Hyperon polarization in SIDIS: Matching between TMD and twist-3 factorizations

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We study the consistency between the transverse momentum dependent (TMD) factorization and the collinear twist-3 factorization for the transversely polarized hyperon production in semi-inclusive deep inelastic scattering (SIDIS). The TMD approach describes the polarization in the small region of the hyperon's transverse momentum P_{hT}, while the twist-3 approach covers that in the large P_{hT} region. In the intermediate region of P_{hT} where both frameworks are valid, they should match consistently. This has been confirmed for the contributions from the Boer-Mulders function and the twist-3 distribution function. Using the recently obtained complete leading-order cross section for the twist-3 fragmentation function contribution to $ep \to e\Lambda^{\uparrow}X$, we show that it also matches consistently with the polarizing fragmentation function contribution to this process. This supports the idea that the two frameworks represent a unique QCD effect for the polarization in different kinematic regions.

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1. Introduction

The transverse single spin asymmetry (SSA) has been observed in various processes. Most famous examples of those SSAs include the transverse polarization of produced hyperons in unpolarized proton-proton collisions [1] and the left-right asymmetry in the inclusive pion production with a polarized proton beam [2], both of which were first observed in 1970s. The SSAs in these processes reach a couple of ten percent in the large rapidity region, which is much larger than what was expected by the parton model and the perturbative quantum chromodynamics (QCD) in the twist-2 level. Large SSAs have also been observed in lepton-nucleon semi-inclusive deep inelastic scatterings (SIDIS). For the last few decades, lots of theoretical studies have been made to clarify the underlying mechanism of the SSAs, which also revealed new features of the nucleon structure beyond the conventional parton model, such as intrinsic transverse momentum of partons and multiparton correlations in the nucleon. Among various SSAs we study in this paper the hyperon polarization in SIDIS.

Two frameworks have been successful in describing the SSAs in SIDIS. One is the collinear twist-3 factorization formalism which is valid when a transverse momentum of a produced hadron P_{hT} is much larger than $\Lambda_{\rm QCD}$, i.e., $\Lambda_{\rm QCD} \ll P_{hT} \leq Q$, where Q is the virtuality of the virtual photon. In this framework, the twist-3 multiparton correlation functions of the target proton and the produced hadron play an important role for the large SSAs. The other is the transverse momentum dependent (TMD) factorization formalism that covers the SSAs in which P_{hT} is much smaller than Q and may be of the order of $\Lambda_{\rm QCD}$ (i.e., $\Lambda_{\rm QCD} \leq P_{hT} \ll Q$). There the intrinsic transverse momentum of partons inside the hadrons causes the large SSAs. It has been reported that the two frameworks match consistently in the intermediate region of P_{hT}, i.e., $\Lambda_{QCD} \ll P_{hT} \ll Q$, for the pion production in SIDIS, $ep^\uparrow \to e\pi X$ [3–5], and the Drell-Yan process, $pp^\uparrow \to e^+e^-X$ [6, 7].

The complete differential cross section for the transversely polarized hyperon production in SIDIS $ep \to e\Lambda^\uparrow X$ on the basis of the twist-3 formalism was derived only recently in Refs. [8, 9] in the leading order (LO) with respect to the QCD coupling constant. The responsible twist-3 effects can be classified into two types: (i) the twist-3 parton distribution functions (PDFs) in the target proton p combined with the twist-2 transversity fragmentation function (FF) for the transversely polarized hyperon $\Lambda^\uparrow$, and (ii) the twist-3 FFs for $\Lambda^\uparrow$ combined with the twist-2 unpolarized PDF in p. In the second case, the twist-3 FFs are furthermore classified into the (a) quark and (b) gluon types of twist-3 FFs. It has been shown that the twist-3 cross section from the above (i) matches with the contribution from the Boer-Mulders function in the TMD factorization formalism in the intermediate P_{hT} [10]. On the other hand, the matching issue between the twist-3 FFs contributions (ii)(a) and (ii)(b) and the TMD FF contribution is not yet established, on which we shall work in this paper.

