

## Longitudinal spin transfer of semi-inclusive $\Lambda$ production in deep inelastic scattering

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We study the longitudinal spin transfer of  $\Lambda$ -hyperon production in semi-inclusive deep inelastic scattering with both current and target fragmentation mechanisms. For existing fixed-target experiments, such as JLab, COMPASS and HERMES, the events from the current region and those from the target fragmentation region are not clearly separated. We find that the contribution from the target fragmentation can significantly suppress the spin transfer to the  $\Lambda$  measured at existing experiments. We also perform an estimation based on the spectator diquark model to quantitatively demonstrate this effect. The predictions are consistent with existing experimental data from COMPASS, HERMES, and CLAS12, and will be examined in future EIC experiments.

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## 1. Introduction

The study of hadron polarization in semi-inclusive deep inelastic scattering (SIDIS) process serves as a powerful tool for exploring the spin structure of nucleons and spin-dependent hadronization mechanism. The polarization of  $\Lambda$  can be relatively easily measured due to its self-analyzing property, making it an ideal candidate for investigating spin effects [1].

In principle, final-state hadrons in SIDIS can be generated through the current and target fragmentation mechanisms. From a straightforward physical perspective, particularly for heavier hadrons in SIDIS, like  $\Lambda$  hyperons, the primary source originates from the residual components of the target particles in target fragmentation region (TFR) rather than quark fragmentation in current fragmentation region (CFR). However, TFR research is generally limited by the scarcity of experimental data. Theoretically, these two regions can be distinguished by the sign of the Feynman variable  $x_F$ , but in some cases the hadron is mainly produced near  $x_F = 0$ , e.g., the  $\Lambda$  at the COMPASS experiment [2]. It is hard to tell the origin of the  $\Lambda$  [3]. Without a quantitative understanding of the impact of these two fragmentation mechanisms on polarization, it is difficult to analyze the spin effects within the nucleon from either a theoretical or experimental standpoint.

In this talk, we conduct a phenomenological study on the longitudinal spin transfer  $D_{LL}$  of  $\Lambda$  and  $\bar{\Lambda}$  at leading-order in SIDIS, focusing on two fragmentation regions. Particularly, we perform a model estimation to quantitatively compute the target fracture functions in TFR, which can quantify the contribution of target fragmentation well. This will contribute to a comprehensive understanding of the hadronization mechanisms in SIDIS.

## 2. Longitudinal spin transfer

We consider the process of producing a longitudinally polarized  $\Lambda$  or  $\bar{\Lambda}$  hyperon in SIDIS,  $l(\ell) + p(P) \rightarrow l(\ell') + \Lambda/\bar{\Lambda}(P_h, \lambda_h) + X$ . The four-momenta corresponding to the particles are provided in parentheses, and  $\lambda_h$  is the  $\Lambda/\bar{\Lambda}$  longitudinal components of the spin polarization vector. The standard invariant variables in SIDIS are usually defined as

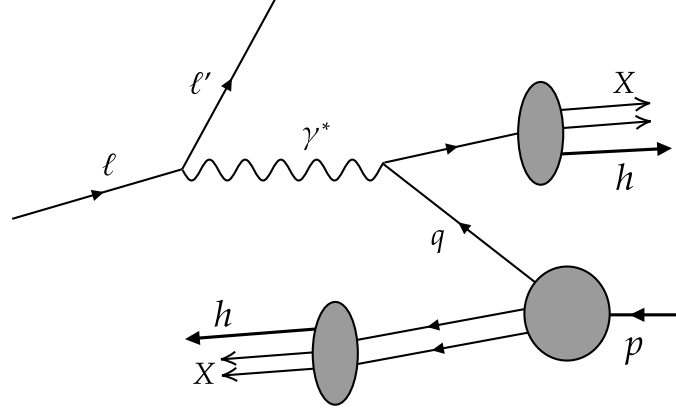
$$\begin{aligned} Q^2 = -q^2, \quad x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z_h = \frac{P \cdot P_h}{P \cdot q}, \\ \gamma = \frac{2x_B M}{Q}, \quad \epsilon = \frac{1 - y - \gamma^2 y^2 / 4}{1 - y + y^2 / 2 + \gamma^2 y^2 / 4}, \end{aligned} \quad (1)$$

