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TMD phenomenology with the HSO approach

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Transverse momentum dependent (TMD) observables are typically classified in terms of their contributions coming from different regions in transverse momentum. The low transverse momentum behavior is often ascribed to intrinsic nonperturbative properties of the hadron described by TMD factorization, while the large transverse momentum region can be computed using fixed order collinear perturbation theory. Combining both pictures in a consistent way presents challenges, for practical calculations as well as the interpretation of results. We discuss a recent approach that is designed to retain a physical interpretation in terms of hadron structure while alleviating tension with techniques used at much higher energies. The approach is organized to allow for convenient nonperturbative model building in a way that incorporates both perturbative and non perturbative contributions in the parametrization of TMD densities while guaranteeing the matching in the large transverse momentum region.

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1. Introduction

This talk summarizes recent work done in Ref. [1], which showed how to resolve long-standing problems in applications of TMD phenomenology when the intention is to interpret measurements in terms of hadronic or nonperturbative structure. That paper focused on the SIDIS process as its test subject and it showed an explicit implementation of the new approach (called "hadron structure oriented" (HSO)) that is based on providing a consistent parametrization for TMD distributions. The general framework and discussion about the HSO approach are found in a previous work [2]. The talk also featured some results obtained in a Yukawa field theory [3], which became a valuable source for explicit checks and insights. The talk addressed issues that are often unacknowledged in traditional TMD phenomenological implementations that focus solely on very high or solely on moderate energies. Specifically, we point out certain theoretical inconsistencies in conventional ways of identifying nonperturbative structure in TMD parametrizations. We advocate for the following theoretical constraints, derivable from the operator definitions of the TMD pdfs, to be imposed directly on the parametrization of the TMD:

1. An integral relation exists that connects TMD and standard collinear pdfs and takes the form

$$\int \mathrm{d}^2\mathbf{k}_\mathrm{T} f(x, k_T; \mu, \mu^2) = f(x; \mu) + \Delta \,, \tag{1}$$

   where $f(x;\mu)$ is a collinear pdf in a standard renormalization scheme like $\overline{\mathrm{MS}}$ and Δ is a correction term. In the naive picture that treats (TMD) pdfs as literal number densities, $\Delta = 0$ and the transverse momentum integral convergences. In QCD, the integral requires a UV regulator of order μ, in which case Δ is nonzero but can be calculated perturbatively at leading power in $O(m/Q)$ in collinear factorization. In practice, this relation is typically taken either in the naive form (in phenomenology that focuses on nonperturbative structures) or it is ignored with regard to the nonperturbative transverse momentum dependence (in many high energy applications). However, it is an important constraint that permits the quasi-probabilistic interpretation.

2. The large transverse momentum behavior ($k_\mathrm{T} \approx Q$) of any TMD distribution is dictated by fixed order collinear factorization, i.e.

$$f(x, k_\mathrm{T} \approx Q) = C(x, k_\mathrm{T} \approx Q; Q, Q^2) \otimes f(x; Q). \tag{2}$$

3. Smooth interpolation between small and large transverse sizes and elimination of a sharp "b_max" separating perturbative and nonperturbative transverse momentum dependence.

2. Conventional approaches and their limitations

In the conventional TMD/CSS methodology, a well known trick is used to sequester the small b_T approximation from the rest, often called the nonperturbative part carried by the g-functions,

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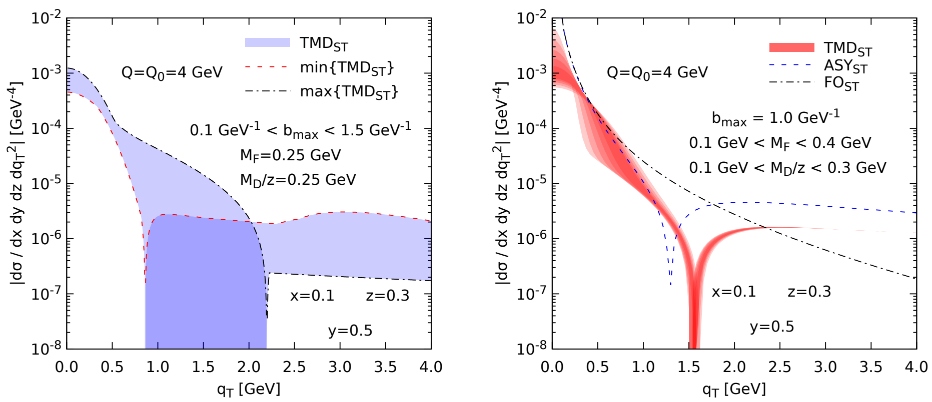
namely

