PROCEEDINGS OF SCIENCE

# On cosmologies with "strong gravity in the past"

**Yulia Ageeva**[a,*]

[a] *Institute for Nuclear Research of the Russian Academy of Sciences, 60th October Anniversary Prospect, 7a, 117312 Moscow, Russia*

*E-mail:* ageeva@inr.ac.ru

This short report is devoted to the study of cosmological solutions without initial singularity in scalar-tensor theories of gravity and the legitimacy of their classical treatment. Non-singular epoch – genesis – was constructed in a certain subclass of the Horndeski theory. Considered solution is stable at all times, and perturbations propagate subluminally. Moreover, it was show that in a specific range of Lagrangian parameters there is no strong coupling regime in the constructed model at early times, i.e. the classical field theory description is applicable. For this analysis of strong coupling problem we have used "naive" dimensional analysis. However, this analysis may sometimes badly fail in estimating the strong coupling scale. Indeed, examining the potential strong coupling problem at early times in a contracting cosmological model , which is conformally related to inflation, from naive dimensional analysis in the Jordan frame one would conclude that the quantum strong coupling energy scale can be lower than the classical energy scale, but from the Einstein frame prospective this should not be the case. We illustrate this point by calculation in the Jordan frame which shows cancellations of the dangerous contributions in the tree level amplitude. Therefore, it is necessary to use more accurate analysis of the strong coupling problem using unitarity bounds. To this end, useful unitarity relations and unitary bounds were found in a theory that contains scalar fields with different sound speeds.

*International Conference on Particle Physics and Cosmology (ICPPCRubakov2023)*
*02-07, October 2023*
*Yerevan, Armenia*

*Speaker





PoS(ICPPCRubakov2023)026

## 1. Introduction

In scalar-tensor theories of modified gravity there is a possibility of bouncing or genesis cosmologies in the Jordan frame, with the effective Planck mass depending on time and tending to zero in the asymptotic past ("strong gravity in the past"). These models have been discussed [1–3], for example, in the framework of Horndeski theories [4–7], where it has been proposed to avoid instabilities, i.e. avoid corresponding no-go theorems [1, 8]. As the effective Planck mass tends to zero in the asymptotic past, one may worry that the theory is in the strong coupling regime at early times, so the classical treatment of the background is not legitimate. One way to approach this issue is to make use of naive dimensional analysis of the interacting theory [9]. Nevertheless, we point out that naive dimensional analysis may sometimes badly fail in estimating the strong coupling scale. Our example is the contracting Universe in the Jordan frame which is conformally related to the inflationary Universe in the Einstein frame [10]. For an appropriate inflationary scalar potential, the Einstein frame picture guarantees that the classical treatment of the background is fully legitimate. We will illustrate that, on the other hand, the naive dimensional analysis in the Jordan frame would show the opposite. This is the problem of the dimensional analysis: the explicit calculations of tree level amplitude (for scalar perturbations only) in the Jordan frame show strong cancellations yielding the consistency with the Einstein frame inflationary considerations.

In cosmological scenarios, however, one always meets several different types of perturbations: for example, scalar and tensor modes. This motivates us to move forward and obtain some useful generalization of common unitarity relation. Using these unitarity relations, one can derive also the unitarity bounds. As we have mentioned above, unitarity bounds are particularly useful for evaluating the quantum strong coupling scale in corresponding EFT (see, e.g., Ref. [11], there we consider Horndeski bounce model).

This paper is organized as follows. We show the simple model of the Universe with genesis in Sec. 2, where we also discuss general properties – stability and strong coupling issue. In Sec. 3 we stick to some simple model of contracting Universe and consider the strong coupling problem firstly at the level of naive dimensional analysis and then calculate the tree level amplitude in this case in order to accurately analyse the legitimacy of classical treatment. Finally, in Sec. 4 we briefly discuss the generalization of untarity relations and bound in the case of theory with several scalar fields with different sound speeds. We conclude in Sec. 5.