with the momentum of the photon  $q = \ell - \ell'$ , and  $h$  stands for  $\Lambda$  or  $\bar{\Lambda}$  hyperon. For the SIDIS process of longitudinally polarized  $\Lambda/\bar{\Lambda}$  produced by a longitudinally polarized lepton beam and unpolarized target, the differential cross section can be expressed as:

$$\frac{d\sigma(\lambda_e, \lambda_h)}{dx_B dy dz_h d^2\mathbf{P}_{h\perp}} = \frac{4\pi\alpha_{em}^2}{x_B y Q^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right) \left\{F_{UU} + \lambda_e \lambda_h \sqrt{1-\epsilon^2} F_{UL} + \dots\right\}, \quad (2)$$

where  $\mathbf{P}_{h\perp}$  is the transverse momentum of the final hadron  $\Lambda$  or  $\bar{\Lambda}$ .  $F$  denotes the structure function with the subscripts specifying the polarization of the target and final state hadron, respectively.  $F_{UU}$  is the spin-averaged structure function, while  $F_{UL}$  corresponds to the spin-dependent structure function contributing to longitudinal spin transfer.

The differential cross section in Eq.(2) can be divided into two parts according to the kinematic regions where the final hadrons are mainly produced: from the scattered quark fragmentation and the nucleon target remnants. Theoretically, the Feynman variable  $x_F = 2P_{hL}/W$  is proposed to distinguish between the CFR and the TFR, where  $P_{hL}$  is the projection of the final-state hadron momentum onto the direction of the virtual photon  $\gamma^*$ -momentum in the  $\gamma^*p$  center-of-mass system, and  $W$  is the invariant mass of the hadronic final state. As shown in a simple schematic diagram Fig. 1, the ability to differentiate between the two fragmentation regions arises from the ideal condition that, in the scenario of very high center-of-mass energy, the longitudinal momentum of hadrons generated in these distinct regions is opposite.



**Figure 1:** Schematic diagram of semi-inclusive production of the hadron  $h$  in CFR and TFR.

In the case of  $x_F > 0$ , which indicates that hadrons move along the direction of the virtual photon, the predominant origin is likely to be associated with the fragmentation of the struck quark, which is known as the CFR. Conversely, hadrons moving along the direction of the nucleon with  $x_F < 0$  reside in the rear hemisphere of the  $\gamma^*p$  rest frame, indicating their origin from the target remnants, corresponding to the TFR. However, if the momentum carried by the virtual photon in this process is not large, the final-state hadrons produced by the struck quark may not necessarily continue to move along the direction of the virtual photon. Therefore, for existing fixed-target experiments, such as COMPASS [4], HERMES [5, 6], and CLAS12 [7], the events from the CFR and those from the TFR are not clearly separated by  $x_F$ .

The longitudinal spin transfer  $D_{LL}$  has been measured in various experiments. It relates the longitudinal polarization of the hyperon  $P_L^h$  to the polarization of the incoming lepton beam  $P_b$  [8]. This can also be obtained from the asymmetry in scattering cross sections and expressed as the ratio of the longitudinal polarized structure function  $F_{UL}$  to the unpolarized  $F_{UU}$ ,

$$D_{LL}^h \equiv \frac{P_L^h}{P_b \cdot D(y)} = \frac{F_{UL}}{F_{UU}}, \quad (3)$$

where the depolarization factor  $D(y) = \sqrt{1 - \epsilon^2}$ .

According to the factorized formalism, the structure function can be described as a convolution of the transverse momentum dependent (TMD) parton distribution function (PDF) and the TMD

fragmentation function (FF) in CFR [9, 10] or the fracture function in TFR [11]:

$$\begin{aligned} F_{UU} &= \mathcal{I}[f_{1q}D_{1q}^h] + x_B \left| \frac{\partial \zeta}{\partial z_h} \right| \sum_q e_q^2 M_q^h, \\ F_{UL} &= \mathcal{I}[f_{1q}G_{1Lq}^h] + x_B \left| \frac{\partial \zeta}{\partial z_h} \right| \sum_q e_q^2 \Delta M_{Lq}^h, \end{aligned} \quad (4)$$

where  $f_{1q}$  is the unpolarized TMD PDF, and  $D_{1q}^h$ ,  $G_{1Lq}^h$  are the unpolarized and longitudinal polarized TMD FFs. Similarly,  $M_q^h(x, \zeta, \mathbf{P}_{h\perp})$  and  $\Delta M_{Lq}^h(x, \zeta, \mathbf{P}_{h\perp})$  represent unpolarized and longitudinal polarized TMD fracture functions, respectively. We use the notation

