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New tests of general relativity

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General relativity (GR) was tested in many experiments and observations and always its predictions were confirmed. However, in a majority of tests GR was checked and confirmed in a weak gravitational field regime. In 2005 it was predicted that a shadow formed near a supermassive black hole at the Galactic Center could be reconstructed from observations of ground based global VLBI system or ground – space interferometer acting in mm or sub-mm bands. In 2022 the Event Horizon Telescope (EHT) collaboration reconstructed the shadow for the black hole at the Galactic Center (GC), therefore our prediction was confirmed. In 2019 the EHT collaboration presented the first image reconstruction around the shadow for the supermassive black hole in M87*. In 2021 the EHT collaboration constrained parameters ("charges") of spherical symmetrical metrics of black holes from an allowed interval for shadow radius. Earlier, we obtained analytical expressions for the shadow radius as a function of charge (including a tidal one) in the case of Reissner–Nordström metric. Based on results of the shadow size evaluation for M87* done by the EHT team we constrain a tidal charge. Similarly we constrained a tidal charge for the black hole at the Galactic Center based on shadow reconstruction done by the EHT collaboration in 2022. We discussed opportunities to use shadows to test alternative theories of gravity and alternative theories for galactic centers. We used also observational data for trajectories of bright stars near the Galactic Center to test gravity theories and theoretical models for the Galactic Center.

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1. Introduction

As it is known general relativity (GR) equations were found by A. Einstein and D. Hilbert in November 1915 [1, 2][1]. Slightly earlier, but also in November 1915 Einstein explained the perihelion anomaly for the Mercury orbit [4] (the problem was formulated by U. J. J. Le Verrier in the mid of the XIX century). In one of his GR paper D. Hilbert appreciated V. Fredericks (Fréedericksz)[2] for useful communications. Scientific knowledge about the universe in which we live is extremely important (at least within the framework of simplified models). The construction of realistic mathematical models of the structure of the Universe turned out to be possible only after A. Einstein created the general theory of relativity and built the first (static) model of the Universe [5]. Friedmann's cosmological solutions, which he found in 1922 and 1924, were of great importance [6, 7] and these results are among the most outstanding theoretical discoveries in physics.

2. Investigations in relativistic cosmology

As it was noted, the first cosmological model of the universe was suggested in the Einstein model, but the first evolving cosmological model was proposed by Soviet mathematician and physicist A. A. Friedmann [6, 7]. Soon after that G. Lemaitre[3] showed that signs of an expanding model of the universe can be detected from astronomical data on the redshifts of distant galaxies [8], as discovered by E. Hubble [9] (an interesting discussion of the historical aspects of the discovery of gravitational redshift is available in [10]). However, it was only by about 1973 that the explanation of the redshifts of distant galaxies by the expansion of the universe became predominant. In 1930s G. Lemaitre introduced and promoted a hot Universe model, which was called the "Primeval Atom" [11]. Lemaitre promoted his cosmological approach for several years and summarized his studies on the subject in [12] (the French edition of the book was published earlier). In the framework of his approach Lemaitre predicted that "If all the atoms of the stars were equally distributed through space there would be about one atom per cubic yard, or the total energy would be that of an equilibrium radiation at the temperature of liquid hydrogen" [13]. As it was noted in [14] practically it was predicted an existence background radiation, many researchers called G. Lemaitre as the Big Bang theory father in spite of the fact that a hot Universe model was called as the Big Bang by Sir Fred Hoyle several years later. In 1933 after the Lemaitre's lecture at Mount Wilson Observatory where E. Hubble worked at this time Einstein said "This is the most beautiful and satisfactory

[1]An intensive correspondence between these two great scientists led to this wonderful discovery [3].

[2]Friedmann's interest in GR and cosmological problems related to this theory was initiated by V. K. Fredericks, who worked as an assistant to D. Hilbert in Göttingen during the First World War and, came back to Russia in 1918. Fredericks was one of the from the pioneers of relativistic research in Soviet Russia. Practically, Fredericks and Friedmann established an outstanding relativity school in Petrograd (Leningrad) and V. A. Fock, G. A. Gamow, D. D. Ivanenko, A. D. Alexandrov belonged to this school. Frederics significantly contributed in liquid crystal physics and in particular, Frederics and his co-authors discovered a phenomenon which is called now Freedericksz (Fredericks) transitions.

