PROCEEDINGS OF SCIENCE

False vacuum decay around black holes

Ratmir Gazizov[a,b,*]

[a] *Institute for Nuclear Research of the Russian Academy of Sciences, 60th October Anniversary Prospect 7a, 117312, Moscow, Russia*

[b] *Faculty of Physics, Lomonosov Moscow State University, 1-2 Leninskiye Gory, 119991, Moscow, Russia*

E-mail: gazizov.rl18@physics.msu.ru

We numerically compute the semiclassical probability of false vacuum decay at a nonzero temperature in the Schwarzschild space-time. We consider a four-dimensional model of a scalar field with toy potential. Our result suggests that if the temperature of the field equals that of the black hole, the decay proceeds via thermal activation mediated by sphalerons.

International Conference on Particle Physics and Cosmology (ICPPCRubakov2023)
02-07, October 2023
Yerevan, Armenia

[*] Speaker

https://pos.sissa.it/

PoS(ICPPCRubakov2023)028

1. Introduction

According to experimental data and calculations within the Standard Model, the Higgs vacuum is metastable [1] and can decay due to quantum tunneling. The standard approach to calculating the probability of false vacuum decay in flat space-time is based on instantons – classical solutions of field equations in Euclidean space-time [2]. The Euclidean actions of the instantons give exponential suppression of the probability of false vacuum decay $P \sim e^{-S}$. If decay occurs in the thermal bath with temperature T, the instantons are periodic in Euclidean time with period $\beta = 1/T$. At high temperatures, these periodic solutions become time-independent sphalerons, and the respective probabilities take the form $P = \exp(-E_{sph}/T)$, where E_{sph} is the sphaleron energy. In this case, false vacuum decay proceeds via thermal activation.

It has been suggested that small black holes (BH) may increase the probability of false vacuum decay in their vicinity [3]. This is closely related to the fact that small BHs have high temperatures $T_H = M_{Pl}^2/8\pi M_{BH}$ and heat the surrounding medium. However, this conclusion is based on specific assumptions, which were criticized in the literature [4]. Thus, a calculation from first principles is needed [5].

In this work, we briefly describe a consistent semiclassical approach and perform numerical calculations in the simplified case of thermal equilibrium between the black hole and its environment.

2. Formulation of the problem

We start with the functional integral for the decay probability [6, 7]:

$$P = \int D\phi_f D\phi_i D\phi_i' \, \langle \phi_f | \hat{S} | \phi_i \rangle \langle \phi_i | \hat{\rho} | \phi_i' \rangle \langle \phi_i' | \hat{S}^\dagger | \phi_f \rangle \, , \tag{1}$$

where ϕ_i and ϕ_i' are the initial configurations of the scalar field $\phi(t, x)$, ϕ_f is the final field configuration corresponding to the true vacuum, $\hat{\rho}$ is the density matrix describing the initial state with temperature T, and $\hat{S}$ is the quantum evolution operator. Fields $\phi(t, x)$ and $\phi'(t, x)$ can be united as one field on the double-bent time contour in Fig.1.

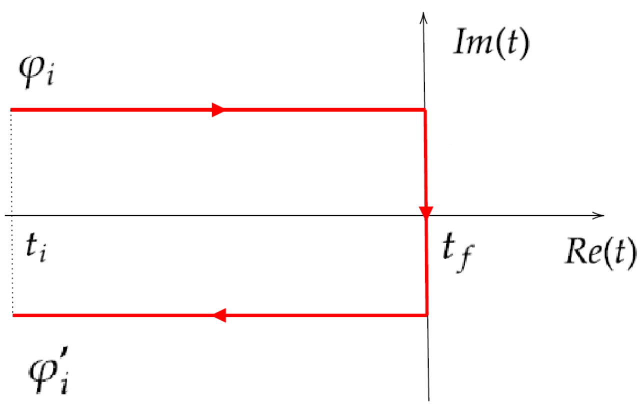


Figure 1: The contour in the complex time plane for the calculation of the decay probability.

Performing functional integration in the saddle-point approximation, we obtain:

$$P \sim e^{iS[\phi_{cl}]+B[\phi_{cl}]} \tag{2}$$

where ϕ_{cl} is the classical solution on the double-bent contour with certain boundary conditions and $B[\phi_{cl}]$ denotes the contribution of the initial density matrix in the integrand of (1).