$$\mathcal{I}[f_q D_q^h] \equiv \sum_q e_q^2 \int d^2 \mathbf{p}_T d^2 \mathbf{k}_T \delta^2(z_h \mathbf{p}_T + \mathbf{k}_T - \mathbf{P}_{h\perp}) x_B f_q(x_B, \mathbf{p}_T) D_q^h(z_h, \mathbf{k}_T), \quad (5)$$

with  $\mathbf{p}_T$  and  $\mathbf{k}_T$  denoting the transverse momenta of quarks.  $\zeta = P_h/P$  is the momentum fraction in TFR, and the factor  $|\partial \zeta / \partial z_h|$  comes from the Jacobian transformation. Based on the above analysis, by inserting Eq. (4) into Eq. (3), the longitudinal spin transfer considering both CFR and TFR can be obtained. If the contribution from the target fragmentation is neglected, i.e., target fracture functions are set to zero, the structure functions revert to the familiar region of current fragmentation, characterized by the convolution of TMD PDFs and TMD FFs.

### 3. Model calculation

We can estimate the magnitudes of the longitudinal spin transfer  $D_{LL}^{\Lambda(\bar{\Lambda})}$  by using available parametrizations of TMD PDFs and TMD FFs. However, fracture functions have limited theoretical and experimental research, and lacked parametrizations. Here we calculate the TMD fracture functions from the perspective of correlation function in the spectator diquark model.

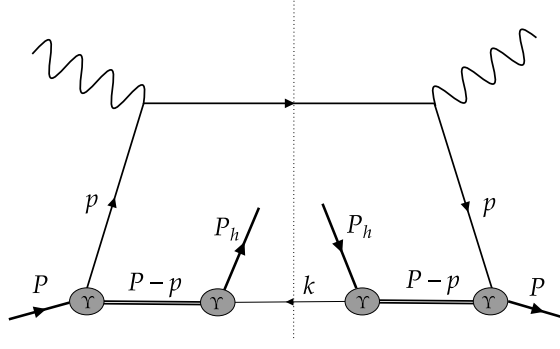
The main idea for the spectator diquark model is based on the naive picture of the quark-parton model, where the nucleon remnants, after the emission of a quark, are viewed as a spectator system with a spin-0 (scalar) or spin-1 (axial-vector) state [12, 13]. The fragmentation process to a  $\Lambda$ , as shown in Fig. 2, can be modeled as  $p(uud) \rightarrow u + R(ud)$  and then  $R(ud) \rightarrow \Lambda(uds) + \bar{s}$ , where only scalar diquark survives according to the SU(6) wave function of  $\Lambda$  hyperon. Since a scalar diquark cannot produce a polarized  $\Lambda$ , the polarized fracture functions including  $\Delta M_{Lq}^h$  in Eq.(4) are zero.

Starting from the fracture correlator  $\mathcal{M}$ , it is defined as the partonic structure in which the target particle remnants fragment into the hadron after the emission of a quark [11]:

$$\begin{aligned} \mathcal{M}_{ij}(p; P, S; P_h, S_h) &= \sum_X \int \frac{d^3 \mathbf{P}_X}{(2\pi)^3 2E_X} \int \frac{d^4 \xi}{(2\pi)^4} e^{i p \cdot \xi} \times \\ &\langle P, S | \bar{\psi}_j(0) | P_h, S_h; X \rangle \langle P_h, S_h; X | \psi_i(\xi) | P, S \rangle. \end{aligned} \quad (6)$$

Following Refs. [12, 13], in the spectator approximation when we insert a completeness relation into the fracture correlation function  $\mathcal{M}$  and truncate the summation of final states to an on-shell quark with mass  $m_{\bar{q}}$ , the fracture correlation Eq. (6) at the tree level can be expressed as

$$\mathcal{M}_{ij}(p; P, S; P_h, S_h) = \frac{\delta(k^2 - m_{\bar{q}}^2)}{(2\pi)^3} \theta(k^+) \langle P, S | \bar{\psi}_j(0) | P_h, S_h; k \rangle \langle P_h, S_h; k | \psi_i(0) | P, S \rangle. \quad (7)$$