This paper is organized as follows: In Sec. 2, we summarize the TMD FFs and the twist-3 quark and gluon FFs contributing to $ep \to e\Lambda^\uparrow X$. In Sec. 3, we study the behavior of the collinear twist-3 FF contribution and the TMD FF contribution in their valid overlapping region, i.e., $\Lambda_{\rm QCD} \ll P_{hT} \ll Q$ and resolve the matching issue between the two approaches. Sec. 4 is devoted to a brief summary.

2. TMD and Twist-3 Fragmentation functions for $ep \to e\Lambda^\uparrow X$

Relevant TMD FFs for this process is defined from the following fragmentation matrix element:

$$\Delta_{ij}(z_f, \vec{k}_T) = \sum_X \int \frac{d\xi^+ d^2\vec{\xi}_T}{2z_f(2\pi)^3} e^{ik\cdot\xi} \langle 0|\psi_i(\xi)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|\bar{\psi}_j(0)|0\rangle\Big|_{\xi^-=0}$$
$$= \frac{1}{2M_h P_h^-}\epsilon^{\alpha P_h k_T S_\perp}(\gamma_\alpha)_{ij} D_{1T}^\perp(z_f, z_f^2\vec{k}_T^2) + \cdots, \tag{1}$$

where i and j are the spinor indices of the quark fields ψ and $\bar{\psi}$, and $|\Lambda^\uparrow\rangle$ denotes the state for the transversely polarized hyperon with mass M_h, momentum P_h and the transverse spin vector $S_\perp$ ($S_\perp^2 = -1$). We use the shorthand notation such as $\epsilon^{\alpha P_h k_T S_\perp} \equiv \epsilon^{\alpha\beta\gamma\delta} P_{h\beta} k_{T\gamma} S_{\perp\delta}$. $D_{1T}^\perp(z_f, z_f^2\vec{k}_T^2)$ is called the polarizing FF and the parametrization follows Ref. [11]. Here and below gauge-link operators which restore the gauge invariance of the FFs are implicit. In (1) we omit the twist-2 transversity FF H_{1T}, which contributes to $ep \to e\Lambda^\uparrow X$ combined with the Boer-Mulders function $h_1^\perp$ of the target proton. It was shown in [10] that the cross section involving them in the TMD factorization approach matches consistently with the twist-3 distribution contribution to this process.

Next we summarize the twist-3 quark FFs according to the notation in [12, 13]. Since we are interested in the FFs contributing to $ep \to e\Lambda^\uparrow X$, we write naively T-odd FFs only. First type of the twist-3 quark FFs are defined from the light cone correlation function in terms of the quark fields as

$$\Delta_{ij}(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|\psi_i(0)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|\bar{\psi}_j(\lambda w)|0\rangle = M_h \epsilon^{\alpha S_\perp w P_h}(\gamma_\alpha)_{ij}\frac{D_T(z)}{z} + \cdots, \tag{2}$$

where N is the number of quark colors for color SU(N), w is the lightlike vector satisfying $P_h \cdot w = 1$ and P_h can be treated as a lightlike vector in the twist-3 accuracy. $D_T(z)$ is one of the *intrinsic* twist-3 quark FFs. Other FFs which do not contribute to the hyperon polarization are omitted by the ellipsis $(\cdots)$. Second type of quark FFs are obtained from the derivative of the two quark correlator as

$$\Delta_{\partial ij}^\alpha(z) = \frac{1}{N}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|\psi_i(0)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|\bar{\psi}_j(\lambda w)|0\rangle\overleftarrow{\partial}^\alpha$$
$$= -iM_h\epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij}\frac{D_{1T}^{\perp(1)}(z)}{z} + \cdots, \tag{3}$$

where the derivative with the left arrow is defined by $\bar{\psi}_j(\lambda w)\overleftarrow{\partial}^\alpha \equiv \lim_{\xi\to 0}(d/d\xi_\alpha)\bar{\psi}_j(\lambda w + \xi)$. $D_{1T}^{\perp(1)}(z)$ is one of the *kinematical* twist-3 quark FFs and can be written as the $k_\perp^2/M_h^2$-moment of the polarizing FF $D_{1T}^\perp$ defined in (1):

$$D_{1T}^{\perp(1)}(z) = z^2 \int d^2p_\perp \frac{\vec{p}_\perp^{\,2}}{2M_h^2} D_{1T}^\perp(z, z^2p_\perp^2). \tag{4}$$