## 2. An example of cosmology with strong gravity in the past: The Universe beginning with genesis

If one uses general relativity (GR) to describe gravity, then an important characteristic is the null energy condition (NEC) for the matter energy-momentum tensor $T_{\mu\nu}$, which reads $T_{\mu\nu}k^\mu k^\nu \geq 0$ for every null vector $k^\mu$. Once the NEC holds in the cosmological context, then (assuming flat spatial case) it follows from the Einstein equations that $dH/dt \leq 0$, where $H$ is the Hubble parameter. This implies that there is a singularity in the past of the expanding universe. Therefore, one should either modify gravity or violate the NEC to build non-singular cosmology.

It is known since 2010 [7, 12] that the best candidate for NEC violation is the Horndeski theories [4] (for reviews see, e.g., Refs. [13, 14]). It is sufficient for our purposes to consider a subclass of Horndeski Lagrangians instead of full one,

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi)R, \quad (1)$$
$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi,$$

where $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$ and $(\nabla_\mu\nabla_\nu\phi)^2 = \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi$, and $R$ is the Ricci scalar. The metric signature is $(-,+,+,+)$. In this Section we consider this theory at large negative times and study spatially flat backgrounds. However, in Ref. [3] we show how to construct explicit Horndeski models with strong gravity in the past. There we introduce several Horndeski cosmologies, which are stable at all times. We ensure that these models are free of the strong coupling problem. Such cosmologies are complete in the sense that at late times the Universe expands in a standard way: at large positive $t$, the models turn into general relativity with a conventional massless scalar field that drives the expansion. We also make sure that the speed of the perturbations about our backgrounds does not exceed the speed of light. That is why, these cosmologies are exotic but healthy (surely, modulo possible pathologies at nonlinear level).

It is convenient to use the freedom of field redefinition and choose the background field $\phi$ as $e^{-\phi} = -\sqrt{2Y_0}t$, where $Y_0$ is a constant. In this paper we stick to the unitary gauge (i.e. $\delta\phi = 0$), in which the field $\phi$ has the latter form. The metric, with perturbations included, is

$$ds^2 = -N^2dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right),$$

where

$$\gamma_{ij} = a^2 e^{2\zeta}\left(e^h\right)_{ij}, \quad (e^h)_{ij} = \delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj} + \frac{1}{6}h_{ik}h_{kl}h_{lj} + \cdots,$$

where $\zeta$ and transverse traceless matrix $h = [h_{ij}]$ are scalar and tensor metric perturbations, respectively, while the lapse and the shift functions involving perturbations are $\delta N = \alpha$, $\delta N_i = \partial_i\beta$, which actually are not physical ones. To make contact with Ref. [1], and also for the convenience, let us rewrite the Lagrangian (1) in terms of ADM variables [14]:

$$\mathcal{L} = A_2(t,N) + A_3(t,N)K + A_4(t,N)(K^2 - K_{ij}^2) + B_4(t,N)R^{(3)}, \quad (2)$$

where ${}^{(3)}R_{ij}$ is the Ricci tensor made of $\gamma_{ij}$, $\sqrt{-g} = N\sqrt{\gamma}$, $K = \gamma^{ij}K_{ij}$, ${}^{(3)}R = \gamma^{ij}\ {}^{(3)}R_{ij}$ with $K_{ij}$ being the extrinsic curvature. One can find all details about the connection between ADM and covariant formalisms in, e.g., Ref. [3].

The equations for background are obtained by setting $N = N(t)$, $N^i = 0$, $\gamma_{ij} = a^2(t)\delta_{ij}$ in (2) and are shown in [15]. An explicit construction of genesis stage is conveniently described in the ADM language. An example we study here is given in Ref. [1]:

$$A_2 = f^{-2\alpha-2-\delta}a_2(N), \quad A_3 = f^{-2\alpha-1-\delta}a_3(N), \quad B_4 = -A_4 = f^{-2\alpha}, \quad (3)$$

where $\alpha$ and $\delta$ are constant parameters satisfying

$$2\alpha > 1 + \delta\,, \qquad \delta > 0\,, \quad (4)$$

and $f(t)$ is some function of time, $a_2$ and $a_3$ are some functions of $N$, which were chosen so that the following solution to equations of motion exists at early times, $t \to -\infty$:

$$H \equiv \frac{\dot{a}}{Na} \approx \frac{\chi}{(-t)^{1+\delta}}\,, \quad a \approx 1 + \frac{\chi}{\delta(-t)^\delta}\,, \quad N \approx 1\,,$$

where $\chi$ is some combination of the Lagrangian parameters, see [16]. Thus, the setup (3) indeed admits the genesis solution at early times.