**Figure 2:** The lowest order diagram for target fragmentation in spectator diquark model.

For  $\Lambda$  hyperons we only consider the spin-0 diquark, and the matrix element can be calculated in the following:

$$\langle P_h, S_h; k | \psi(0) | P, S \rangle = \bar{U}(P_h, S_h) \Upsilon_{2s} v(k) \frac{i}{(P-p)^2 - M_s^2} \frac{i}{\not{p} - m} \Upsilon_{1s} U(P, S). \quad (8)$$

Here  $U(P, S)$ ,  $U(P_h, S_h)$  and  $v(k)$  are the Dirac spinor of the proton, the final-hadron and the antiquark, respectively. The hadron-quark-scalar diquark vertex can be expressed as  $\Upsilon_s = g_s \mathbf{1}$ , where  $g_s$  is a Gaussian form factor [13]. The symbols  $m$  and  $M_s$  are the masses of the quark and the spectator diquark.

By inserting the matrix element Eq. (8) into the fracture correlator Eq. (7) and projecting on  $\gamma^+$  matrices, we can get the unpolarized fracture function [11]:

$$\begin{aligned} M_q^h &\equiv \frac{1}{4\zeta} \int \frac{dp^-}{(2\pi)^3} \text{Tr} [\mathcal{M}_U \gamma^+] \Big|_{p^+ = x_B P^+} \\ &= \int \frac{d^2 \mathbf{p}_T}{(2\pi)^6} \frac{g_{1s}^2 g_{2s}^2 x [(m + xM)^2 + \mathbf{p}_T^2]}{2\zeta^2 (1 - \zeta - x)^2 (p^2 - m^2)^2} \frac{[(1 - x - \zeta)M_h - \zeta m_q]^2 + [(1 - x)\mathbf{P}_{h\perp} + \zeta \mathbf{p}_T]^2}{x(1 - x)M^2 - xM_s^2 - (1 - x)p^2 - \mathbf{p}_T^2}, \end{aligned} \quad (9)$$

where the subscript  $U$  specifies the unpolarized process. To determine the diquark model parameters, we fit the model calculated unpolarized PDF  $f_{1q}(x)$  with the JR14 parametrization [14], and obtain the fitted results as:

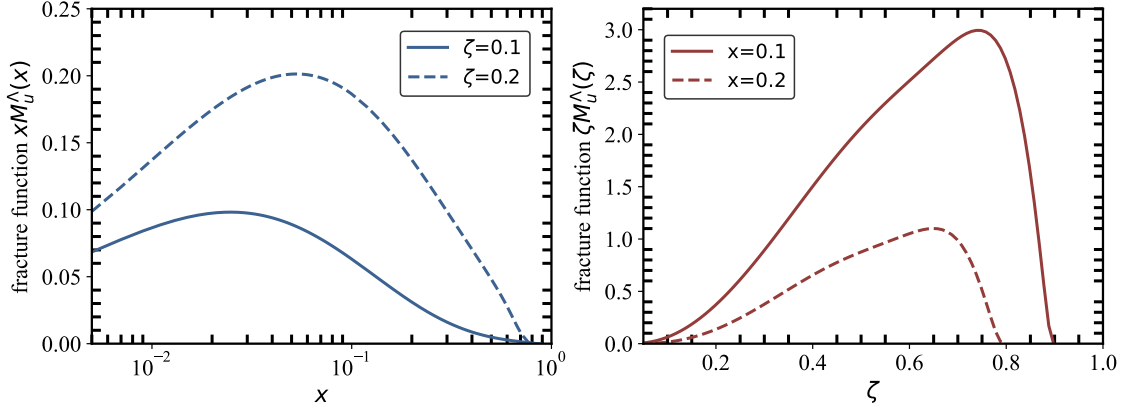
$$M_s = 1.2 \text{ GeV}, \quad \Lambda_s = 2.3 \text{ GeV}, \quad g_s = 14.98. \quad (10)$$

Using these as inputs we can compute the numerical results of the fracture function  $M_u^\Lambda(x)$  or  $M_u^\Lambda(\zeta)$ , as shown in Fig. 3.