Third type of the quark FFs are defined from the quark-gluon correlation function:

$$\Delta_{Fij}^\alpha(z, z_1) = \frac{1}{N}\sum_X \iint \frac{d\lambda d\mu}{(2\pi)^2} e^{-i\frac{\lambda}{z_1}} e^{-i\mu\left(\frac{1}{z}-\frac{1}{z_1}\right)} \langle 0|\psi_i(0)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|\bar{\psi}_j(\lambda w) gF^{\alpha w}(\mu w)|0\rangle$$
$$= M_h\epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij}\frac{\widehat{D}_{FT}^*(z, z_1)}{z} - iM_h S_\perp^\alpha(\gamma_5\not{P}_h)_{ij}\frac{\widehat{G}_{FT}^*(z, z_1)}{z} + \cdots, \tag{5}$$

where $F^{\alpha w} \equiv F_a^{\alpha w} t_a$ is the gluon's field strength tensor with t^a the generator of color SU(N). $\widehat{D}_{FT}(z, z_1)$ and $\widehat{G}_{FT}(z, z_1)$ are the *dynamical* twist-3 quark FFs and are complex functions. Their real parts are naively T-even, while their imaginary parts are naively T-odd and contribute to the hyperon polarization.

The twist-3 gluon FFs are also classified into the *intrinsic, kinematical and dynamical* ones in parallel to the twist-3 quark FFs. Here we omit the intrinsic one, $\Delta\widehat{G}_{3\overline{T}}(z)$, since it can be replaced with the other FFs by the QCD equation-of-motion (EOM) relation [13]. The kinematical twist-3 gluon FFs are defined from the derivative of the 2-gluon correlation function as

$$\widehat{\Gamma}_{\partial}^{\alpha\beta}(z) = \frac{1}{N^2-1}\sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0|F_a^{w\beta}(0)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|F_a^{w\alpha}(\lambda w)|0\rangle \overleftarrow{\partial}^{\gamma}$$
$$= \frac{-i}{2} M_h g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp \gamma}\widehat{G}_T^{(1)}(z) - \frac{i}{8}M_h\left(\epsilon^{P_h w S_\perp\{\alpha}g_\perp^{\beta\}\gamma} + \epsilon^{P_h w\gamma\{\alpha}S_\perp^{\beta\}}\right)\Delta\widehat{H}_T^{(1)}(z) + \cdots, \quad (6)$$

where $g_\perp^{\alpha\beta} \equiv g^{\alpha\beta} - P_h^\alpha w^\beta - P_h^\beta w^\alpha$. These $\widehat{G}_T^{(1)}(z)$ and $\Delta\widehat{H}_T^{(1)}(z)$ can also be written as the transverse momentum moment of the TMD FFs. The dynamical twist-3 gluon FFs are defined from the 3-gluon correlation functions with the strength tensors contracted by the antisymmetric or symmetric structure constants, $-if_{abc}$ or d_{abc}, as

$$\widehat{\Gamma}_{FA}^{\alpha\beta\gamma}\left(\frac{1}{z_1},\frac{1}{z_2}\right)$$
$$= \frac{-if_{abc}}{N^2-1}\sum_X \iint \frac{d\lambda d\mu}{(2\pi)^2} e^{-i\frac{\lambda}{z_1}} e^{-i\mu\left(\frac{1}{z_2}-\frac{1}{z_1}\right)} \langle 0|F_b^{w\beta}(0)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|F_a^{w\alpha}(\lambda w) g F_c^{w\gamma}(\mu w)|0\rangle$$
$$= -M_h\left(\widehat{N}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\alpha\gamma}\epsilon^{P_h w S_\perp\beta} + \widehat{N}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\beta\gamma}\epsilon^{P_h w S_\perp\alpha} - \widehat{N}_2\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp\gamma}\right), \quad (7)$$

