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Meson and Baryon spin-dependent GPDs via Quantum Computing

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Testing detailed predictions of QCD and searching for phenomena at the LHC requires knowing spin dependent Parton Distribution Functions for quarks and gluons. For some observables Generalized or Transverse Momentum pdf's are needed. Calculating these distributions from QCD, ab initio, is prohibitively resource intensive and depends on non-perturbative techniques. Quantum simulation on a quantum computer of quantum field theories offers a new way to investigate properties of the fundamental constituents of matter. We develop quantum simulation algorithms based on the light-front formulation of relativistic field theories, particularly QCD in 2+1D. We compute pdf's and GPD's for a model of pion-like mesons and quark-diquark baryons.

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1. Introduction

In a 2020 paper by Michael Kreshchuk et. al. [1], a formalism was laid out to utilize quantum computation to simulate quantum field theory and generate parton distribution functions (PDFs). This paper aims to extend this formalism to higher dimensions and generate generalized parton distributions (GPD) via quantum computing. GPDs describe the momentum distribution of partons in terms of both longitudinal momentum fractions, as well as transverse momentum fractions. We lay out a method to utilize quantum computation to solve for hadronic bound states and use these states to create GPDs.

A main branch of hadronic physics is to study the internal structure of the nucleon. This was first done via deep inelastic scattering (DIS) ($lp \to lX$), in which a lepton is scattered off of a nucleon. Measuring the scattered electron's energy and deflection angle gives information about the internal structure of the hadron [2]. This process leads to a one dimensional probability distribution, the PDF. The PDF gives the probability to find a quark inside of a hadron with a fraction of the total hadronic momentum.

The kinematic variables of the GPD are $\{x, \xi, t\}$ where x is the longitudinal momentum fraction of the parton, ξ is the deviation of longitudinal momentum fraction in the process, and t is the total momentum transfer [3]. The support of the GPD is defined on $(x, \xi) \in [-1, 1]^2$. This leads to three distinct regions: $x \in [\xi, 1]$, $x \in [-\xi, \xi]$, and $x \in [-1, \xi]$. [4] The first and third cases are called the DGLAP region which describe the emission and reabsorption of a quark and antiquark respectively while the second case is called the ERBL region, describing the emission of a quark-antiquark pair. In this paper, we are interested in zero-skewness GPDs ($\xi = 0$) for which only the DGLAP region exists. In particular, we show the GPD for a quark inside of a pion, which corresponds to the region $x \in [0, 1]$ only.

This paper exploits light cone coordinates which describe the motion of a massless particle moving at the speed of light on the light cone. This allows quantum field theory to be formulated similarly to quantum chemistry, which has been simulated on a quantum computer [5]. Additionally, the vacuum in this formulation is trivial, as the lightfront momentum P^+ is bounded below as $P^+ > 0$ strictly. Thus, there is no confusion towards the vacuum state $|0\rangle$ referring to the state with no particles present, and not the state of some hadron with momentum $P = 0$ [6].

GPDs produce the most illuminating structure of hadrons that we have access to, and it is important to simulate GPDs to supplement or confirm experimental data. A common way of doing this is via Lattice QCD [7]. The problem with this method, however, is that PDFs (and thus GPDs) are defined as non-local light cone correlators and thus are intrinsically not Euclidean (which is the space that the lattice is defined on) [8]. Quasi-PDFs [9] are the objects studied on the lattice, however, can we simulate these distribution functions while keeping their inherent Minkowskian nature? It turns out that quantum computing is an effective way to accomplish this goal.

2. Formalism

A fermionic field [10], ψ, interacting with a gluon gauge field, A_μ, can be described by the QCD Lagrangian

PoS(SPIN2023)018

$$\mathcal{L} = \frac{1}{2}\bar{\psi}(i\gamma^{\mu}D_{\mu} - m_q)\psi - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} \tag{1}$$

where m_q is the bare fermionic mass (for single flavor particles), $G_{\mu\nu}$ is the gluon field strength tensor, $D_\mu = \partial_\mu - igT_aA^a_\mu$ is the covariant derivative, and g describes the coupling between the fields. Light-front (LF) coordinates [**?**] describe the motion of a (massless) particle moving at the speed of light on the light-cone and will be utilized throughout this paper. Given equal time coordinates $x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$, the coordinate transformation to LF coordinates is: $x^\pm = x^0 \pm x^3$.