Next, in the theory (1), the quadratic action for tensor and scalar perturbations reads [15]

$$\mathcal{S}_{hh} + \mathcal{S}_{ss} = \int N dt\, a^3 d^3x \left[ \mathcal{G}_T \frac{\dot{h}_{ij}^2}{8N^2} - \frac{\mathcal{F}_T}{8a^2} h_{ij,k} h_{ij,k} + \mathcal{G}_S \frac{\dot{\zeta}^2}{N^2} - \frac{\mathcal{F}_S}{a^2} \zeta_{,i}\zeta_{,i} \right],$$

where

$$\mathcal{G}_T = -2A_4, \quad \mathcal{F}_T = 2B_4,$$
$$\mathcal{F}_S = \frac{1}{aN}\frac{d}{dt}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T, \quad \mathcal{G}_S = \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T,$$

with $\Sigma$ and $\Theta$ being some combinations of $H$ and Lagrangian functions $A_2, A_3, A_4, B_4$ and their derivatives with respect to $N$ [15]. To avoid ghost and gradient instabilities, one requires that

$$\mathcal{F}_S, \mathcal{G}_S, \mathcal{F}_T, \mathcal{G}_T > 0.$$

We also require that the speed of perturbations does not exceed the speed of light.

Now let us briefly turn to the following obstacle, connected with the construction of the completely healthy genesis model in Horndeski theories, known as the "no-go theorem" [1, 8]. Namely, if the background is non-singular at all times, the functions $\mathcal{F}_S, \mathcal{G}_S, \mathcal{F}_T, \mathcal{G}_T$ do not vanish at any time and, crucially, the integral

$$\int_{-\infty}^{t} a(t)[\mathcal{F}_T(t) + \mathcal{F}_S(t)]dt\,, \tag{5}$$

is divergent at the lower limit of integration. The defining property of genesis is $a(t) \to 1$ as $t \to -\infty$, therefore a sufficient condition for the latter property is that $\mathcal{F}_T$ and $\mathcal{F}_S$ are finite as $t \to -\infty$. The no-go theorem states that under these assumptions, there is a gradient or ghost instability at some stage of the cosmological evolution. However, as it was suggested in Refs. [1, 2, 17], one (working with unextended Horndeski theories) can require that the integral (5) is convergent:

$$\int_{-\infty}^{t} a(t)[\mathcal{F}_T(t) + \mathcal{F}_S(t)]dt < \infty\,, \tag{6}$$

so this implies that $\mathcal{F}_T \to 0$, $\mathcal{F}_S \to 0$ as $t \to -\infty$. Therefore, the necessary condition (6) for evading the no-go theorem (together with the genesis condition $a(t) \to 1$ as $t \to -\infty$) means that $G_4(\phi)$ sufficiently rapidly tends to zero as $t \to -\infty$.[1] The requirement that $G_4 \to 0$ as $t \to -\infty$ immediately implies that the strong coupling energy scale tends to zero in the asymptotic past: $G_4$ serves as an effective Planck mass squared.

In the setup (3), we have the following early-time asymptotics for quadratic order action couplings

$$\mathcal{F}_T \propto (-t)^{-2\alpha}, \qquad \mathcal{G}_T \propto (-t)^{-2\alpha}\,, \quad \text{as} \quad t \to -\infty\,, \tag{7}$$

$$\mathcal{F}_S \propto (-t)^{-2\alpha+\delta}, \qquad \mathcal{G}_S \propto (-t)^{-2\alpha+\delta}\,, \quad \text{as} \quad t \to -\infty\,. \tag{8}$$

[1] This follows from the connection between functions in ADM and covariant formalisms, i.e. in this case we have $G_4 = -A_4 = B_4$, see [1, 16].