We obtain the classical field equation from the action of the scalar field:

$$S = \int \sqrt{-g} d^4x \left(\frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi)\right) \tag{3}$$

where $g_{\mu\nu}$ is the external Schwarzschild metric with the interval

$$ds^2 = f(r)\, dt^2 - \frac{dr^2}{f(r)} - r^2 d\Omega^2, \; f(r) = 1 - \frac{2M}{r}. \tag{4}$$

Hereafter, we use Planck units with $M_{Pl} = 1$.

Assuming that the field ϕ is spherically symmetric, we rescale it and introduce the tortoise coordinate x:

$$\phi = \frac{\varphi(t,x)}{r}, \quad x = r + 2M \ln(r - 2M) \tag{5}$$

Then the action takes the form:

$$S = 4\pi \int dt dx \left(\frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2}(\partial_x \varphi)^2 - \frac{1}{2} U(x)\, \varphi^2 - r^2 f(r)\, V\left(\frac{\varphi}{r}\right)\right), \tag{6}$$

where $U(x) = f(r) f'(r)/r = 2Mr^{-3}(1 - 2M/r)$. The field equation can be simply obtained from this action.

In numerical calculations, we use the potential:

$$V(\phi) = \frac{m^2}{2}\phi^2 - \frac{m\sqrt{\lambda}}{2}\phi^3 + \frac{\lambda}{8}(1-\epsilon)\,\phi^4 \tag{7}$$

plotted in (Fig. 2). We exclude parameters m and λ from the action (6) by changing the variables $\phi \to m\phi/\sqrt{\lambda}$, $x \to m^{-1}x$, $t \to m^{-1}t$, $r \to m^{-1}r$. Also, we use $\epsilon = 0.1$, making thin wall approximation marginally valid.

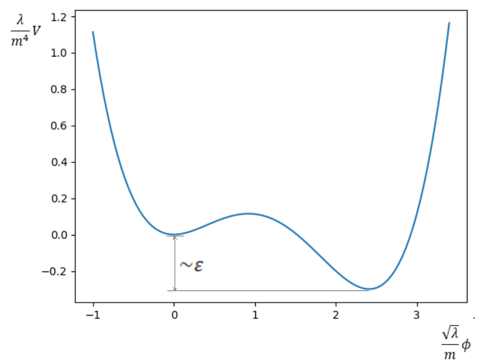


Figure 2: The potential $V(\phi)$ in (7).

The boundary conditions are derived from the functional integral (1) and imposed on the Fourier coefficients:

$$\phi_i(x) = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} \sum_{I=R,L} \left(a_{I,\omega} f_{I,\omega}(x) e^{-i\omega t_i} + b_{I,\omega} f^*_{I,\omega}(x) e^{i\omega t_i} \right) \tag{8}$$

$$a_{I,\omega} = b^*_{I,\omega} e^{-\omega\beta_I} \tag{9}$$

$$\beta_I = \begin{cases} \beta_H, I = R \\ \beta_E, I = L \end{cases} \tag{10}$$

where letters R and L denote right and left waves respectively, $f_{I,\omega}(x)$ are the mode functions derived from the linearized field equation, $\beta_H = 8\pi M_{BH}/M_{Pl}^2$ is the Hawking temperature of the BH, and β_E is the temperature of the environment.

This is a complicated problem to obtain a solution with boundary conditions (9) in the general case. However, if the temperature of the BH and the environment are equal, then the expression for the decay probability is simplified. In this case, the solution on the particular contour is real, and the boundary term $B[\phi_{cl}]$ in (2) vanishes. The solution $\phi_{cl}(t, x)$ is periodic in Euclidean time with period $\beta = \beta_H = \beta_E$ (Fig. 3). The decay probability is simply $P = e^{-S_E}$, where S_E is the action of the periodic instanton ϕ_{cl}.