$$\widehat{\Gamma}_{FS}^{\alpha\beta\gamma}\left(\frac{1}{z_1},\frac{1}{z_2}\right)$$
$$= \frac{d_{abc}}{N^2-1}\sum_X \iint \frac{d\lambda d\mu}{(2\pi)^2} e^{-i\frac{\lambda}{z_1}} e^{-i\mu\left(\frac{1}{z_2}-\frac{1}{z_1}\right)} \langle 0|F_b^{w\beta}(0)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|F_a^{w\alpha}(\lambda w) g F_c^{w\gamma}(\mu w)|0\rangle$$
$$= -M_h\left(\widehat{O}_1\left(\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\alpha\gamma}\epsilon^{P_h w S_\perp\beta} + \widehat{O}_2\left(\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\beta\gamma}\epsilon^{P_h w S_\perp\alpha} + \widehat{O}_2\left(\frac{1}{z_2}-\frac{1}{z_1},\frac{1}{z_2}\right) g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp\gamma}\right). \quad (8)$$

$\widehat{N}_{1,2}(1/z_1, 1/z_2)$ and $\widehat{O}_{1,2}(1/z_1, 1/z_2)$ are complex functions, and their imaginary parts are naively T-odd while their real parts are naively T-even. We have another type of dynamical FFs defined by

$$\widetilde{\Delta}_{ij}^{\alpha}\left(\frac{1}{z_1},\frac{1}{z_2}\right) = \frac{1}{N}\sum_X \iint \frac{d\lambda d\mu}{(2\pi)^2} e^{-i\frac{\lambda}{z_1}} e^{-i\mu\left(\frac{1}{z_2}-\frac{1}{z_1}\right)} \langle 0|gF_a^{w\alpha}(\mu w)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|\bar{\psi}_j(\lambda w)t^a\psi_i(0)|0\rangle$$
$$= M_h\epsilon^{\alpha P_h w S_\perp}(\not{P}_h)_{ij}\widetilde{D}_{FT}\left(\frac{1}{z_1},\frac{1}{z_2}\right) + iM_h S_\perp^\alpha(\gamma_5\not{P}_h)_{ij}\widetilde{G}_{FT}\left(\frac{1}{z_1},\frac{1}{z_2}\right). \quad (9)$$

$\widetilde{D}_{FT}(1/z_1, 1/z_2)$ and $\widetilde{G}_{FT}(1/z_1, 1/z_2)$ are complex functions. As is the case for the other dynamical FFs, only their imaginary parts contribute to the hyperon polarization. One can also show that they are related to the purely gluonic twist-3 FFs through the EOMs and the Lorentz invariance relations (LIRs)[13].

3. Matching between the twist-3 and the TMD FF contributions

For the hyperon polarization process, $e(\ell) + p(p) \to e(\ell') + \Lambda^{\uparrow}(P_h, S_\perp) + X$, the complete LO twist-3 FF contribution to the cross section was derived in [8, 9], whose azimuthal dependence takes the following form:

$$\frac{d^6\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \mathcal{F}_1 \sin\Phi_S + \mathcal{F}_2 \sin\Phi_S \cos(\phi-\chi) + \mathcal{F}_3 \sin\Phi_S \cos 2(\phi-\chi) + \mathcal{F}_4 \cos\Phi_S \sin(\phi-\chi) + \mathcal{F}_5 \cos\Phi_S \sin 2(\phi-\chi), \tag{10}$$