$$
\begin{aligned}
\tilde{f}(x, b_{\mathrm{T}}; \mu, \mu^2) &= \tilde{f}(x, b_*; \mu, \mu^2) \frac{\tilde{f}(x, b_{\mathrm{T}}; \mu, \mu^2)}{\tilde{f}(x, b_*; \mu, \mu^2)} \\
&= \widetilde{C}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f(x; \mu_{b_*}) \exp\left\{ \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left(\gamma\left(a_S(\mu')\right) - \gamma_K\left(a_S(\mu')\right) \ln \frac{\mu}{\mu'} \right) + \ln \frac{\mu}{\mu_{b_*}} \widetilde{K}\left(b_*; \mu_{b_*}\right) \right\} \\
&\times \exp\left\{ -g(x, b_{\mathrm{T}}, b_*) - g_K(b_{\mathrm{T}}, b_*) \ln \frac{\mu}{\mu_{Q_0}} \right\} + O(\Lambda_{\mathrm{QCD}}^2 b_{\max}^2),
\end{aligned}
\tag{3}
$$

where from the first to the second line the TMD pdf at the scale μ_{b_*} has been approximated by its well known OPE expansion. The function $b_*(b_{\mathrm{T}}, b_{\max})$ is constructed to behave like b_{T} for small values and freeze to $b_{\max}$ and $\mu_{b_*} \equiv 2e^{-\gamma_E}/b_*$. The functions g and g_K are universal functions defined to describe the remaining region from $b_{\max}$ to infinity, and in applications they are modeled with an appropriate ansatz generally chosen to vanish like a power of b_{T} for $b_{\mathrm{T}} \to 0$. The introduction of the auxiliary parameter $b_{\max}$ is manifestly arbitrary, and the full parametrization must be independent of its choice. In most practical implementations, however, this is not found. Indeed, in our example shown in Fig. 1(a) there is a strong $b_{\max}$ dependence on the SIDIS cross section. Particularly, we notice how the choice of $b_{\max}$ affects the large q_{T} region, which should in principle be solely characterized by collinear factorization and should be minimally affected by the small transverse momentum content of the TMDs.



(a) (b)

Figure 1: (a) W (TMD) term of the SIDIS cross section for different values of $b_{\max}$ for fixed parameters and Gaussian core model. (b) SIDIS differential cross section showing W (TMD), FO and Asy terms calculated according to the conventional approach for a range of model mass parameters.

An additional limitation is manifested when one tries to describe the cross section over the full kinematic range in the observed transverse momentum q_{T}. Processes like SIDIS, Drell-Yan and e^+e^- into two back-to-back hadrons are described by TMD factorization in the small q_{T} region (the so-called W term), while the large q_{T} region is purely dictated by collinear factorization (also

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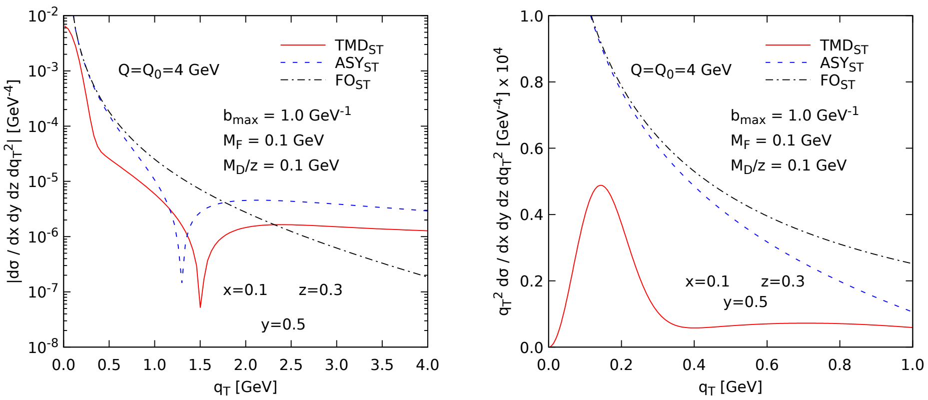
known as the Fixed Order or FO term). In general it is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_{\mathrm{T}}\dots} = \underbrace{T_{\mathrm{small}}\frac{\mathrm{d}\sigma}{\mathrm{d}q_{\mathrm{T}}\dots}}_{\mathrm{W}} - \underbrace{T_{\mathrm{small}}T_{\mathrm{large}}\frac{\mathrm{d}\sigma}{\mathrm{d}q_{\mathrm{T}}\dots}}_{\mathrm{Asy}} + \underbrace{T_{\mathrm{large}}\frac{\mathrm{d}\sigma}{\mathrm{d}q_{\mathrm{T}}\dots}}_{\mathrm{FO}} + O\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right). \tag{4}$$