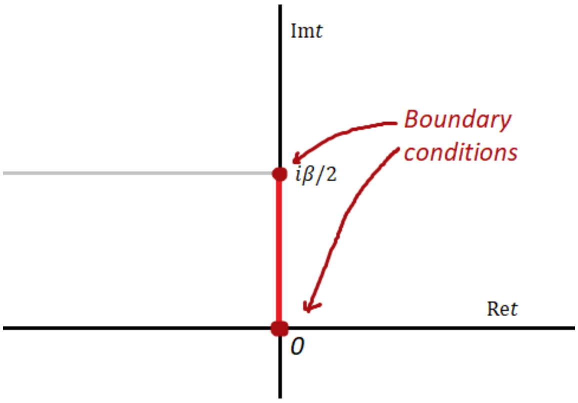


Figure 3: Euclidean part of the contour.

So, we need to solve the classical field equations in Euclidean time with boundary conditions:

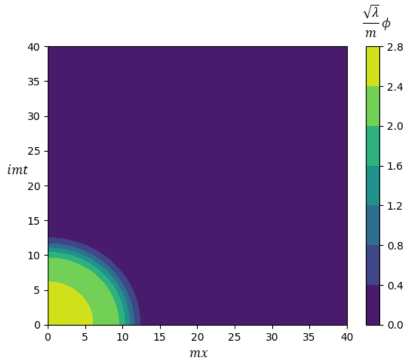
$$\partial_\tau \phi_{cl}(0, x) = \partial_\tau \phi_{cl}(\beta/2, x) = 0 \tag{11}$$

$$\partial_x \phi_{cl}(t, -\infty) = \partial_x \phi_{cl}(t, \infty) = 0 \tag{12}$$

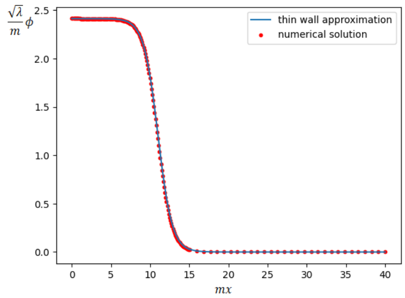
3. Numerical results

We solve the system of discretized field equations using the Newton-Raphson method.

At first, we obtain the instanton in flat space. This solution at $\epsilon \ll 1$ is compared in Fig. 4 to the thin-wall profile (points versus solid line). The false vacuum bubble has radius $mr_b \simeq 10$.



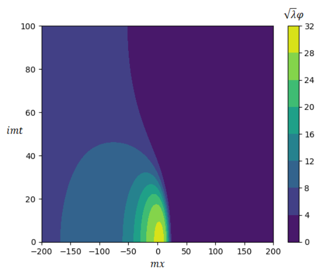
(a)



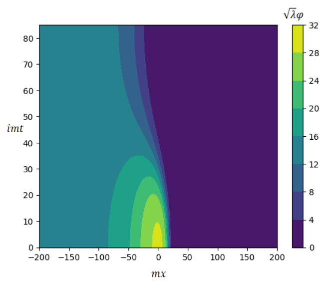
(b)

Figure 4: (a) The instanton $\phi(t, x)$ in flat Euclidean space-time, (b) The profile of the some solution at $t = 0$ (dots) is compared to the thin-wall result (line). Both figures are obtained using lattice $N_t \times N_x = 150 \times 150$.

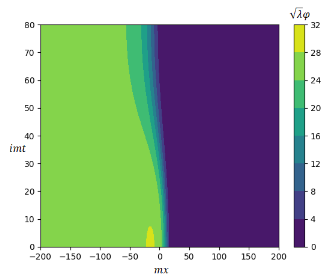
Next, we numerically compute the periodic instantons with different periods β in the presence of a BH in thermal equilibrium with the environment. In Fig.5, the latter has radius $mr_h = 2Mm/M_{Pl}^2 = 12$. Notably, these solutions are not realistic instantons since their period does not match that of the BH $m\beta/2 \neq m\beta_H/2 = 8\pi Mm/2M_{Pl}^2 = 75.4$, but the geometry of space is curved by the BH with mass M. By changing the period, we approach the physical $m\beta_H/2$ in accordance with the formula for the Hawking radiation temperature. In this limit, the instanton smoothly approaches a time-independent sphaleron, as is clear from Fig.6, showing dependence of the action on β. We also note that the radius and the temperature of a BH are related by $mr_h = m\beta_H/4\pi$. So the sizes of the BH and the false vacuum bubble are comparable.