In view of (4), (7) and (8), the integral (6) is convergent indeed, but the price to pay is that $\mathcal{F}_T$, $\mathcal{G}_T$, $\mathcal{F}_S$, $\mathcal{G}_S$ vanish in the asymptotic past, which may signalize the strong coupling problem coming from either scalar, tensor, or mixed scalar-tensor sector.

So, to see whether the classical treatment of this stage is legitimate, we make use of the naive dimensional analysis and find the early time asymptotics of the strong coupling energy scales dictated by various cubic (and also quadratic) terms in the Lagrangian for perturbations. We compare these scales with the energy scale characteristic of the classical evolution $E_{class} \propto \frac{\dot{H}}{H} \propto (-t)^{-1}$ (another classical energy scale $H$ is lower). Thus, if the strong coupling energy scales decrease slower than $(-t)^{-1}$ as $t \to -\infty$, the classical treatment of the background evolution is legitimate, assuming that interactions of higher than third order do not induce lower energy scales than cubic ones.

In this paper we consider interaction terms in scalar sector only, while the analysis of other ones (mixed and tensor) can be found in [16]. Each term in cubic action involving scalar perturbations schematically has the following form [16]:

$$\mathbb{L}^{(i)}_{\zeta\zeta\zeta} \propto \Lambda_i \cdot \zeta^3 \cdot (\partial_t)^{a_i} \cdot (\partial)^{b_i} \ , \tag{9}$$

where $a_i$ and $b_i$ are the numbers of temporal and spatial derivatives, respectively. There are 17 terms in cubic order action for scalars [16] and all couplings $\Lambda_1, \dots, \Lambda_{17}$ have power-law behavior at early times $t \to -\infty$:

$$\Lambda_i \propto (-t)^{x_i},$$

where $x_i$ are combinations of the parameters $\alpha$ and $\delta$, see [16] for details.

Next, one introduces the canonically normalized field $\pi$ instead of $\zeta$. Since $a(t)$ and $N(t)$ tend to constants as $t \to -\infty$, and $\mathcal{F}_S \propto \mathcal{G}_S$, we have (modulo a time-independent factor):

$$\pi = \sqrt{2\mathcal{G}_S}\zeta \propto (-t)^{-\alpha+\delta/2}\zeta \ .$$

The fact that the coefficient here tends to zero as $t \to -\infty$ is crucial for what follows. In terms of the canonically normalized field $\pi$ one rewrites (9) as[2]

$$\mathbb{L}^{(i)}_{\zeta\zeta\zeta} \propto \hat{\Lambda}_i \cdot \pi^3 \cdot (\partial_t)^{a_i} \cdot (\partial)^{b_i} \ , \tag{10}$$

where

$$\hat{\Lambda}_i = \Lambda_i \mathcal{G}_S^{-3/2} = \Lambda_i (-t)^{-\frac{3}{2}(\delta-2\alpha)} \propto (-t)^{x_i - \frac{3}{2}(\delta-2\alpha)} \ .$$

Now, the dimension of $\hat{\Lambda}_i$ is $1 - a_i - b_i$, so the strong coupling energy scale associated with the term $\mathbb{L}^{(i)}_{\zeta\zeta\zeta}$ is

$$E^{\zeta\zeta\zeta,(i)}_{strong} \propto \hat{\Lambda}_i^{-\frac{1}{a_i+b_i-1}} \propto (-t)^{-\frac{x_i+3\alpha-3\delta/2}{a_i+b_i-1}} \ .$$

By requiring that $E_{class} \ll E^{\zeta\zeta\zeta,(i)}_{strong}$, where $E_{class}$ is the energy scale of the classical evolution, we find the condition for the legitimacy of the classical treatment of the early evolution,

$$x_i + 3\alpha - \frac{3}{2}\delta < a_i + b_i - 1 \ , \qquad \text{for all} \quad i = \overline{1, 17} \ .$$

[2] In [16] we comment that it is sufficient to consider the Lagrangian (10) only.