The asymptotic (Asy) term acts as a natural interpolator between the two contributions in the region where the two approximations start to fail, i.e. $\Lambda_{\mathrm{QCD}} \ll q_{\mathrm{T}} \ll Q$. This is exactly the region where the three terms should share the same behavior and be roughly equal. However, as shown in Fig. 1(b) and better in Fig. 2, this is generally not the case. Due to unconstrained parametrizations conventionally used, the existence of a "matching region" is not guaranteed.



(a) (b)

Figure 2: (a) Same as Fig. 1(b) but for fixed parameters. (b) Same as (a) but with linear axes and multiplied by q_{T}^2.

3. HSO approach

3.1 Input scale

The HSO approach cures these issues. It preserves the integral constraint in Eq. (1) and the large k_{T} tail constraint in Eq. (2) explicitly at the input scale Q_0. A realization of the HSO parametrization was studied in [1] where the TMD at the input scale $\mu_{Q_0} = Q_0$ reads (similarly for the FF)

$$\begin{aligned} f_{\mathrm{inpt},i/p}(x, \boldsymbol{k}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2) &= \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2 + m_{f_{i,p}}^2}\left[A_{i/p}^{f}(x;\mu_{Q_0}) + B_{i/p}^{f}(x;\mu_{Q_0}) \ln\frac{Q_0^2}{k_{\mathrm{T}}^2 + m_{f_{i,p}}^2}\right] \\ &+ \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2 + m_{f_{g,p}}^2} A_{i/p}^{f,g}(x;\mu_{Q_0}) \\ &+ C_{i/p}^{f}\, f_{\mathrm{core},i/p}(x, \boldsymbol{k}_{\mathrm{T}}; Q_0^2)\,, \end{aligned} \tag{5}$$

where the first two lines follow from the large k_{T} tail region with the addition of mass parameters (only the order α_S was implemented but generalizations to higher orders is straightforward). The

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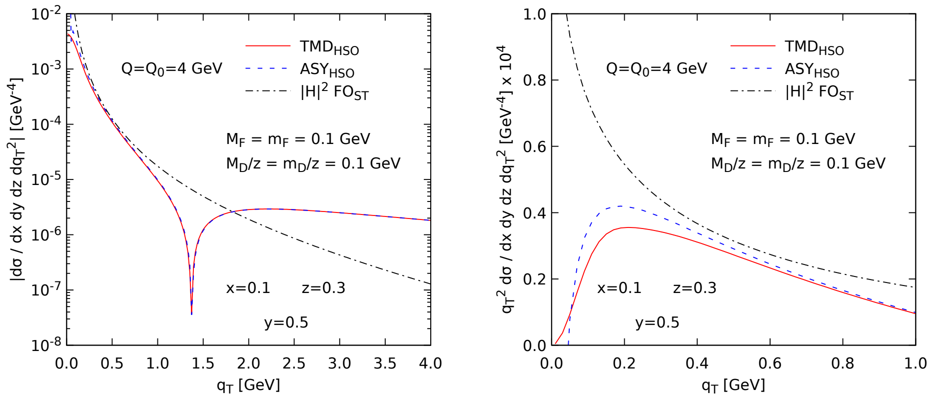


(a) (b)

Figure 3: HSO approach to the SIDIS differential cross section. (a) Logarithmic scale. (b) Linear scale.

last contribution $\tilde{f}_{\text{core}}$ is any small k_{T} model which can be easily swapped with another. A crucial part is the $C^{f}_{i/p}$ factor which is not simply a trivial normalization constant, but it is defined in such a way that the two constraints mentioned in Eq. (1) and Eq. (2) are satisfied. In particular, the integral relation between TMDs and collinear distribution is satisfied with a cutoff scheme

$$f^{c}_{i/p}(x;\mu_{Q_0},\mu_f) \equiv \int_0^{\mu_f} \mathrm{d}^2\boldsymbol{k}_{\text{T}} f_{\text{inpt},i/p}(x,\boldsymbol{k}_{\text{T}};\mu_{Q_0},Q_0^2) = f(x;\mu_{Q_0}) + \Delta(x;\mu_{Q_0},\mu_f), \tag{6}$$

where $f(x;\mu_{Q_0})$ is the usual renormalized collinear pdf in the $\overline{\text{MS}}$ scheme, for instance, and Δ is calculable in perturbation theory up to power suppressed contributions. Notice how the above is inherently independent of any auxiliary parameters such as $b_{\max}$ and $b_{\min}$.