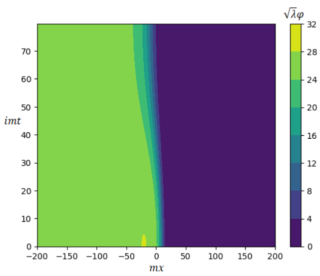
(a) $m\beta/2 = 100$



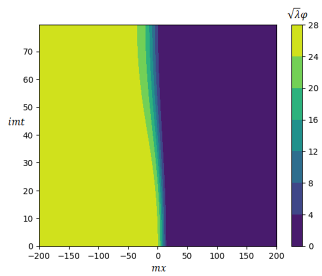
(b) $m\beta/2 = 85$



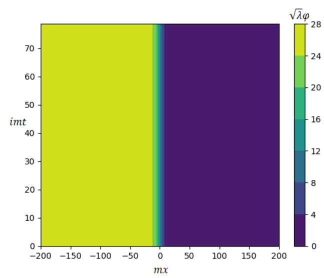
(c) $m\beta/2 = 80$



(d) $m\beta/2 = 79.7$

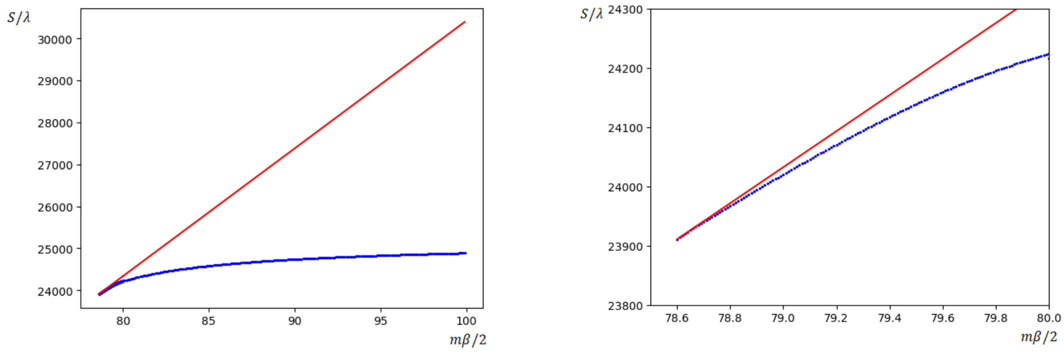


(e) $m\beta/2 = 79.6$



(f) $m\beta/2 = 78.6$

Figure 5: Periodic instantons at different β in the presence of a BH with radius $mr_h = 2Mm/M_{Pl}^2 = 12$, lattice $N_t \times N_x = 150 \times 300$ is used.



(a) **(b)**

Figure 6: Euclidean action S as a function of period β. We consider nontrivial periodic instantons (blue) and time-independent sphaleron (red) at $mr_h = 2Mm/M_{Pl}^2 = 12$, and $m\beta_H/2 = 75.4$.

We conclude that in the physical limit, periodic instantons turn into sphalerons. Dependence of the sphaleron action on the inverse BH temperature is shown in Fig.7, where the black dots and blue line correspond to presence and absence of a BH, respectively. In the infinity – temperature limit $\beta \longrightarrow 0$, the size of the BH approaches zero, and the BH and flat-space results coincide. In other words, small BHs do not significantly change the probability of decay in the very hot environment. Conversely, as the temperature approaches zero, large massive BH significantly changes the geometry of space, affecting the sphaleron solutions. This happens at $m\beta \simeq 10$, i.e. when the radius of the BH achieves $mr_h \simeq 0.1 \times mr_b$.