The fields [10] are given as

$$A_\mu(x) = \sum_{n=1}^{\Lambda} \frac{1}{(\Omega n)^{\frac{1}{2}}}(a_n\epsilon_\mu(k,\lambda)e^{-ik_n^\nu x_\nu} + a_n^\dagger\epsilon_\mu^*(k,\lambda)e^{ik_n^\nu x_\nu}) \tag{2}$$

and

$$\psi(x) = \sum_{n=1}^{\Lambda} \frac{1}{(\Omega)^{\frac{1}{2}}}(u(k)b_n e^{-ik_n^\nu x_\nu} + v(k)d_n^\dagger e^{ik_n^\nu x_\nu}). \tag{3}$$

Hadronic bound states in light front coordinates can be found by solving the "Schroëdinger-like" equation [11]

$$P^\mu P_\mu \left|\Psi\right\rangle = M^2 \left|\Psi\right\rangle$$

which comes from the Lorentz invariant quantity for four-momentum vectors. In this light front frame, $P^\mu P_\mu \equiv H_{LC} = P^+P^- - \vec{P}^\perp$. We call this quantity the effective Hamiltonian such that $H_{\text{eff}} = P^+P^- - \vec{P}^\perp$. The effective Hamiltonian can be decomposed into two parts: $H_{\text{eff}} = H_0 + H_{\text{int}}$, where H_0 contains the kinetic piece while H_{int} contains the interaction terms, which in this case, we take to come from the particular form of the Lagrangian we proposed in equation 1. The wavefunctions can be expanded in terms of the basis functions, $\left|q\bar{q}\right\rangle, \left|q\bar{q}g\right\rangle$ etc.:

$$\left|\Psi\right\rangle = \sum_s \int \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3}\psi_s^{m_j}(x,k_\perp) \times \left|n\right\rangle.$$

In DLCQ (discretized light cone quantization)[10], the wavefunctions $\psi_s^{m_j}$ are plane waves, while in BLFQ [12], the wavefunctions are expanded in basis functions that are symmetry-adapted towards the problem we are interested. With BLFQ, we use the AdS/CFT correspondence of QCD [13] where a holographic QCD Hamiltonian is used. The holographic QCD Hamiltonian is

$$H_0 = \frac{\vec{k}_\perp^2 + m_q^2}{+}\frac{\vec{k}_\perp^2 + m_d^2}{x} + \kappa^4 x(1-x)\vec{r}_\perp^2$$
$$- \frac{\kappa^4}{(m_q + m_d)^2}\partial_x(x(1-x)\partial_x).$$

If we separate this expression into the longitudinal and transverse pieces, we see that the transverse basis states are 2D harmonic oscillators:

$$\phi_{nm}(\vec{q}_\perp) = \frac{1}{\kappa}\sqrt{\frac{4\pi n!}{(n+|m|)!}}\left(\frac{q_\perp}{\kappa}\right)^{|m|} \times e^{-q_\perp^2/(2\kappa^2)} L_n^{|m|}\left(\frac{q_\perp^2}{\kappa^2} e^{im\theta_q}\right)$$

while the longitudinal basis functions are written in terms of Jacobi polynomials:

$$\chi(x;\alpha,\beta) = x^{\beta/2}(1-x)^{\alpha/2} P_l^{(\alpha,\beta)}(2x-1) \times \sqrt{4\pi(2l+\alpha+\beta+1} \times \sqrt{\frac{\Gamma(l+1)\Gamma(l+\alpha+\beta+1)}{\Gamma(l+\alpha+1)\Gamma(l+\beta+1)}}.$$