So, each term from cubic Lagrangian provides some condition on Lagrangian parameters [16], and the strongest constraints are

$$0 < \delta < \frac{1}{4}, \qquad 2 - 3\delta > 2\alpha > 1 + \delta \, ,$$

where we also recall (4). The choice of such $\alpha$ and $\delta$, satisfying these constraints, opens up the possibility that the Universe starts up with very low quantum gravity energy scale (the effective Planck mass asymptotically vanishes as $t \to -\infty$), and yet its classical evolution is so slow that the classical field theory description remains valid.

Nevertheless, in the following Section we comment the problem connected with such a dimensional analysis of strong coupling issue.

## 3. Strong coupling problem: the necessity of accurate analysis involving unitarity bound

In order to illustrate the mentioned issue about dimensional analysis of strong coupling regime, in this Section we examine the early times in a contracting cosmological model with "strong gravity in the past" (Jordan frame), which is conformally related to inflation (Einstein frame). Begin with the action in the Jordan (bounce) frame is given by [10]

$$\mathcal{S}_b = \int d^4x\sqrt{-g}\left[P(\phi, X) + \frac{M_P^2 f^2(\phi)}{2}R\right] \, , \quad P(\phi, X) = \omega(\phi)X - V(\phi), \tag{11}$$

where $M_P = (8\pi G)^{-1/2}$ is reduced Planck mass, $R$ is Ricci scalar and

$$\omega(\phi) = f^2 - 6M_P^2\left(\frac{df}{d\phi}\right)^2 \, , \qquad V(\phi) = f^4(\phi)V_I(\phi) \, .$$

Here $f(\phi)$ is a yet undetermined function [3], and $V_I(\phi)$ is the scalar potential in the Einstein frame. We do not use special notation for quantities in the Jordan frame.

By conformal transformation $g_{\mu\nu} = f^{-2}(\phi)g_{I\,\mu\nu}$ the theory (11) is related to the following inflationary model in the Einstein (inflation) frame:

$$S_I = \frac{1}{2}\int d^4x\sqrt{-g_I}\left[M_P^2 R_I - g_I^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2V_I(\phi)\right] ,$$

where subscript "I" refers to quantities in the Einstein frame.

We consider inflation potential that flattens out at large fields, so that the energy density is always sub-Planckian. Viewed from the Einstein frame, the classical description of inflating background and semiclassical treatment of cosmological perturbations are perfectly legitimate. Inflation solution

$$\frac{d\phi(\tau)}{d\tau} = -\frac{M_P V_I'}{\sqrt{3V_I}} \, , \qquad H_I = \sqrt{\frac{V_I}{3}}\frac{1}{M_P} \, ,$$

[3] We note, that this function $f(\phi)$ is not somehow related to the function $f(t)$ from Section 2 – this is just a coincidence of notations.

occurs in the slow roll regime at early times, $\epsilon \ll 1$, $\eta \ll 1$, where we use the standard notations for slow roll parameters [18].

Choosing appropriate function $f(\phi)$ defining the conformal transformation one can obtain contracting Universe in the Jordan frame. We choose such $f(\phi)$, so that $f \to 0$ as $t \to -\infty$. So, the Jordan frame Hubble parameter

$$H = f\frac{d}{d\tau}\ln(a_I f^{-1}) = -f \cdot \frac{\alpha}{M_P}\sqrt{\frac{V_I}{3}}\ , \tag{12}$$

vanishes in the asymptotic past. The Jordan frame effective Planck mass $M_P^{(eff)} = fM_P$ also tends to zero as $t \to -\infty$, so we meet "strong gravity in the past" again.