[3]At the beginning of XX century Lemaitre spent some time in the USA and knew Slipher's results on positive redshifts for distant galaxies and perhaps this knowledge helped Lemaitre to connect a theory with observations and to derive the cosmological expansion law which is called now the Hubble law. As a recognition of the Lemaitre contribution at the Thirtieth General Assembly of the International Astronomical Union (Vienna, August 2018) astronomers suggested to rename the Hubble law as the Hubble – Lemaitre law and it was accepted that "from now on the expansion of the universe be referred to as the Hubble–Lemaitre law".

explanation of creation to which I have ever listened!” [15]. Since, thanks to journalists, this phrase of Einstein became widely known, it had an extremely negative impact on the free discussion of various cosmological theories in Soviet Union and thus, the discussion of realistic evolutionary cosmological models in the USSR was, to put it mildly, not welcomed, which led to the fact that Soviet scientists ceased to be leaders in this field of science. Some time ago, V. P. Vizgin noted that since Soviet physicists were involved in Soviet atomic project and the nuclear shield protected physicists from ideologically motivated critics. However, in contrast to other branches of physics Soviet cosmology was controlled by ideologists and philosophers and they repeated the official dogmas to make sure that they did not deviate from the general line of the Communist Party on this issue. Nowadays it may seem strange, but in the USSR in the 30s - 50s of the twentieth century, the official point of view on cosmology was that the universe is non-evolutionary and should be infinite in time and space, and it was these factors that distinguished Soviet cosmology from "bourgeois" (which, according to Soviet ideologists, was obviously wrong). Apparently, Friedmann’s idea of the "birth of the universe" seemed to representatives of Marxist-Leninist philosophy too similar to the biblical ideas about the creation of the world to recognize it as acceptable. In particular, in article ”Cosmology” [16] in the Great Soviet Encyclopedia (GSE, Second Edition) it was written ”Modern bourgeois Cosmology is characterized by the transfer to the entire universe of the properties of the known part of the Metagalaxy, which are also highly schematized. This transfer, with the Doppler interpretation of the red shift, creates the "theory of the expanding Universe" (Belgian physicist Abbé G. Lemaitre et al.). Supporters of this "theory" were, in particular, illegally extended to the entire universe the solutions of Einstein’s equations of gravity found by Friedmann, including equations with cosmic constant (bourgeois relativistic Cosmology).” This ideological ban on cosmological studies in Soviet Union produced a very negative impact on a development of research in this field. For instance, as it is known [17, 18], the CMB radiation (which is one of the signatures of hot Universe model developed by G. Gamow in 1940s and 1950s) was discovered by T. Shmaonov at the Pulkovo Observatory several years before Penzias[4] and Wilson (who were awarded the Nobel prize in 1978). However, Shmaonov’s achievements were not known for many years, as they did not have an appropriate cosmological interpretation. There is a popular opinion that no one in the Soviet Union knew about the Gamow model of the hot Universe and the predictions of this model. However, this interpretation is at least incomplete. As mentioned earlier, dynamic models of the Universe (including Gamow models) were considered inadequate descriptions of the universe and their consideration was not welcomed by official ideology and philosophy. In addition, despite the fact that Gamow was one of the most famous Soviet theoretical physicists, he did not return from a business trip abroad without the permission of the authorities. Thus, the mention of Gamow’s works could be interpreted as support for his disloyal attitude towards the Soviet government. Therefore, even if some experts understood that Shmaonov’s achievements had a cosmological interpretation, they preferred not to demonstrate their understanding in order to avoid the danger of being condemned for supporting the provisions of physical cosmology, which were criticized by Soviet philosophers. On February 6, 1962, P. L. Kapitsa made a report at the general meeting of the Soviet Academy of Sciences, where he noted "that the application of dialectics in the field of natural sciences requires an exceptionally deep knowledge of experimental facts and their theoretical

[4]Arno Penzias passed away on January 22, 2024.

generalization" and on the basis of this report an article was written in the Economic Gazette of March 26, 1962. Thus, Kapitsa argued that only experiment, and not the theoretical provisions of any "advanced" theory, is the criterion for the truth of the laws of nature. At the beginning of the 60s of the last century, observational confirmations of the Friedmann model of the expanding Universe already had enough confirmations, although according to R. Feynman (who gave lectures on gravity at Caltech in 1962 -1963), the Bondi-Hoyle model of the stationary Universe could not yet be completely excluded. Remarkable British astronomer Fred Hoyle supported his cosmological model until the end of his life in 2001. In July 1963, the journal "Soviet Physics – Uspekhi" published a special issue of the journal dedicated to the 75th anniversary of A. A. Friedmann, which contains articles by famous scientists such as P. Ya. Polubarinova – Kochina, V. A. Fock, Ya. B. Zeldovich, E. M. Lifshitz and I. M. Khalatnikov, as well as Russian translations of Friedman's articles on cosmology published in German in 1922 and 1924. Moreover, in his review Zeldovich started to quote Gamow's papers. Initially, Zeldovich critized Gamov's papers on a hot Universe model, but Zeldovich immediately recognized a hot Universe model as a correct cosmological approach after the CMB discovery done by Penzias and Wilson. Thus, it can be said that in 1963, the Soviet Union lifted the ban on discussing realistic cosmological models where it was considered the origin and an evolution of the Universe. In 1966, the collected works of A. A. Friedmann were published.