Here, $\vec{q_\perp} = \vec{k}_\perp/\sqrt{x(1-x)}$, $\theta_q = \arg \vec{q}_\perp$, and $L_n^a(z)$ is the generalized Laguerre polynomial. For the longitudinal basis functions, $\alpha = 2m_d(m_q+m_d)/\kappa^2$ and $\beta = 2m_q(m_q+m_d)/\kappa^2$. Together, the light-front wave function (LFWF) of the hadron, $\psi_{s_q}^{m_k}$ in equation **??**, is written:

$$\psi_{s_q}^{m_j}(\vec{k}_\perp, x) = \sum_{nml} \tilde{\psi}(n,m,l)\phi_{nm}(\vec{q}_\perp)\chi_l(x) \tag{4}$$

where $\tilde{\psi}(n,m,l)$ is a coefficient.

In particular, the DLCQ-approach will be utilized when studying $|q\bar{q}\rangle$ mesons while BLFQ will be utilized with $|qd\rangle$ (quark-diquark) baryons. The main difference in this is the form of the Hamiltonian. For the DLCQ meson, the Hamiltonian is $H = T + S$ where

$$T = \sum_q \frac{m_q^2 + k_\perp^2}{x}\left(a_q^\dagger a_q + b_q^\dagger b_q + d_q^\dagger d_q\right)$$

while in BLFQ, the Hamiltonian is the AdS/QCD Hamiltonian above plus the interaction piece

$$P^-_{qd\to qd} = \frac{1}{2}g^2 \int dx^- d\vec{x}^\perp \bar{\psi}\gamma^+\psi \frac{1}{(i\partial^+)^2}\left(\partial^+\varphi\right)^\dagger \varphi$$

where ϕ is the scalar diquark field.

3. Obtaining the ground state of H_{LC} with a Quantum Computer

One commonly used algorithm to find the ground state of a Hamiltonian is the Variational Quantum Eigensolver (VQE) [14][5]. This algorithm relies on the variational principle of quantum mechanics which states that the expectation value of the Hamiltonian for a given state, $|\psi\rangle$, must be greater than or equal to the ground state energy:

$$\langle\psi|H|\psi\rangle \geq E_0.$$

We can generate a parameterized state on a quantum computer, $|\psi(\vec{\theta})\rangle$, such that as we alter the values of $\vec{\theta} = (\theta_1, \theta_2, ..., \theta_n)$, we expect to obtain a better estimate on the ground state energy of H. Known as a hybrid algorithm for utilizing both a classical and quantum processor to solve for the ground state, the steps are the following:

1. Define a state/operator mapping from the physical states in second-quantized space to qubit states.
2. Choose an initial ansatz state that can be easily prepared with a quantum computer $|\psi_0\rangle$ (Here this is taken as a fock state in the (K, Q)-sector of the Hamiltonian).
3. Create a parameterized circuit $U(\vec{\theta})$ that prepares the parameterized state via $|\Psi(\vec{\theta})\rangle = U(\vec{\theta})\,|\psi_0\rangle$
4. Calculate $E(\vec{\theta}) = \langle\psi(\vec{\theta})|H|\psi(\vec{\theta})\rangle$ on a quantum processor for a given set of parameters $\vec{\theta}$.
5. Use a classical processor to update $\vec{\theta}$ based on the current value of $E(\vec{\theta})$.
6. Repeat this process until convergence (within a set tolerance) to the ground state E_0

After mapping our Fock states to qubit states (which is done in detail in [1], [15]), we can variationally prepare ground bound states of the hadrons of interest, $|q\bar{q}\rangle$ and $|qd\rangle$.