From now on we work in the Jordan frame. In order to proceed the dimensional analysis firstly, we need to consider quadratic and cubic order action for perturbations. We consider only scalar perturbations of the metric. The full metric in the Jordan frame cosmic time is [19]

$$ds^2 = -[(1+\alpha)^2 - a^{-2}\mathrm{e}^{-2\zeta}(\partial\psi)^2]dt^2 + 2\partial_i\psi\, dt dx^i + a^2\mathrm{e}^{2\zeta}d\mathbf{x}^2\ ,$$

where $\alpha$ and $\psi$ are perturbations of the lapse and shift. We work in unitary gauge. Upon solving the constraints, one arrives at the unconstrained action written in terms of $\zeta$:

$$\mathcal{S}^{(2)}_{\zeta\zeta} = \int dt d^3x a^3 \mathcal{G}_S\left[\dot{\zeta}^2 - \frac{1}{a^2}\zeta_{,i}\zeta_{,i}\right],$$

where the coupling is given by [18]

$$\mathcal{G}_S = \frac{1}{2}\frac{\dot{\phi}^2}{H_I^2} = \frac{f^2}{2H_I^2}\left(\frac{d\phi}{d\tau}\right)^2\ ,$$

while in the slow roll case one has

$$\mathcal{G}_S = f^2 \cdot \frac{M_P^4(V_I')^2}{2V_I^2}\ .$$

The terms in the cubic action for scalars [19–21], are

$$\mathcal{S}^{(3)}_{\zeta\zeta\zeta} = \int dt d^3x\, a^3\left\{C_1\zeta\dot{\zeta}^2 + \frac{1}{a^2}C_2\zeta(\partial\zeta)^2 + C_4\dot{\zeta}\,(\partial_i\zeta)\,(\partial_i\mathcal{X}) + C_5\partial^2\zeta(\partial\mathcal{X})^2\right\}\ , \tag{13}$$

where $\partial^2 = \partial_i\partial_i$ and $\partial^2\mathcal{X} = \dot{\zeta}$. The coefficients are straightforwardly calculated. To the leading order in the slow roll parameters we have

$$C_1 = f^2 \cdot \frac{M_P^6(V_I')^2}{4V_I^4}(4V_IV_I'' - 3(V_I')^2),\quad C_2 = f^2 \cdot \frac{M_P^6(V_I')^2}{4V_I^4}(5(V_I')^2 - 4V_IV_I''), \tag{14a}$$

$$C_4 = f^2\frac{M_P^6(V_I')^4}{16V_I^6}(M_P^2(V_I')^2 - 8V_I^2),\quad C_5 = f^2\frac{M_P^8(V_I')^6}{32V_I^6}\ . \tag{14b}$$

Using these expressions, we now proceed with the naive dimensional analysis of the strong coupling problem. The classical energy scale is of order of the Hubble parameter (12),

$$|E^{(class)}| = |H| \sim \frac{f\sqrt{V_I}}{M_P}\ . \tag{15}$$

Next, we introduce canonically normalized field

$$\zeta_c = \sqrt{2\mathcal{G}_S}\zeta \ ,$$

so the cubic action still has the form (13) with the replacement $\tilde{C}_i = (2\mathcal{G}_S)^{-3/2}C_i$, so that

$$\tilde{C}_1 = \frac{1}{f}\cdot\frac{(-3(V_I')^2 + 4V_I V_I'')}{4V_I V_I'}, \quad \tilde{C}_2 = \frac{1}{f}\cdot\frac{(5(V_I')^2 - 4V_I V_I'')}{4V_I V_I'},$$

$$\tilde{C}_4 \sim \frac{1}{f}\cdot\frac{V_I'}{V_I} \ , \quad \tilde{C}_5 \sim \frac{1}{f}\cdot M_P^2\left(\frac{V_I'}{V_I}\right)^3 \ .$$

All operators in the resulting cubic Lagrangian are dimension-5, so one immediately finds naive estimates for the associated strong coupling scales, $E_i^{(naive)} \sim |\tilde{C}_i|^{-1}$. Naively, the most relevant of these scales are the lowest ones, which are associated with the largest $C_i$.

For asymptotically flat inflaton potential, one typically has $\eta \gg \epsilon$, i.e. $V_I V_I'' \gg (V_I')^2$, so the largest couplings in (14) are $C_1$ and $C_2$, see [18] for details. The two naive strong coupling scales are of the same order:

$$E^{(naive)} \sim f\frac{V_I'}{V_I''}. \tag{16}$$

Thus, depending on the shape of the inflaton potential, classical energy scale (15) may exceed strong coupling energy scale (16).