3. The Galactic Center

In 1933 the Galactic Center (GC) has been discovered by Karl Jansky as one of the first extraterrestrial radio source. The discovery was based on his observations with a radio antenna (Bell Labs) located in Holmden, N. J.[5] and it was acting at 14.5 m. However, professional astronomers did not follow up on the discovery for several years. Later, GC was observed in different bands including radio, optical, IR ones, X-rays and γ.

In 1963 quasars have been discovered or in other words, very bright sources located outside of our Galaxy have been found. These sources must be very compact due to rapid variations of these sources. Therefore, the mechanism of gigantic energy releases in these objects presented a big problem. Different models for so huge energy release were discussed at the First Texas Symposium on Relativistic Astrophysics (held in Austin (TX) in 1963), soon after it and later.

Analyzing the energy release of quasars in 1969 D. Lynden-Bell suggested that binding energy of dead quasars may be transformed in electromagnetic radiation in accretion of gas and a total binding energy is around 0.057 mc^2 (where m is a particle mass). Soon after, J. Bardeen modified this idea and noted that in the case of Kerr black hole with the extreme rotation ($a = 1$), the binding energy is 0.432 mc^2 or in other words, an energy release may be almost a magnitude larger then the value for Schwarzschild black hole case. Now it is a generally accepted opinion that there are supermassive black holes in centers of galaxies including our Milky Way. A mass of the black hole in our Galactic Center was estimated as a few million solar masses [19] and consequent observations and their analysis confirmed this estimate.

[5]In 1964, at the same place Arno Penzias and Robert Woodrow Wilson of Bell Labs discovered evidence for CMB radiation. In 1978 Penzias and Wilson received the Nobel Prize for this discovery (indeed, each of these scientists received a quarter of the Nobel Prize, and P. L. Kapitsa received half of the prize in the same year).

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4. GR tests

At the initial stage of GR development, there were only three effects when taking into account the theory of relativity leads to the observed phenomena. For instance, former Einstein's assistant P. Bergmann noted in 1942, GR was needed only to explain the pericenter advance for the Mercury orbit, the defection of light passing near Sun, redshifts of spectral lines for emitted light from white dwarfs. In 1964 I. Shapiro proposed the fourth test of GR. Namely, he proposed to send a radar signal toward Venus and Mercury at the instants when these planets are located almost behind the Sun. In this case it will be possible to detect the arrival instants for echoes of these signals. Estimates showed that time delays in these cases should be around 10^{-4} sec. Shapiro also proposed a radar to send a signal and Arecibo telescope as receiver. In 1968 Shapiro and his team reported on results of their measurements these time delays. An overview of experiments and a discussion of the development of the theory of gravity over its more than a century-old history is given in the book [20]. An existence of gravitational radiation was predicted by Einstein in 1916. The binary PSR 1913+16 was detected at the Arecibo Observatory and observations of this system showed that the orbit was shrinking. The first detection of gravitational waves was done by the LIGO Scientific Collaboration on September 14, 2015 and this event is called GW150914 [21]. In the first publication about this discovery the authors reported on discovery of gravitational waves and binary black holes and in addition, the authors constrained graviton mass $m_g < 1.2 \times 10^{-22}$ eV. Soon after that a solid confirmation for a detection of gravitational wave was obtained in GW170817 event [22], where it was observed an electromagnetic counterpart as a result a kilonova explosion in binary neutron star merger. Many years ago M. V. Sazhin proposed a way to detect ultra-long gravitational waves from distant supermassive binary black hole systems analysing times of arrival (TOAs) for pulsars in our Galaxy [23]. At the Arecibo radio telescope observations of TOAs for pulsar array were started in 1980s. Recently, using the Sazhin's idea the NANOgrav collaboration reported a detection of stochastic gravitational wave background with probability $p = 10^{-3}$ [24].