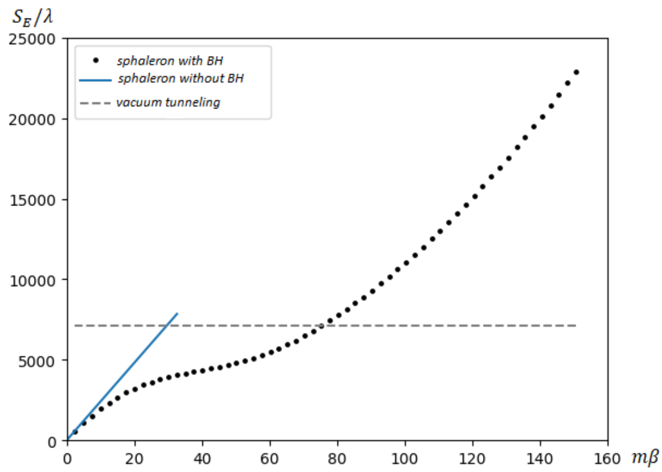


Figure 7: Euclidean action S_E of shalerons as a function of the inverse temperature β. Black dotted and solid blue lines correspond to presence of BH with $m\beta_H = m\beta = 8\pi mM/M_{Pl}^2$ and to decay in flat space-time. The gray line shows the Euclidean action of flat space instanton at zero temperature.

4. Conclusions and further research

In this talk, we considered decay of false vacuum at nonzero temperature in the presence of a BH in thermal equilibrium with the environment. We had explicitly shown that the physical solutions describing this process are static sphalerons. This means that the respective transitions proceed via thermal activation, i.e. overbarrier jumps caused by thermal fluctuations. Besides, we saw that small BHs with $mr_h \ll mr_b$ do not significantly change the sphaleron in the hot environment and the respective probabilities too. On the other hand, large BH with $mr_h \not\ll mr_b$ changes the geometry of space and, consequently, the sphaleron solutions and the decay probability.

More interesting is the nonequilibrium case, when the BH is immersed in the environment at different temperature. Our solutions with $\beta \neq \beta_H$ do not describe this situation. Rather, the physical solutions are defined on the contour in Fig.8 and satisfy sophisticated boundary conditions. It is also of interest to consider more realistic potentials, such as $V(\phi) = -\lambda\phi^4/4$, to estimate the probability of Higgs vacuum decay induced by BH.

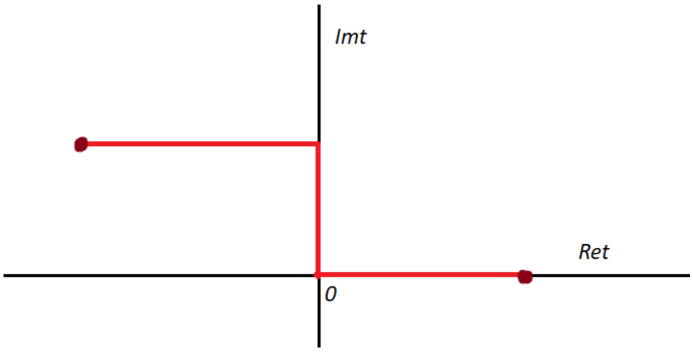


Figure 8: A time contour in general case

Acknowledgments The author acknowledges support of the scientific program of the National Center for Physics and Mathematics, sector 5 "Particle Physics and Cosmology", stage 2023-2025

References

[1] D. Buttazzo, G. Degrassi, P.P. Giardino, G.F. Giudice, F. Sala, A. Salvio et al., *Investigating the near-criticality of the higgs boson*, *Journal of High Energy Physics* **2013** (2013) .

[2] S.R. Coleman, *The Fate of the False Vacuum. 1. Semiclassical Theory*, *Phys. Rev. D* **15** (1977) 2929.

[3] P. Burda, R. Gregory and I.G. Moss, *The fate of the higgs vacuum*, *Journal of High Energy Physics* **2016** (2016) .

[4] D. Gorbunov, D. Levkov and A. Panin, *Fatal youth of the universe: black hole threat for the electroweak vacuum during preheating*, *Journal of Cosmology and Astroparticle Physics* **2017** (2017) 016–016.

[5] A. Shkerin and S. Sibiryakov, *Black hole induced false vacuum decay from first principles*, *Journal of High Energy Physics* **2021** (2021) .

[6] S. Khlebnikov, V. Rubakov and P. Tinyakov, *Periodic instantons and scattering amplitudes*, *Nuclear Physics B* **367** (1991) 334.

[7] V.A. Rubakov and M.E. Shaposhnikov, *Electroweak baryon number non-conservation in the early universe and in high-energy collisions*, *Physics-Uspekhi* **39** (1996) 461–502.