where $\mathcal{F}_i$ $(i = 1, \cdots, 5)$ are the structure functions for each azimuthal component, $q^\mu = \ell^\mu - \ell'^\mu$ is the 4-momentum of the virtual photon, $Q^2 = -q^2$, $x_{bj} = Q^2/(2p\cdot q)$, $z_f = p\cdot P_h/(p\cdot q)$, and $q_T = \sqrt{-q_t^2}$ with $q_t^\mu = q^\mu - \frac{P_h\cdot q}{p\cdot P_h}p^\mu - \frac{p\cdot q}{p\cdot P_h}P_h^\mu$. In a frame where p and q are collinear, q_T is related to the transverse momentum of P_h, P_{hT}, by $q_T = P_{hT}/z_f$. Φ_S is the azimuthal angle of the transverse spin vector $\vec{S}_\perp$ measured from the hadron plane around $\vec{P}_h$, and ϕ and χ are, respectively, the azimuthal angles of the lepton plane and the hadron plane around $\vec{q}$. (See [8, 9] for the details.) In order to investigate the matching with the TMD factorization approach, we consider the behavior of (10) at $\Lambda_{\rm QCD} \ll q_T \ll Q$, where both approaches are valid. At $q_T \ll Q$, we found that the twist-3 FFs contribution gives rise only to $\mathcal{F}_1 \sin\Phi_S$ in (10) in the leading power of q_T/Q. Using the result for the cross section in [8, 9], we have obtained $\mathcal{F}_1$ in the following form:

$$\mathcal{F}_1 = \frac{-4\alpha_s M_h \sigma_0}{4\pi^2 q_T^3}\left[f_1(x_{bj}) \int \frac{dz}{z}\left(A - \frac{1}{4}B\right) + C_F D_{1T}^{\perp(1)}(z_f) \int \frac{dx}{x} f_1(x) \frac{1+\hat{x}^2}{(1-\hat{x})_+} + 2C_F f_1(x_{bj}) D_{1T}^{\perp(1)}(z_f) \ln\frac{Q^2}{q_T^2}\right], \tag{11}$$

where $\hat{x} = x_{bj}/x$, $\alpha_s = g^2/(4\pi)$ is the strong coupling constant, $C_F = (N^2-1)/(2N)$, $\sigma_0 = \alpha_{em}^2(1-y+y^2/2)/Q^4$ with the DIS inelasticity parameter $y = p\cdot q/(p\cdot\ell)$, $f_1(x)$ is the unpolarized twist-2 PDF. A and B are, respectively, written in terms of the four twist-3 quark FFs (D_T, $D_{1T}^{\perp(1)}$, $\widehat{D}_{FT}$ and $\widehat{G}_{FT}$) and the eight twist-3 gluon FFs ($\widehat{G}_T^{(1)}$, $\Delta\widehat{H}_T^{(1)}$, $\widehat{N}_{1,2}$, $\widehat{O}_{1,2}$, $\widetilde{D}_{FT}$ and $\widetilde{G}_{FT}$) defined in Sec. 2 as follows:

$$\begin{aligned} A =\ & \frac{D_T(z)}{z}\left(-C_F(1+2\hat{z}) - \frac{1}{2N}\frac{1+\hat{z}^2}{\hat{z}}\right) \\ &+ \left(\frac{\partial}{\partial(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\right)\left(-\frac{1}{2N}\frac{1+\hat{z}^2}{\hat{z}}\right) + D_{1T}^{\perp(1)}(z)\left(C_F\frac{\hat{z}(1+\hat{z})}{(1-\hat{z})_+}\right) \\ &+ \int d\left(\frac{1}{z'}\right)\frac{1/z}{1/z-1/z'}\Im\widehat{D}_{FT}(z,z')\left\{\frac{1}{1/z'}\frac{1}{2N}\frac{2-\hat{z}}{\hat{z}} + \frac{1}{1/z'-1/z_f}\left(C_F+\frac{1}{2N}\right)(1+\hat{z})\right\} \\ &+ \int d\left(\frac{1}{z'}\right)\frac{1/z}{1/z-1/z'}\Im\widehat{G}_{FT}(z,z')\left\{\frac{1}{1/z'}\frac{1}{2N} - \frac{1}{1/z'-1/z_f}\left(C_F+\frac{1}{2N}\right)(1-\hat{z})\right\} \end{aligned} \tag{12}$$

