dx^j = d\chi^2 + S^2_{\mathcal{K}}(\chi)d\Omega^2$ where

$$\mathrm{S}_{\mathcal{K}}(\chi) := \begin{cases} \sin(\sqrt{\mathcal{K}}\chi)/\sqrt{\mathcal{K}} & (\text{ closed: } \mathcal{K} > 0) \\ \chi & (\text{ flat: } \mathcal{K} = 0) \\ \sinh(\sqrt{-\mathcal{K}}\chi)/\sqrt{-\mathcal{K}} & (\text{ open: } \mathcal{K} < 0) \end{cases}$$

Let us perform a similar flat perturbation analysis in the case of a closed universe. Substituting the above metric into the action and varying over it, we obtain the background equations of motion in the form:

$$\mathcal{E}_0 + \mathcal{E}_{\mathcal{K}} = 0$$
$$\mathcal{P}_0 + \mathcal{P}_{\mathcal{K}} = 0$$

where $\mathcal{E}_0$ and $\mathcal{P}_0$ are curvature independent (these are exactly the equations from [2]), and $\mathcal{E}_{\mathcal{K}}$ and $\mathcal{P}_{\mathcal{K}}$ are proportional to $\mathcal{K}$ and are represented as:

$$\mathcal{E}_{\mathcal{K}} = -3\mathcal{G}_T\frac{\mathcal{K}}{a^2}, \quad \mathcal{P}_{\mathcal{K}} = \mathcal{F}_T\frac{\mathcal{K}}{a^2}, \tag{1}$$

with time-dependent coefficients:

$$\mathcal{F}_T = 2\left[G_4 - X(\ddot{\phi}G_{5X} + G_{5\phi})\right], \tag{2}$$
$$\mathcal{G}_T = 2\left[G_4 - 2XG_{4X} - X(H\dot{\phi}G_{5X} - G_{5\phi})\right]. \tag{3}$$

Where we used standart notation for Horndesky theories, which you can check in [2]. Thus, the effect of curvature on the background equations of motion is determined.

2.1 Scalar perturbations

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To analyze the stability and obtain the spectrum, a quadratic action for scalar perturbations in the presence of curvature is needed. Similarly to [2] can be obtained [7]:

$$\begin{aligned} S_S^{(2)} = \int dt d^3x\sqrt{\gamma}a^3 \Big\{ & -3\mathcal{G}_T\dot{\zeta}^2 - \frac{\mathcal{F}_T}{a^2}\zeta\mathcal{D}^2\zeta \\ & + \Sigma\delta n^2 - 2\Theta\delta n\frac{\mathcal{D}^2\chi}{a^2} + 2\mathcal{G}_T\dot{\zeta}\frac{\mathcal{D}^2\chi}{a^2} + 6\Theta\delta n\dot{\zeta} \\ & - 2\mathcal{G}_T\delta n\frac{\mathcal{D}^2\zeta}{a^2} - 3\mathcal{F}_T\zeta^2\frac{\mathcal{K}}{a^2} - 6\mathcal{G}_T\delta n\zeta\frac{\mathcal{K}}{a^2} \\ & - \frac{\mathcal{G}_T}{2a^4}\left[(\mathcal{D}^2\chi)^2 - (\mathcal{D}_i\mathcal{D}^j\chi)^2\right]\Big\}, \end{aligned} \tag{4}$$

where $\mathcal{D}_i$ is the covariant derivative related to the metric γ_{ij}, and

$$\Theta = \Theta_0 + \Theta_{\mathcal{K}}, \quad \Sigma = \Sigma_0 + \Sigma_{\mathcal{K}}, \tag{5}$$

the flat and curve parts are again emphasized, here Θ_0 and Σ_0 are exactly the same as in flat case and

$$\Theta_{\mathcal{K}} := -\dot{\phi} X G_{5X} \frac{\mathcal{K}}{a^2} \tag{6}$$

$$\begin{aligned}\Sigma_{\mathcal{K}} := 6(&XG_{4X} + 2X^2 G_{4XX} - XG_{5\phi} - X^2 G_{5\phi X} \\ &+ 2H\dot{\phi} X G_{5X} + H\dot{\phi} X^2 G_{5XX})\frac{\mathcal{K}}{a^2}.\end{aligned} \tag{7}$$