The W term at the input scale is readily constructed

$$\left.\frac{\mathrm{d}\sigma_{\text{SIDIS}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}q_{\text{T}}^2}\right|_{q_{\text{T}}\ll Q_0} = \frac{2\pi^2 z^2 \alpha_{e.m}^2}{Q_0^2}\left[y + 2\left(\frac{1-y}{y}\right)\right]\sum_j \|H\|_j^2 \times \int \mathrm{d}^2\boldsymbol{k}_{\text{T}} f_{\text{inpt,j/p}}(x,k_{\text{T}};\mu_{Q_0},Q_0^2) D_{\text{inpt,h/j}}(x, z(k_{\text{T}}+q_{\text{T}});\mu_{Q_0},Q_0^2) \tag{7}$$

as well as the asymptotic term (Asy) which is nothing but the large q_{T} approximation to the above expression and it is completely determined by fixed order collinear factorization. The full expression is rather lenghty and can be found in Ref. [1] but the important point for this talk is that, by construction, it follows the large q_{T} tail of the W term, in contrast to the conventional approach (see Fig. 1-2). Furthermore, plotting the new W and asymptotic terms along with the FO one (see Fig. 3), now yields a clear improvement. That is, the existence of a matching region is now manifested in the matching behavior of the curves in the region $0.4 \lesssim q_{\text{T}} \lesssim 1$ GeV, which is more or less where it is expected that to happen.

3.2 Evolution

Once the constraints in Eq. (1) and Eq. (2) are manifestly imposed at the input scale, the evolution is implemented according to the usual RG equations. The Fourier conjugate space to transverse momentum is the natural space to implement evolution since the latter is solely contained in an exponential factor whose b_T dependence is uniquely due to the CS kernel. The full solution is

$$\begin{aligned}\tilde{f}(x, b_\mathrm{T}; \mu, \mu^2) &= \tilde{f}(x, b_\mathrm{T}; \mu_{Q_0}, \mu_{Q_0}^2) \\ &\times \exp\left\{\int_{\mu_{Q_0}}^{\mu} \frac{\mathrm{d}\mu'}{\mu'}\left(\gamma\left(\alpha_S(\mu')\right) - \ln\left(\frac{\mu}{\mu'}\right)\gamma_K\left(\alpha_S(\mu')\right)\right) + \ln\left(\frac{\mu}{\mu_{Q_0}}\right)\widetilde{K}\left(b_\mathrm{T}; \mu_{Q_0}\right)\right\} \\ &\equiv \tilde{f}(x, b_\mathrm{T}; \mu_{Q_0}, \mu_{Q_0}^2) E(\mu_{Q_0} \to \mu),\end{aligned} \tag{8}$$

where we have defined the evolution factor as $E(\mu_{Q_0} \to \mu)$ with μ_{Q_0} the input scale and μ any other higher scale we wish to evolve to. Naturally, we give the recipe to build the Collins-Soper kernel in the HSO spirit. That is, an input scale parametrization that smoothly interpolates between the small b_T OPE expansion and a large b_T "core" model $\widetilde{K}_\mathrm{core}(b_\mathrm{T})$ of our choice. At order $\alpha_S(\mu)$ it reads

$$\widetilde{K}_\mathrm{input}(b_\mathrm{T}; \mu_{Q_0}) = 2C_F \frac{\alpha_S(\mu_{Q_0})}{\pi}\left(K_0(m_K \mathrm{b_T}) + \ln\left(\frac{\mathrm{m_K}}{\mu_{\mathrm{Q_0}}}\right)\right) + \widetilde{K}_\mathrm{core}(b_\mathrm{T}), \tag{9}$$

where m_K is a mass parameter. Under evolution it behaves as expected, i.e.

$$\widetilde{K}(b_\mathrm{T}; \mu) = \widetilde{K}_\mathrm{input}(b_\mathrm{T}; \mu_{Q_0}) - \int_{\mu_{Q_0}}^{\mu} \frac{\mathrm{d}\mu'}{\mu'}\gamma_K\left(\alpha_S(\mu)'\right). \tag{10}$$

Implementing the same steps for higher orders in the QCD coupling is straightforward, and we give the full NLO parametrization for the Collins-Soper kernel in A.