5. Bright stars near the Galactic Center: recent bright confirmations of GR predictions

As it was noted earlier there is a supermassive black hole at our Galactic Center. To determine a gravitational potential at the GC astronomers monitored trajectories of bright stars for decades. The European group led by R. Genzel started its observations of bright stars at the GC in 1992 with the 3.5-metre New Technology Telescope located at the La Silla observatory in Chile. Since 2002 this group started observations with 8.2-metre VLT telescope on Cerro Paranal in Chile [25]. Currently, since 2018 this group uses the GRAVITY interferometer formed with VLT telescopes with adaptive optics [26]. Another group led by Andrea Ghez monitored bright stars moving around the black hole at GC using twin 10-metre Keck telescopes equipped by adaptive optics. Observations of both groups showed that elliptical orbits with foci at the position of the black hole at GC (the center of force) are nice first approximations for bounded orbits of bright stars. Therefore, it is possible to conclude that a Newtonian potential of point like mass is a good approximation for gravitational field inside a region where astronomers monitored trajectories of bright stars. If we take into account relativistic corrections for orbits of bright stars near GC (ignoring a bulk distribution of matter in the from of a stellar cluster, gas, dust and dark matter) we obtain a relativistic pericenter advance (the Schwarzschild precession) which was firstly calculated by A. Einstein for the Mercury orbit. On the other hand, if we consider a bulk distribution of matter it causes a negative shift in respect to relativistic one [27, 28]. The orbit of S2 star is interesting to check relativistic effects, its semi-major axis is of about 970 au (au is astronomical unit), a pericenter distance is around 120 au its eccentricity is $e = 0.88$, its period is around 16 years. In the pericenter passage in May 2018 the GRAVITY collaboration evaluated gravitational redshifts for the S2 star and the authors showed that the obtained redshifts are nicely fitted by the first post-Newtonian correction but the Newtonian gravity fit for redshifts should be rejected at 5σ level [29, 30]. Slightly later, these results were confirmed by the Keck group [31]. These observational results and the corresponding analysis confirmed the natural expectations that the laws of gravity are universal and they are the same both in the Solar System and in the vicinity of the Galactic Center. These conclusions are very important because recently theories of gravity have been proposed in which the laws of gravity may be different in different astronomical systems. In addition with orbits of bright stars the GRAVITY collaboration monitored hot spots near innermost stable circular orbit (ISCO) of the supermassive black hole at GC [29]. In these studies it was assumed that hot spots are moving along time-like geodesics while photons are moving along isotropic geodesics in a black hole metric. In last years a number of alternative theories of gravity has been proposed. A majority of them were motivated by attempts to explain dark matter and dark energy problems as gravitational effects. Observational data for trajectories of bright stars were used to constrain some of alternative theories of gravity. For instance, using observational data for S2 star in [32] $f(R)$ was constrained, in particular, in [33] Yukawa gravity parameters were bounded, in [34] a graviton mass was constrained as $m_g < 2.9\times10^{-21}$ eV or in other words, at a level comparable with the initial LIGO constraint[6]. The Keck group showed that the obtained graviton mass bound could be improved taking in consideration additional observational data for S2 star orbit [35]. In [36] perspectives to improve the current bounds of graviton mass were found from observations of bright stars. Constraints on a tidal charge of the supermassive black hole at the Galactic Center with trajectories of bright stars were found in [37]. In 2020 the GRAVITY collaboration discovered that Schwarzschild precession for S2 star orbit is very close to its theoretical estimate calculated in the framework of GR [38] assuming a presence a Schwarzschild black hole at GC. Recently, the GRAVITY collaboration showed that a bulk distribution of matter inside the S2 orbit is less than 3000 $M_\odot$ (1σ), or less than 0.1% of the black hole mass at GC [39]. Using this observational result the improved constraints on Yukawa gravity parameters were found in [40], while the improved graviton mass bound was given in [41]. Sometime ago a model of dark matter distribution with a dense core and diluted halo have been proposed [42]. Recently, it was declared that this model (it is started to call the RAR-model) could provide a better fit of trajectories of bright stars [43]. If test bodies are moving in a spherically symmetric ball with constant density, they move in harmonic oscillator potential and they move along elliptical trajectories but the centers of ellipses coincide with the center of force while for observed trajectories of bright stars foci of ellipses coincide with the center of force [44, 55]. According to Bertrand's theorem (1873), if in a central potential field all bounded trajectories are closed, then there are only two potentials

[6]Currently, the LIGO – Virgo – KARGA collaboration significantly improved the first LIGO estimate and after data analysis of three observational runs $m_g < 1.27 \times 10^{-23}$ eV.

satisfying these conditions, namely the harmonic oscillator potential $U(r) = kr^2$ ($k > 0$) and the Newtonian potential $U(r) = -\alpha/r$ ($\alpha > 0$). In addition, we could say concerning an opportunity to distinguish the harmonic oscillator and the Newtonian potentials that radial velocity curves for these two models of GC should be different and the shadow for the dense core would be different from the observed shadow for Sgr A*.