We conclude that naive dimensional analysis in the Jordan frame suggests that there is a quantum strong coupling energy scale which, for appropriate inflaton potential, is below the classical scale. If not for the Einstein frame considerations, one would be tempted to dismiss such a model.

To end up with dimensional analysis, we notice that the third and fourth terms in the integrand in (13) per se do not imply strong coupling, even naively. Thus, we do not have to consider the terms with couplings $C_4$ and $C_5$ in our analysis of the amplitudes.

Now, turn to more accurate analysis of strong coupling. Making use of the first and second terms in the cubic action (13), with $C_{1,2}$ replaced by $\tilde{C}_{1,2}$ and $\zeta$ by canonically normalized $\zeta_c$, it is straightforward to calculate $2 \to 2$ scattering amplitude. Before giving the result, we note that if we set, for the sake of argument, $\tilde{C}_2 = 0$, then our naive expectation would be confirmed; indeed, the corresponding matrix element

$$M_{\tilde{C}_1\,;\,\tilde{C}_2=0} = -\frac{E^2}{f^2}\cdot\frac{(9x^2-5)\big(3(V_I')^2 - 4V_I V_I''\big)^2}{64(x^2-1)V_I^2(V_I')^2} \ , \quad x \equiv \cos\theta \ ,$$

so the partial wave amplitudes (PWA)

$$a^{(l)} = \frac{1}{32\pi}\int dx\, P_l(x)\, M_{\tilde{C}_1\,;\,\tilde{C}_2=0} \ ,$$

where $P_l$ is the Legendre polynomials, would hit the unitarity bound $|a^{(l)}| = 1/2$ at $E \sim E^{(naive)}$ [22]. The same situation would occur if we set $\tilde{C}_1 = 0$. However, there are strong cancellations. Indeed, the matrix elements in $s$-, $t$- and $u$-channels are, respectively

$$M_s = -\frac{E^2}{4}(3\tilde{C}_1 + \tilde{C}_2)^2 \ , \quad M_t = \frac{E^2}{2(1-x)}\Big[\tilde{C}_1 + \tilde{C}_2(2-x)\Big]^2 \ , \quad M_u = \frac{E^2}{2(1+x)}\Big[\tilde{C}_1 + \tilde{C}_2(2+x)\Big]^2 \ ,$$

and the resulting matrix element is

$$M = M_s + M_t + M_u = \frac{E^2}{f^2} \cdot \frac{(41x^2 - 45)(V_I')^2 - 40(x^2-1)V_I V_I''}{16(x^2-1)V_I^2} \ .$$

We see that the strong coupling scale is actually given by $E^{(strong)} \sim f \cdot \left(\frac{V_I}{V_I''}\right)^{1/2} \sim f \cdot \frac{M_P}{\eta^{1/2}}$. As anticipated, this scale is much higher than the classical energy scale (15) for $V_I \ll M_P^4$. Our calculation of the amplitude confirms the absence of the strong coupling problem in Jordan frame.

## 4. The generalisation of unitarity relations and unitarity bound

Motivated with the discussion from the previous Section about the necessity of more accurate analysis of strong coupling as well as the fact that one always meets different types of perturbations in cosmoogical context, in this Section we show the unitarity relation for $2 \to 2$ scattering processes in theories with scalar fields $\phi_i$ whose sound speeds $u_i$ are different and the generalization of unitarity bound. All details about calculations one can find in Ref. [23]. In the set up with

$$S = \sum_i S_{\phi_i} \ , \quad S_{\phi_i} = \int d^4x \left( \frac{1}{2}\dot{\phi_i}^2 - \frac{1}{2}u_i^2 (\vec{\nabla}\phi_i)^2 \right),$$

the generalized PWA unitarity relation are as follows

$$-\frac{i}{2}\left(a_{l,\alpha\beta} - a^*_{l,\beta\alpha}\right) = \sum_\gamma g_\gamma a_{l,\alpha\gamma} a^*_{l,\beta\gamma} \ ,$$

where

$$g_\gamma = \frac{2}{u_{5\gamma}u_{6\gamma}(u_{5\gamma} + u_{6\gamma})} \quad \text{distinguishable} \ , \quad g_\gamma = \frac{1}{2u_\gamma^3} \quad \text{identical} \ ,$$