#### 4. Results and discussion

We can study the contribution of target fragmentation to explore the hadronization mechanisms in SIDIS through longitudinal spin transfer, which for  $\Lambda$  can be expressed explicitly as following based on the analysis in the previous sections,

$$D_{LL}^\Lambda(x, z) = \frac{\sum_q e_q^2 \int d^2 \mathbf{P}_{\Lambda\perp} \mathcal{I} \left[ f_{1q}(x, p_T^2) G_{1Lq}^\Lambda(z, k_T^2) \right]}{\sum_q e_q^2 \int d^2 \mathbf{P}_{\Lambda\perp} \left[ \mathcal{I} \left[ f_{1q}(x, p_T^2) D_{1q}^\Lambda(z, k_T^2) \right] + x \left| \frac{\partial \zeta}{\partial z} \right| M_q^\Lambda(x, z, \mathbf{P}_{\Lambda\perp}) \right]}. \quad (11)$$



**Figure 3:** The results for  $xM_u^\Lambda(x)$  and  $\zeta M_u^\Lambda(\zeta)$  on  $x$  and  $\zeta$  dependences using the spectator diquark model.

We examine  $D_{LL}$  on the  $x$ ,  $z$  and  $x_F$  dependencies, which is a direct experimental observable in COMPASS [4], HERMES [5, 6] and CLAS12 [7], and compare our numerical results with the data in these experiments. To implement the numerical calculation we need the TMD functions as inputs in Eq. (11). To this end we use JR14 parametrization for the unpolarized collinear PDF  $f_{1q}(x)$ , DSV parametrization [15] for collinear FFs including  $D_{1q}^\Lambda(z)$  and  $G_{1Lq}^\Lambda(z)$ , and take the Gaussian ansatz to estimate the TMD PDF and TMD FFs roughly,

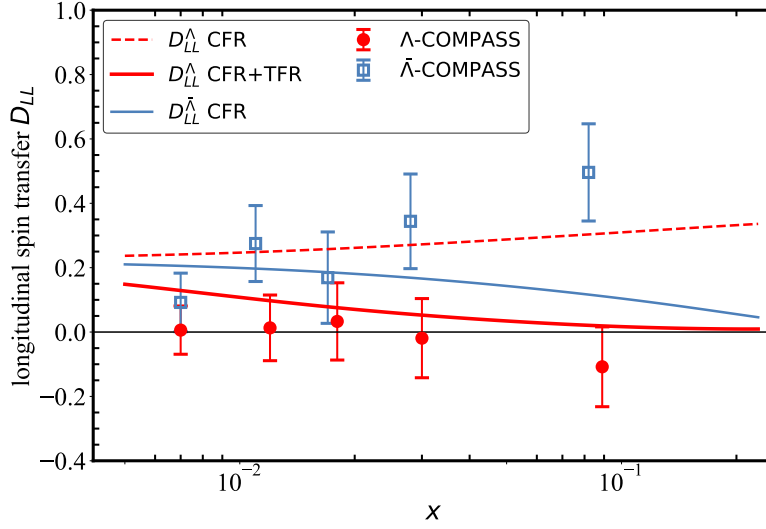
$$f_{1q}(x, p_T^2) = f_{1q}(x) \frac{e^{-\frac{p_T^2}{\Delta_p^2}}}{\pi \Delta_p^2}, \quad D_{1q}^\Lambda(z, k_T^2) = D_{1q}^\Lambda(z) \frac{e^{-\frac{k_T^2}{\Delta_\Lambda^2}}}{\pi \Delta_\Lambda^2}, \quad G_{1Lq}^\Lambda(z, k_T^2) = G_{1Lq}^\Lambda(z) \frac{e^{-\frac{k_T^2}{\Delta_\Lambda^2}}}{\pi \Delta_\Lambda^2}, \quad (12)$$

where  $\Delta_p^2$  and  $\Delta_\Lambda^2$  represent the corresponding Gaussian widths. This ansatz is commonly used and we take  $\Delta_p^2 = 0.57$ ,  $\Delta_\Lambda^2 = 0.118$  following Refs. [16, 17]. For the unpolarized TMD fracture function  $M_q^\Lambda(x, z, \mathbf{P}_{\Lambda\perp})$  we use the spectator model result Eq. (9) and the fitted parameter inputs Eq. (10).