and

$$
\begin{aligned}
B = 2C_F z^2 \Bigg[& \frac{(1-\hat{z})(-2+\hat{z}^2)}{\hat{z}^2}\widehat{G}_T^{(1)}(z) - 2\frac{(1-\hat{z})}{\hat{z}}\Delta\widehat{H}_T^{(1)}(z) + \int d\left(\frac{1}{z'}\right)\frac{1}{1/z-1/z'}\frac{(1-\hat{z})}{\hat{z}^2} \\
& \times\Im\left\{4(2-3\hat{z}+\hat{z}^2)\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right) + 2(2-3\hat{z})\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right) - 2(2-3\hat{z}+2\hat{z}^2)\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\} \\
& + \int d\left(\frac{1}{z'}\right)\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\frac{(1-\hat{z})}{\hat{z}^2} \\
& \times\Im\left\{(4-4\hat{z}+\hat{z}^2)\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right) + (4-4\hat{z}+\hat{z}^2)\widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right) - 2(2-2\hat{z}+\hat{z}^2)\widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\} \\
& + \int d\left(\frac{1}{z'}\right)\frac{1}{1/z-1/z'}\frac{(1-\hat{z})}{\hat{z}^2} \\
& \times\Im\left\{4(2-\hat{z})\widehat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right) + 2(4-3\hat{z}+\hat{z}^2)\widehat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right) + 2(4-3\hat{z}+\hat{z}^2)\widehat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\} \\
& + \int d\left(\frac{1}{z'}\right)\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\frac{(1-\hat{z})}{\hat{z}^2} \\
& \times\Im\left\{(4-4\hat{z}+\hat{z}^2)\widehat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right) + (4-4\hat{z}+\hat{z}^2)\widehat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right) + 2(2-2\hat{z}+\hat{z}^2)\widehat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\right\} \\
& + \frac{1}{C_F}\int d\left(\frac{1}{z'}\right)\Im\widetilde{D}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right) \\
& \times\left\{-4\frac{(1-\hat{z})}{\hat{z}^2}(2-3\hat{z}+\hat{z}^2) + \frac{1}{N}\frac{1}{z}\frac{1}{1/z-1/z'}\frac{-1}{\hat{z}}(2-\hat{z}) + \frac{1}{N}\frac{1}{1-z_f/z'}(-\hat{z})(-2+\hat{z})\right\} \\
& + \frac{1}{C_F}\int d\left(\frac{1}{z'}\right)\Im\widetilde{G}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left\{\frac{1}{N}\frac{1/z}{1/z-1/z'} + \frac{1}{N}\frac{1}{1-z_f/z'}(-\hat{z}^2)\right\}\Bigg], \qquad (13)
\end{aligned}
$$

where $\hat{z} = z_f/z$.

On the other hand, for $\Lambda_{QCD} \le P_{hT} = z_f q_T \ll Q$, one obtains the polarizing FF $D_{1T}^{\perp}$ contribution in the TMD factorization formalism as [11, 14]

$$
\begin{aligned}
\frac{d^6\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = & -z_f^2\sigma_0 \sin\Phi_S \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\lambda}_\perp \delta^2(\vec{k}_\perp - \vec{p}_\perp + \vec{\lambda}_\perp - \vec{P}_{hT}/z_f) \\
& \times\frac{\vec{p}_\perp^{\,2}}{q_T M_h} f_1(x_{bj},k_\perp^2) D_{1T}^{\perp}(z_f, z_f^2 p_\perp^2) S^{-1}(\lambda_\perp^2) H(Q^2), \qquad (14)
\end{aligned}
$$

where $f_1(x_{bj},k_\perp^2)$, $S(\lambda_\perp^2)$ and $H(Q^2)$ are the unpolarized TMD PDF, a soft factor and a hard factor, respectively. To check the consistency with the twist-3 factorization, we let q_T much larger than $\Lambda_{\rm QCD}$ in (14) ($\Lambda_{\rm QCD} \ll q_T \ll Q$). The leading contribution with respect to q_T/Q in (14) can be obtained by setting one of $-\vec{p}_\perp$, $\vec{k}_\perp$ and $\vec{\lambda}_\perp$ in the δ-function equal to $\vec{P}_{hT}/z_f$, neglecting the other two transverse momenta. To proceed further we need perturbative expressions for each factor in (14). The one-loop perturbative result of the soft factor is given by [3, 6]

$$
S^{-1}(\vec{\lambda}_\perp^2) = -\frac{\alpha_s C_F}{2\pi^2\vec{\lambda}_\perp^2}(\ln\rho^2 - 2), \qquad (15)
$$

where $\rho^2 = (2v\cdot\tilde{v})^2/(v^2\tilde{v}^2)$ with non-lightlike vectors v and $\tilde{v}$ to regulate the light cone singularities. The hard factor takes the form of $H(Q^2) = 1 + O(\alpha_s)$. The transverse momentum dependence of