We can cut ties and get the action in the following form:

$$S_\zeta^{(2)} = \int dt d^3x \sqrt{\gamma} a^3 \left[\mathcal{G}_S \dot{\zeta}^2 + \zeta \frac{\mathcal{F}_S}{a^2} (\mathcal{D}^2 + 3\mathcal{K})\zeta \right], \tag{8}$$

where

$$\mathcal{G}_S := \frac{\mathcal{D}^2 + 3\mathcal{K}}{\mathcal{D}^2 - (\mathcal{G}_T \Sigma/\Theta^2)\mathcal{K}} \mathcal{G}_T \left(\frac{\mathcal{G}_T \Sigma}{\Theta^2} + 3 \right), \tag{9}$$

$$\begin{aligned}\mathcal{F}_S := &\frac{1}{a}\frac{d}{dt}\left[\frac{\mathcal{D}^2 + 3\mathcal{K}}{\mathcal{D}^2 - (\mathcal{G}_T \Sigma/\Theta^2)\mathcal{K}} \left(\frac{a\mathcal{G}_T^2}{\Theta} \right) \right] \\ &- \mathcal{F}_T + \frac{\mathcal{D}^2 + 3\mathcal{K}}{\mathcal{D}^2 - (\mathcal{G}_T \Sigma/\Theta^2)\mathcal{K}} \left(\frac{\mathcal{G}_T^3}{\Theta^2} \frac{\mathcal{K}}{a^2} \right).\end{aligned} \tag{10}$$

In the following, a special case of this action will be used to build a specific model.

3. Model building without initial singularity

Let us proceed to the construction of the model without initial singularity. We will consider a special case of the Horndeski theory, in which the only nonzero functions in the action are $\mathcal{L}_2$ and $\mathcal{L}_4$. To make the proposed model similar to the GR model with the inflaton field, we choose $\mathcal{L}_4 = R$. We obtain the model of gravitation with an arbitrary scalar field. Further we will work in it.

$$S = \int d^4x \sqrt{-g} \left(F(\pi, X) + R \right) \tag{11}$$

3.1 Scale factor

The first step is to choose a scale factor that corresponds to evolution without an initial singularity. As such a scale factor a has been chosen (Fig. 1):

$$a = \frac{\mathrm{e}^x + 1}{1 + \mathrm{e}^{x-1}} + \left(1 + x^2\right)^{\frac{1}{6}} \frac{1}{1 + \mathrm{e}^{-x+1}} \tag{12}$$

Let us discuss what this choice of scale factor corresponds to. In the infinite past, the closed universe was of finite size, after which a period of inflation with exponential expansion began, which passed into a period of hot stage. Note that in this example the characteristic sizes and times are extremely small, below these parameters will be added to the model.

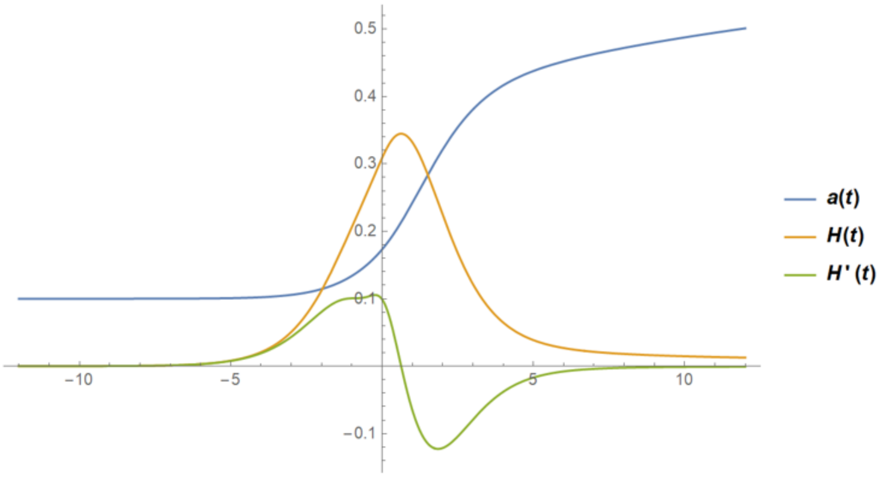

Figure 1: Dependencies $a(t), H(t)\ \ \dot{H}(t)$

3.2 Null Energy Condition

Among the various energy-dominance conditions, or energy conditions, the Null Energy Condition (NEC) plays a special role. This condition means that the energy-momentum tensor $T_{\mu\nu}$ satisfies the relation:

$$T_{\mu\nu}n^{\mu}n^{\nu} > 0$$

for any isotropic (light-like) vector n^{μ}, i.e., such a vector that $g_{\mu\nu}n^{\mu}n^{\nu} = 0$.

By virtue of Einstein's equations, this condition can be rewritten as:

$$G_{\mu\nu}n^{\mu}n^{\nu}n^{\nu} > 0$$

For the constructed metric, we can find the Einstein tensor:

$$G_{\mu\nu} = \begin{pmatrix} \frac{3(e+e^{x})^2}{\left(e+e^{1+x}+e^{x}\left(1+x^2\right)^{1/6}\right)^2} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\sin(\chi)^2 & 0 \\ 0 & 0 & 0 & -\sin(\chi)^2\sin(\tau)^2 \end{pmatrix} \tag{13}$$

It is easy to check that such Einstein tensor satisfies the NEC condition.