Similarly to what is achieved by the scale transformation μ_* in the common approach, the HSO method allows us to perform RG improvements on the input parametrization only relying on a scale transformation we call $\overline{Q}_0(b_\mathrm{T}, Q_0)$ which behaves like $2e^{-\gamma_E}/b_\mathrm{T}$ at small b_T and it rapidly converges to the input scale Q_0 for larger b_T. The specific functional form we choose is

$$\overline{Q}_0 = Q_0\left[1 - \left(1 - \frac{2e^{-\gamma_E}}{Q_0 b_\mathrm{T}}\right)e^{-a^2 b_\mathrm{T}^2}\right], \tag{11}$$

with the choice $a = Q_0$, but any function with the same overall behavior is acceptable.

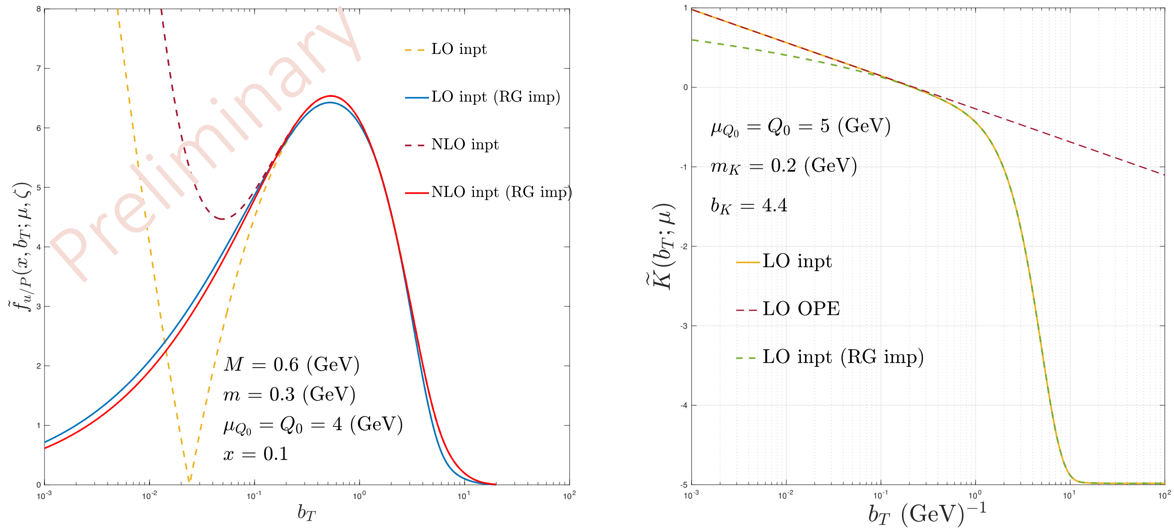
The final expression we use for our analyses is thus Eq. (8) with the replacement

$$\tilde{f}(x, b_\mathrm{T}; \mu_{Q_0}, \mu_{Q_0}^2) \mapsto \tilde{f}_\mathrm{input}(x, b_\mathrm{T}; \overline{Q}_0, \overline{Q}_0^2) E(\overline{Q}_0 \to \mu_{Q_0}) \tag{12}$$

along with

$$\widetilde{K}\left(b_\mathrm{T}; \mu_{Q_0}\right) \mapsto \widetilde{K}_\mathrm{input}\left(b_\mathrm{T}; \overline{Q}_0\right) - \int_{\overline{Q}_0}^{\mu_{Q_0}} \frac{\mathrm{d}\mu'}{\mu'}\gamma_K(\alpha_S(\mu')). \tag{13}$$

An example of how the input parametrization at $Q_0 = 4$ GeV is improved after the $\overline{Q}_0$ prescription is shown in Fig. 4(a). The same improvement procedure is illustrated in Fig. 4(b) for the Collins-Soper kernel at leading order. This is a crucial advantage of the HSO approach as it never imposes an explicit demarcation between what is considered perturbative and what is not unlike the common approach. There, the role of b_max is twofold since it is used to split the space in two as well as to take care of the large logs coming from the small b_T region by making a scale transformation using the RG equations.