For the  $\bar{\Lambda}$  production process, it is relatively difficult to break up two quarks into a particle consisting of three antiquarks. Ignoring the contribution from target fragmentation, the longitudinal spin transfer for  $\bar{\Lambda}$  can be simplified to the following form,

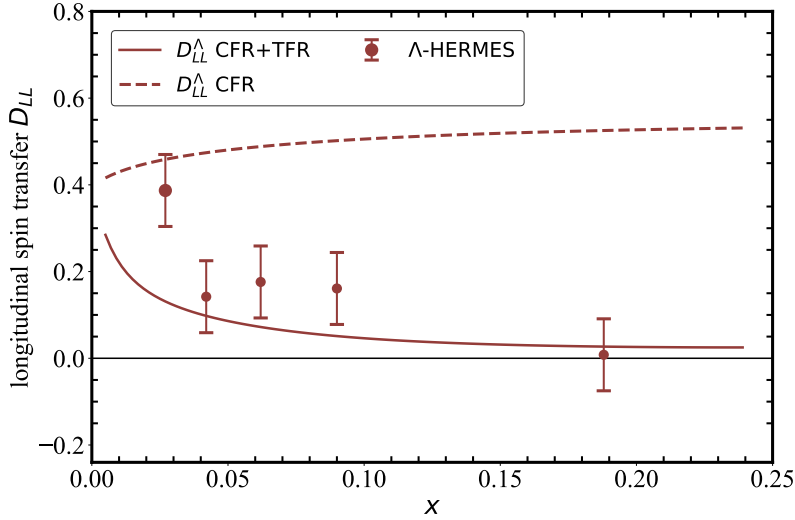
$$D_{LL}^{\bar{\Lambda}}(x, z) = \frac{\sum_q e_q^2 f_{1q}(x) G_{1Lq}^{\bar{\Lambda}}(z)}{\sum_q e_q^2 f_{1q}(x) D_{1q}^{\bar{\Lambda}}(z)}. \quad (13)$$

We plot the result of the  $x$ -dependent longitudinal spin transfer of  $\Lambda$  and  $\bar{\Lambda}$  in SIDIS process in Fig. 4 comparing with COMPASS data [4], with  $\bar{Q}^2 = 3.7 \text{ GeV}^2$  and  $\bar{z} = 0.27$ . For  $\bar{\Lambda}$  production process, since the contribution of the target fragmentation to antiparticles is relatively small, only the contribution from the current fragmentation is considered as we mentioned above, and the numerical results can relatively well describe the experimental data if considering the large uncertainties. However, for  $\Lambda$  production, the calculation results when considering only the current fragmentation mechanism (red dashed curve) show significant deviations from the experimental data. To further elucidate the impact of the target fragmentation in SIDIS process, we simultaneously consider



**Figure 4:** The comparison of  $x$ -dependent results and COMPASS data for  $\Lambda$  and  $\bar{\Lambda}$ . The red and blue represent  $\Lambda$  and  $\bar{\Lambda}$ , respectively. The curves are the theoretical calculation while the dots with error bars represent the experimental data.

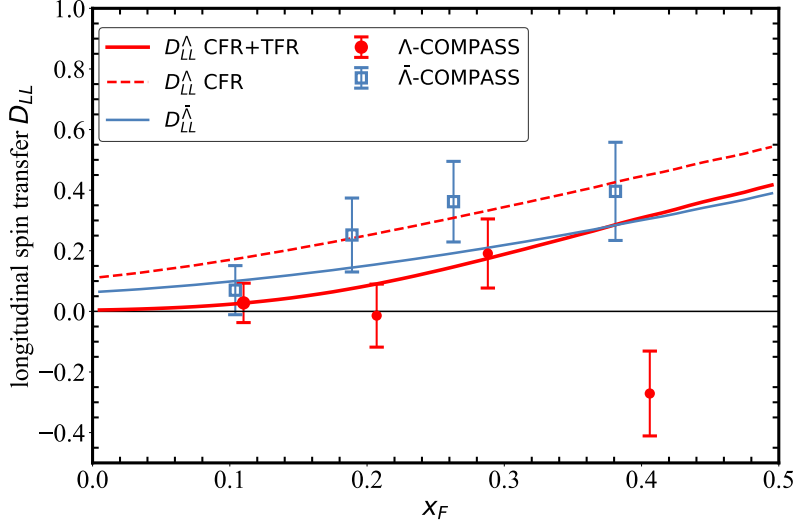
both CFR and TFR to calculate  $D_{LL}$  (red solid curve), providing a reasonable explanation for the COMPASS data. Compared to pure current fragmentation, the calculation results considering the target fragmentation give a good trend for spin transfer.