4. Generalized Parton Distribution Functions

Generalized Parton Distributions are formally defined as [4]

$$H^q(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P'|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})|P\rangle\,|_{z^+=\mathbf{z}=0}.$$

where $q(\frac{z}{2})$ is a quark field operator and γ^+ is one of the LF Dirac matrices defined by $\gamma^+ = \gamma^0 + \gamma^3$. In order to put equation 4 to use, we must write the quark field operators in terms of fermionic and bosonic creation and annihilation operators as well as define the Fock states in 2 + 1D formalism. Note that in the limit that $t = 0$, (i.e. the incoming and outgoing hadronic states are the same: $|P\rangle = |P'\rangle$), we obtain the formal definition of the PDF and hence: $H^q(x,0,0) = f_q(x)$ where $f_q(x)$ is the parton distribution function.

Golec-Biernat and Martin[16] give the explicit form of the GPD (off-diagonal distribution) in terms of creation/annihilation operators by writing the quark field operators $q(\frac{-z}{2})$ and $q(\frac{z}{2})$ (defined in equation 3)in terms of fermionic and bosonic operators. Eq 5 depends on the DGLAP and ERBL regions and we are interested in the quark GPD so we look at the region $\theta(x \geq \xi)$.

$$\begin{aligned} H^q(x,\xi) \quad &= \frac{1}{2\bar{P}^+}\int \frac{d^2k_T}{2\sqrt{|x^2-\xi^2|}(2\pi)^3} \qquad (5)\\ &\sum_\lambda [\langle P'|\, b^\dagger_\lambda((x-\xi)\bar{P}^+, k_T-\Delta_T) b_\lambda((x+\xi)\bar{P}^+, k_T)\,|P\rangle\,\theta(x\geq\xi) \\ &+ \langle P'|\, d^\dagger_\lambda((-x+\xi)\bar{P}^+, -k_T+\Delta_T) b_{-\lambda}((x+\xi)\bar{P}^+, k_T)\,|P\rangle\,\theta(-\xi<x<\xi) \\ &- \langle P'|\, d^\dagger_\lambda((-x-\xi)\bar{P}^+, k_T-\Delta_T) d_\lambda((-x+\xi)\bar{P}^+, k_T)\,|P\rangle\,\theta(x\leq\xi)]. \end{aligned}$$

This is called the off-diagonal distribution because it connects different Fock states $|P\rangle$ and $|P'\rangle$ whereas the PDF (forward distributions) is diagonal and connects the same states. H^q simplifies when we look at the zero skewness case ($\xi = 0$) since we are interested in hadrons with the same resolutions before and after an interaction. This form of the GPD will be useful for DLCQ, where the SWAP test [18] can be used to calculate the GPD on a quantum computer. Alternatively, with BLFQ, we can use the definition of the GPD in terms of light front wavefunctions [17]

$$H(x, \xi = 0, t) = \sum_{\lambda,\lambda'} \int d^2\vec{k}_\perp \psi^*(\vec{k}'_\perp, x, \lambda)\psi(\vec{k}_\perp, x, \lambda).$$

5. GPDs via Quantum Computing Results

After properly encoding and calculating GPDs with techniques on quantum computers, such as the SWAP test or the Hadamard test [19], we can plot the results. Note that these plots are from quantum simulators, rather than true noisy quantum computers; however, they can be translated directly to a real quantum device. For the meson, we obtain the following GPD:

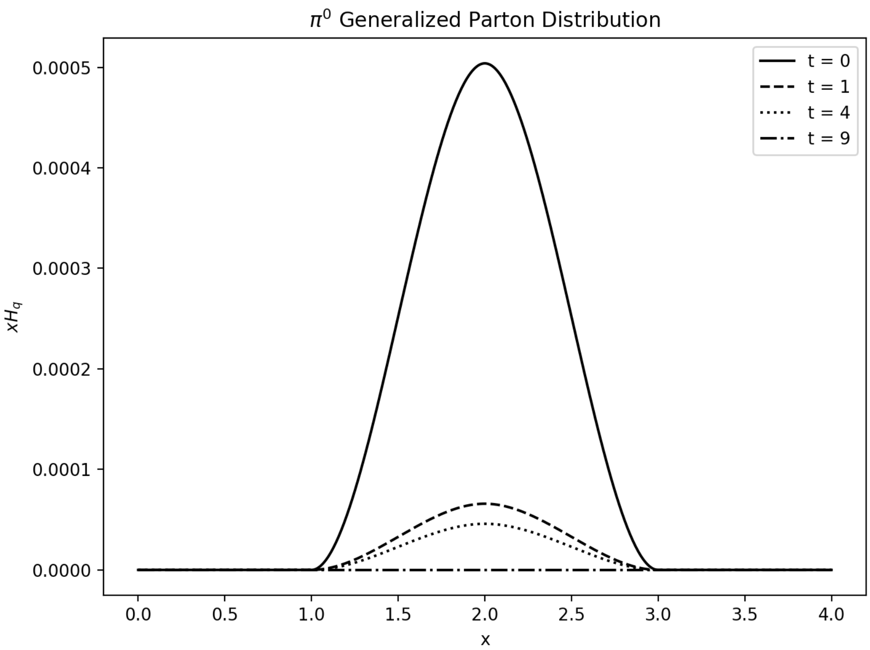

Figure 1: π^0 Meson GPD in DLCQ

while for the baryon, we obtain:

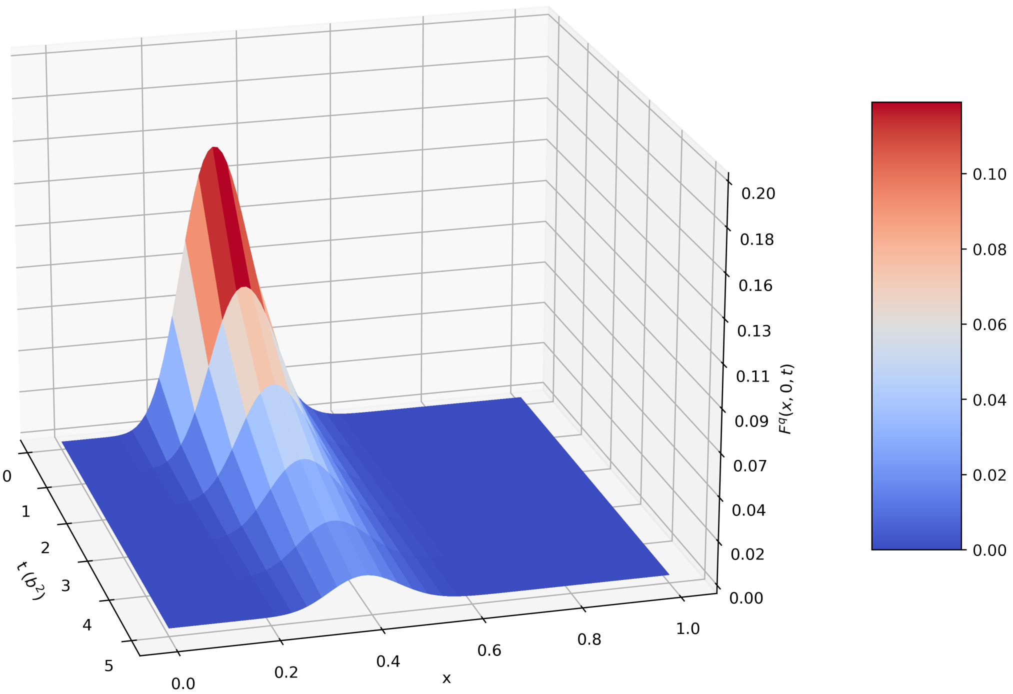

Figure 2: qd Baryon GPD in DLCQ

6. Summary

In Summary, we obtained GPDs via quantum computers for a $|q\bar{q}\rangle$ meson as well as a $|qd\rangle$ baryon. For the meson, we utilized DLCQ, while for the baryon, we used BLFQ, which is seen as a symmetry-adapted version of DLCQ. Ground bound states were prepared via VQE on quantum simulators via Qiskit.

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