where we write down the cases of distinguishable and identical particles in the two-particle intermediate state. Upon redefining

$$a_{l,\alpha\beta} = \frac{\tilde{a}_{l,\alpha\beta}}{\sqrt{g_\alpha g_\beta}} \ ,$$

the most stringent tree level unitarity bound is obtained for the largest eigenvalue of the tree level matrix $\tilde{a}^{(l)}$ (which is real and symmetric) and this bound reads

$$|\text{maximum eigenvalue of } \tilde{a}^{(l)}| \leq \frac{1}{2} \ .$$

So, the latter inequality is particularly relevant when it comes to perturbative unitarity and the estimate of the strong coupling scale, see, e.g. [22].

## 5. Conclusion

In this paper we briefly discuss the different approaches for the analysis of strong coupling issue. We illustrates the main point: naive dimensional analysis may grossly underestimate the



PoS(ICPPCRubakov2023)026

quantum strong coupling energy scale. There may be less trivial situations where this property holds, e.g., due to kinematical or dynamical symmetries. Then, motivated by scalar-tensor gravities, we consider a theory which contains massless scalar fields with different sound speeds and show the unitarity relations for partial wave amplitudes of $2 \to 2$ scattering as well as unitarity bounds in the most general case. As it was mentioned, these bounds can be used for accurate estimating the strong coupling scale of a pertinent effective field theory (EFT).

## References


[1] T. Kobayashi, Phys. Rev. D **94**, no.4, 043511 (2016)

[2] A. Ijjas and P. J. Steinhardt, Phys. Lett. B **764**, 289-294 (2017)

[3] Y. Ageeva, P. Petrov and V. Rubakov, Phys. Rev. D **104**, no.6, 063530 (2021)

[4] G. W. Horndeski, Int. J. Theor. Phys. **10**, 363-384 (1974)

[5] D. B. Fairlie, J. Govaerts and A. Morozov, Nucl. Phys. B **373**, 214-232 (1992)

[6] A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D **79**, 064036 (2009)

[7] C. Deffayet, O. Pujolas, I. Sawicki and A. Vikman, JCAP **10**, 026 (2010)

[8] M. Libanov, S. Mironov and V. Rubakov, JCAP **08**, 037 (2016)

[9] Y. Ageeva, P. Petrov and V. Rubakov, JHEP **12**, 107 (2020)

[10] D. Nandi, Phys. Lett. B **809**, 135695 (2020)

[11] Y. Ageeva, P. Petrov and V. Rubakov, JHEP **01**, 026 (2023)

[12] P. Creminelli, A. Nicolis and E. Trincherini, JCAP **11**, 021 (2010)

[13] V. A. Rubakov, Phys. Usp. **57**, 128-142 (2014)

[14] T. Kobayashi, Rept. Prog. Phys. **82**, no.8, 086901 (2019)

[15] T. Kobayashi, M. Yamaguchi and J. Yokoyama, JCAP **07**, 017 (2015)

[16] Y. Ageeva, O. Evseev, O. Melichev and V. Rubakov, Phys. Rev. D **102**, no.2, 023519 (2020)

[17] S. Nishi and T. Kobayashi, Phys. Rev. D **95**, no.6, 064001 (2017)

[18] Y. Ageeva and P. Petrov, Mod. Phys. Lett. A **37**, no.26, 2250171 (2022)

[19] A. De Felice and S. Tsujikawa, JCAP **04**, 029 (2011)

[20] X. Gao, T. Kobayashi, M. Shiraishi, M. Yamaguchi, J. Yokoyama and S. Yokoyama, PTEP **2013**, 053E03 (2013)

[21] J. M. Maldacena, JHEP **05**, 013 (2003)

[22] C. Grojean, Phys. Usp. **50**, 1-35 (2007)

[23] Y. A. Ageeva and P. K. Petrov, Uspekhi Fizicheskih Nauk **193**, no.11, 1205-1213 (2022)




PoS(ICPPCRubakov2023)026