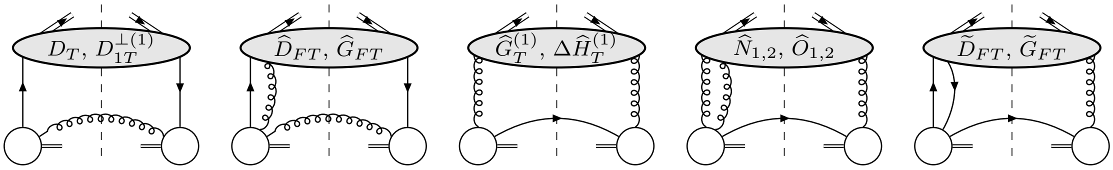

Figure 1: Feynman diagrams contributing to the polarizing FF $D_{1T}^{\perp}$ at $q_T \gg \Lambda_{\rm QCD}$. The mirror diagrams should be also included. Each upper blob denotes the corresponding twist-3 FFs. The white lower blobs represent all of possible diagrams.

the TMD functions can be calculated in perturbative QCD at $q_T \gg \Lambda_{\rm QCD}$, allowing us to express them in terms of appropriate collinear PDFs and FFs. The result for the unpolarized TMD PDF is well known [14]:

$$f_1(x_{bj}, \vec{k}_\perp^2) = \frac{\alpha_s C_F}{2\pi^2 \vec{k}_\perp^2} \int \frac{dx}{x} f_1(x) \left[\frac{1+\hat{x}^2}{(1-\hat{x})_+} + \delta(1-\hat{x}) \left(\ln \frac{x_{bj}^2 \zeta^2}{\vec{k}_\perp^2} - 1 \right) \right], \tag{16}$$

where $\zeta^2 = (2v\cdot p)^2/v^2$. The P_{hT}-dependence of the polarizing FF $D_{1T}^{\perp}(z_f, P_{hT})$ can be obtained by the perturbative calculation of the diagrams shown in Fig. 1, which gives the factorized expression for $D_{1T}^{\perp}(z_f, P_{hT})$ in terms of the collinear twist-3 FFs as

$$D_{1T}^{\perp}(z_f, P_{hT}^2) = \frac{\alpha_s}{2\pi^2} \frac{2M_h^2 z_f^2}{P_{hT}^4} \left[\int \frac{dz}{z} \left(A - \frac{1}{4} B \right) + C_F D_{1T}^{\perp(1)}(z_f) \left(\ln \frac{\hat{\zeta}^2}{P_{hT}^2} - 1 \right) \right], \tag{17}$$

where $\hat{\zeta}^2 = (2P_h \cdot \tilde{v})^2/\tilde{v}^2$, and A and B are, respectively, given in (12) and (13). It is critically important that $D_{1T}^{\perp}$ has been written in terms of the same A and B which appeared in $\mathcal{F}_1$ calculated in the collinear twist-3 factorization. Owing to this property, insertion of (15), (16) and (17) into (14) reproduces (11) exactly. This result confirms that the two approaches lead to the same cross section in the intermediate region of the transverse momentum $\Lambda_{\rm QCD} \ll q_T \ll Q$.

4. Summary

We have examined the consistency between the TMD factorization and the collinear twist-3 factorization for the hyperon polarization in SIDIS with regard to the twist-3 quark and gluon FFs contribution. We have demonstrated that the collinear twist-3 cross section consistently matches the TMD cross section in the intermediate region of the transverse momentum of the final hyperon. This establishes the idea that the two frameworks represent a unique QCD origin for the hyperon polarization.

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