(a) (b)

Figure 4: Left: Example of an HSO parametrization for the up-quark TMD pdf in coordinate space. The dashed yellow line shows the LO input parametrization as in Eq. (5) for $\mu_{Q_0} = Q_0 = 4$ GeV, $x = 0.1$, $m = 0.3$ GeV and $M = 0.6$ GeV (all mass parameters have been set equal to m except fro the one appearing in the core model, which we refer to as M). The “core" model chosen here is a gaussian: $f_{\text{core}}(x, k_{\text{T}}; Q_0) = e^{-k_{\text{T}}^2/M^2}/(\pi M^2)$. Similarly, the dashed purple line implements the NLO version. The two solid lines (blue for LO and red for NLO) show the RG improved TMD distribution as in Eq. (12). Right: Same as before but for the Collins-Soper kernel at the input scale $\mu_{Q_0} = 5$ GeV with the specific “core" model $\widetilde{K}_{\text{core}} = b_K(\exp\{-m_K^2 b_{\text{T}}^2\} - 1)$. Here the dashed green line is the HSO parametrization after the RG improvement of the leading-order input scale parametrization (solid yellow), which asymptotes to its OPE expansion (dashed purple) for small b_{T}.

3.3 Phenomenology

The *hadronic structure* emphasis of the HSO approach is also manifested in the phenomenological methodology we adopt.

With the HSO approach we are able to compare different nonperturbative models, in the low-to-moderate Q region where nonperturbative physics is dominant, in a very direct way. The effect of evolution to increasingly high energies washes out most of the nonperturbative details of the input parametrization, leaving the main description of the TMD distribution to its collinear perturbative expression. This is why, despite being formally equivalent, an approach that focuses on backward evolution introduces a higher uncertainty into the extracted nonperturbative information. The HSO parametrization explicitly interpolates between the perturbative tail and the chosen “core" model, regardless of its shape, ensuring that the moderate energies match the nonperturbative information at small transverse momentum, while consistently agreeing with the large transverse momentum behavior.

For these reasons, we use a different strategy than that of a global fit analysis: we extract the non perturbative parameters of the TMD models from the low-to-moderate energy data, which contain most of the nonperturbative transverse momentum information. Then we evolve the resulting TMDs to postdict the higher energy data sets. By doing this, we test in an unambiguous way the assumptions made for the extractions. See for instance Fig. 5 where we give a preliminary fit for the E288 Drell-Yan experiment, whose results will be tested againts the Z^0 boson production Drell-Yan data. A more detailed analysis of this strategy and its results can be found in our most recent work [4]. An additional advantage of the HSO approach is that the stringent requirements to match the

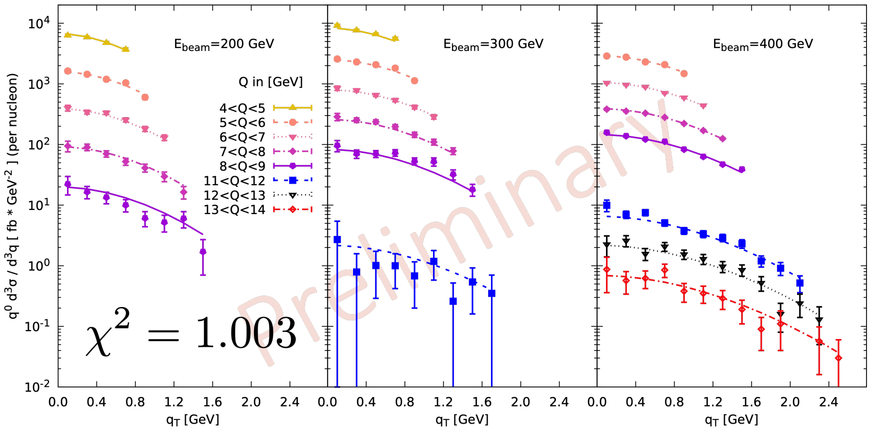


Figure 5: Fit of the E288 Drell-Yan experimental data using the HSO approach.

theoretical consistency constraints (1) and (2) ensure that the extracted fits are not contaminated by models of nonperturbative transverse momentum in the perturbative collinear factorization regions where it would not be sensible. Otherwise, uncontrolled leaking of model dependence may migrate into regions where it is unreasonable and affect interpretations of fit results. Constraints like (1) imply potentially strong correlations between collinear pdf parametrizations and nonperturbative transverse momentum model, an effect that is lost if the constraint is not imposed explicitly. In the latter case, an apparent lack of sensitivity to collinear pdfs may indicate overfitting rather than dominance by nonperturbative transverse momentum dependence.