6. Shadows from gedanken experiments to GR tests

At the initial stage of development of GR and quantum mechanics gedanken (thought) experiments were very popular in a discussion of specific features of new theories. To discuss observations signatured of black holes J. M. Bardeen considered features of an existence of bright screen which is located behind a Kerr black hole in the case of an observer is located in the equatorial plane [46]. In these considerations it was assumed that photons emitted by a luminous screen do not interact with a matter around a black hole. Clearly, this gedanken experiment looked rather artificial since first, there are no luminous screens behind astrophysical black holes, second, masses of black holes were estimated not precisely and a majority of astrophysical black holes were black holes with stellar masses but even now shadows around these black holes are too small to be detected, third, it was not clear how to detect a darkness or to distinguish it from a faintness. In [47] it was proposed to use a shadow around GC as a GR test and to detect this shadow it was suggested to use ground based VLBI acting in mm, sub-mm or X-ray bands since it was noted that secondary images are located near the shadow boundaries (a development of a shadow from a theoretical concept to a GR test was discussed in [48]). In spite of the fact that there are severe doubts that astrophysical black holes have significant electric charges, shadows for Reissner – Nordström black holes were considered and it was found that there is an analytical expression for a shadow radius as a function of charge [49].

If we consider the Randall – Sundrum theory where there is an extra dimension. Using this approach Dadhich and his co-authors found a solution which looks like a Reissner – Nordström metric where parameter q may be negative. It was suggested to call this solution a Reissner – Nordström metric with a tidal charge. In the conventional Reissner – Nordström metric $q = Q^2$ (Q is an electric charge). Later it was proposed to apply this solution with a tidal charge for the black hole at the Galactic Center to test observational signatures, however, it was noted that in the case of a significant negative tidal charge q the shadow size is so large that it is not consistent with existed observational constraints on the shadow size for Sgr A* [50]. A shadow size as a function of a tidal charge was obtained in [51]. We recall these results. The Reissner – Nordström metric can be written in natural units ($G = c = 1$) as

$$ds^2 = -\alpha(r)dt^2 + \alpha^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where $\alpha(r) = 1 - \dfrac{2M}{r} + \dfrac{Q^2}{r^2}$, M is a black hole mass, Q is its charge. E and L are constants of motion for photon geodesics, namely E is a photon energy, L is its angular momentum. If we introduce dimensionless radial coordinate, impact parameter, charge we have $\hat{r} = r/M, \xi = L/(ME)$, $\hat{Q} = Q/M$ and $q = \hat{Q}^2$, then we obtain [51]

$$\xi_{\rm cr}{}^2 = \frac{(8q^2 - 36q + 27) + \sqrt{D}}{2(1-q)}, \tag{2}$$

where $D = -512\left(q - \frac{9}{8}\right)^3$ and $\xi_{\rm cr}$ was a critical impact parameter corresponding to circular photon orbit for a given q parameter. As it was noted earlier in the case of a tidal charge [51] (or the Horndeski version of scalar-tensor theories[37]) a parameter q may be negative.

7. Conclusions

Based on estimates of shadow size for M87* done in [52] it was evaluated a tidal charge, namely $q \in [-1.22, 0.814]$ at 68% C. L. [53] and the upper bound ($q_{upp} = 0.814$) of the interval corresponds to the upper limit $Q_{upp} = \sqrt{q_{upp}} \approx 0.902$ in [52]. The Event Horizon Telescope Collaboration reconstructed a shadow at the Sgr A* and evaluated a shadow size and if similarly to [54] we adopted a shadow diameter $\theta_{\rm sh\ Sgr\ A*} \approx (51.8 \pm 2.3)\mu as$ at 68% confidence level then as it was done in [55] constraints on a tidal charge were obtained $q \in [-0.27, 0.25]$. Therefore, there were found constraints on a tidal charge (and/or a parameter of scalar-tensor gravity of Horndeski type theory) for M87* and Sgr A*. So, bounds for the Randall – Sundrum theory with extra dimension for M87* and Sgr A* were obtained. Thus, as it was predicted in [47] the shadow reconstructions around supermassive black holes provide a new test of GR and some alternative theories of gravity.

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