3.3 Checking stability conditions

In [9-12], conditions on the propagation velocities of scalar and tensor perturbations were obtained, from which it follows that:

$$\mathcal{G}_{\mathcal{T}} \geq \mathcal{F}_{\mathcal{T}} > \epsilon > 0, \quad \mathcal{G}_{\mathcal{S}} \geq \mathcal{F}_{\mathcal{S}} > \epsilon > 0. \tag{14}$$

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For our model, it is convenient to put $\mathcal{G}_{\mathcal{T}} = \mathcal{F}_{\mathcal{T}} = 1$, then $c_{\mathcal{T}}^2 = 1$.

In the framework of the Lagrangian under consideration, the coefficients $\mathcal{G}_{\mathcal{T}}, \mathcal{F}_{\mathcal{T}}$ are greatly simplified. Based on the action for a closed universe, we can obtain:

$$\mathcal{G}_{\mathcal{S}}^{cl} = \mathcal{G}_{\mathcal{S}}^{flat} = \frac{\Sigma}{H^2} + 3, \tag{15}$$

and

$$\mathcal{F}_{\mathcal{S}}^{cl} = \mathcal{F}_{\mathcal{S}}^{flat} + \frac{\kappa}{\Theta^2 a^2} = -\frac{\dot{H}}{H^2 a^2} + \frac{\kappa}{\Theta^2 a^2} \tag{16}$$

Thus, only $\mathcal{F}_{\mathcal{S}}$ depends on the curvature in our case.

Furthermore, it is convenient to take $\mathcal{G}_{\mathcal{S}} = \mathcal{F}_{\mathcal{S}}$, which will correspond to a constant rate of propagation of scalar perturbations, that is, $c_{\mathcal{S}}^2 = 1$. It is easy to sure that the constructed coefficients satisfy conditions (14).

3.4 Reconstruction of the Lagrangian of the theory

Now, knowing the coefficients in the quadratic Lagrangians for the tensor and scalar perturbations, we can reconstruct the Lagrangian of the model. Here we use, first, the equations of motion, and second, the coefficients found above.

For this purpose it is convenient to take the following ansatz for the scalar field:

$$F(\pi, X) = f_0 + f_1 X + f_2 X^2 \tag{17}$$

$F_X = f_1(t) + 2f_2(t), \quad F_{XX} = 2f_2(t)$

Now we have 3 equations for 3 unknown functions f_0, f_1, f_2, respectively, the exact expressions for these functions have been found. The Lagrangian of the theory is now completely known.

3.5 Analysis of scalar perturbations and spectrum

Now it is necessary to analyze the proposed model for consistency with the observed data. It is known that in the inflation model the spectrum of scalar perturbations is flat. Let us verify in this section that the spectrum is also flat in the proposed model.

First, we find the equations of motion for the quadratic action of scalar perturbations:

$$S_s = \int \mathrm{d}t\, \mathrm{d}^3x a^3 \left[\mathcal{G}_{\mathcal{S}} \dot{\alpha}^2 - \mathcal{F}_{\mathcal{S}} \frac{(k\alpha)^2}{a^2} \right] \tag{18}$$

coresponding EOM:

$$\alpha''(t) + \alpha(t) w(t) = 0 \tag{19}$$

where

$$B = \frac{(a^3 \mathcal{G}_{\mathcal{S}})'}{a^3 \mathcal{G}_{\mathcal{S}}} \tag{20}$$

and

$$w(t) = \frac{k^2}{a^2} - \left(\frac{B}{2}\right)^2 - \frac{B'}{2} \tag{21}$$

4. Scalar pertrubation spectrum

We now have the equations of motion for scalar perturbations in a flat universe, which we can solve numerically. We have written a program that calculates the scalar perturbation spectrum numerically for arbitrary equations of motion.

This method has been tested on the inflation theory, which gives a flat spectrum. Using our method, it is indeed possible to obtain the desired dependence of the spectrum on the impulse, i.e., a flat spectrum at a given choice of the scale factor:

$$a(t) = e^{Ht} \tag{22}$$

Next, we did the same procedure for the scale factor in the form (12) and obtained, as expected, a flat spectrum. In our case, we assumed 25 e-foldings to speed up the numerical count, but this parameter can be changed.

5. Conclusion

So far, a model of inflation without initial singularity in the closed universe, passing to the stage dominated by a massless scadjar field ("kination"), satisfying all the above conditions, has been constructed. Moreover, the parameters responsible for the sizes of the universe in the periods before and after inflation were added, and what is especially important - a flat spectrum was obtained, as in the inflation theory. To obtain the spectrum, we numerically solved the equations of motion for scalar perturbations in a large momentum range. The constructed method of spectrum analysis seems to be possible to apply to a wide class of cosmological theories to find a constraint on their parameters, which is one of the vectors for future work. Further analysis of the constructed model is planned in the near future. First of all, it is necessary to analyze more finely the features of the spectrum of scalar and tensor perturbations, due to which additional constraints on the model parameters will be placed from the observed data.

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