4. Conclusion

We have studied transverse momentum observables like the SIDIS differential cross section, guided by the HSO approach. The TMD distributions were constructed to explicitly satisfy theoretical constraints like the integral relation between TMD and collinear distributions as well as the large k_T behavior dictated by collinear factorization. In doing so, we have provided a consistent parametrization designed to maximally exploit the nonperturbative information coming from low-to-moderate Q data and thereby predict higher Q measurements.

We have successfully improved upon the so called “matching problem" by providing a set of tools and instructions that, by construction, make the extracted cross section smoothly interpolate between its small q_T approximator (W term) and its large q_T approximator (the Fixed Order term) at the input scale.

Additionally, we advocate for a shift in philosophy regarding the phenomenological extraction of nonperturbative information about TMD distributions. We point out how the low-to-moderate data should be considered more efficient than high energy data for the extraction of nonperturbative phsyics. The HSO approach uses the latter as a step for postdiction tests which determine the validity of the assumptions made for the fit extractions.

The hadron structure oriented approach we have discussed here is ideal also for extensions to spin dependent observables like the Sivers effect [5], which we plan to address in the future.

A. NLO Collins-Soper kernel

The CS-kernel at NLO in the HSO spirit reads

$$\tilde{K}_{\text{inpt}}(b_T;\mu_{Q_0}) =$$

$$2\pi\left[A_K^{(1)}(\mu_{Q_0}) + A_K^{(2)}(\mu_{Q_0})\right] K_0\left(m_K b_T\right) + 2\pi B_K^{(2)}(\mu_{Q_0}) K_0\left(m_K b_T\right) \ln\left(\frac{\mu_{Q_0}^2 b_T}{2 m_K e^{-\gamma_E}}\right) \tag{14}$$

$$+ \tilde{K}_{\text{core}}(b_T) + D_K(\mu_{Q_0})\,, \tag{15}$$

where

$$A_K^{(1)}(\mu_{Q_0}) = \frac{\alpha_s(\mu_{Q_0}) C_F}{\pi^2}\,, \tag{16}$$

$$A_K^{(2)}(\mu_{Q_0}) = -\frac{\alpha_s(\mu_{Q_0})^2 C_F}{4\pi^3}\left(-\frac{67}{9}C_A + \frac{\pi^2}{3}C_A + \frac{10}{9}n_f\right), \tag{17}$$

$$B_K^{(2)}(\mu_{Q_0}) = -\frac{\alpha_s(\mu_{Q_0})^2 C_F}{4\pi^3}\left(\frac{2}{3}n_f - \frac{11}{3}C_A\right), \tag{18}$$

and

$$\begin{aligned} D_K(\mu_{Q_0}) = &\frac{2\alpha_s(\mu_{Q_0}) C_F}{\pi}\ln\left(\frac{m_K}{\mu_{Q_0}}\right) \\ &+ \frac{C_F \alpha_s(\mu_{Q_0})^2}{2\pi^2}\left[C_A\left(\frac{7}{2}\zeta_3 - \frac{101}{27}\right) + \frac{14}{27}n_f\right] \\ &- \frac{\alpha_s(\mu_{Q_0})^2 C_F}{18\pi^2}\ln\left(\frac{m_K}{\mu_{Q_0}}\right)\left[(33C_A - 6n_f)\ln\left(\frac{m_K}{\mu_{Q_0}}\right) + \left(3\pi^2 - 67\right)C_A + 10 n_f\right]. \end{aligned} \tag{19}$$

The "nonperturbative" function $\tilde{K}_{\text{core}}(b_T)$ can be freely chosen to describe the large b_T behavior of the CS-kernel. The example chosen in Fig. 4(b) reads

$$\tilde{K}_{\text{core}} = b_k\left(e^{-m_K^2 b_T^2} - 1\right), \tag{20}$$

so that

$$\lim_{b_T\to\infty}\tilde{K}_{\text{inpt}}(b_T;\mu_{Q_0}) = -b_K + D_K(\mu_{Q_0}) = -b_K + O\left(\alpha_S\right). \tag{21}$$

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