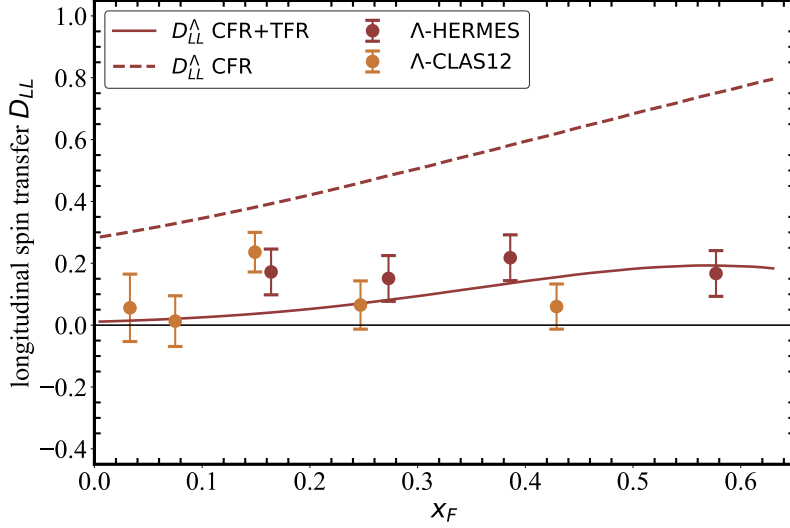
**Figure 5:** The comparison of  $x$ -dependent results and HERMES data for  $\Lambda$ .

In Fig. 5 we show the spin transfer results comparing with the HERMES data [6], with  $\bar{Q}^2 = 2.4 \text{ GeV}^2$  and  $\bar{z} = 0.45$ . One can observe that the contribution from the target fragmentation remarkably reduces the predicted value, and the results match the data much better than the pure contribution from the current fragmentation.

As for the  $x_F$  bins, the calculation is performed at the average value of  $\bar{x} = 0.03$  in COMPASS



**Figure 6:** Similar to Fig. 4, the comparison of  $x_F$ -dependent results and COMPASS data for  $\Lambda$  and  $\bar{\Lambda}$ .



**Figure 7:** The comparison of  $x_F$ -dependent results and HERMES and CLAS12 data for  $\Lambda$ .

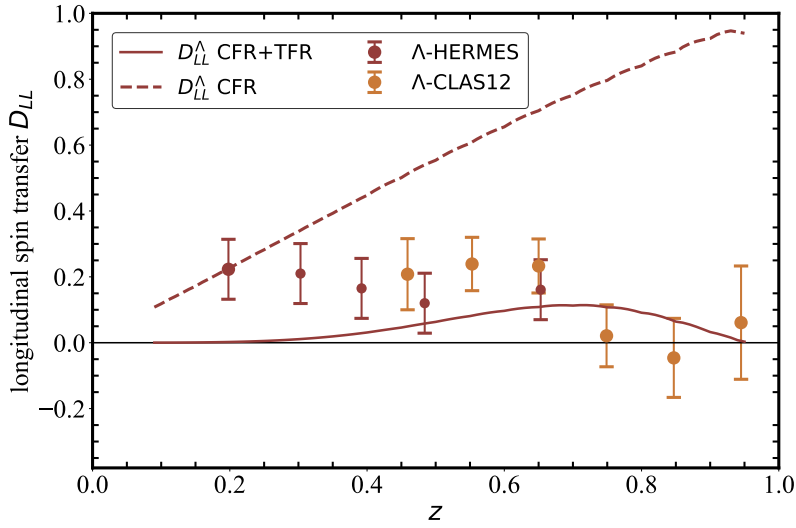
and  $\bar{x} = 0.088$  in HERMES. The calculation results of the  $x_F$  variable dependencies are shown in Fig. 6 and Fig. 7.

We also present the results for the  $z$ -dependent spin transfer calculations in Fig. 8, which are measured in the HERMES and CLAS12 experiments. The results considering both current and target fragmentation are in good agreement with the experimental data.

## 5. Summary

In this study, we compute the longitudinal spin transfer of  $\Lambda$  and  $\bar{\Lambda}$  in SIDIS process by taking into account the target fragmentation besides the current fragmentation. Considering the spin flavor structure of  $\Lambda$ , the presence of target fragmentation significantly suppresses the spin transfer. The





**Figure 8:** The comparison of  $z$ -dependent results and HERMES and CLAS12 data for  $\Lambda$ .

calculations demonstrate that incorporating the fracture functions in TFR alongside fragmentation functions in CFR can well describe the experimental data from COMPASS, HERMES, and CLAS12. We anticipate new and precise experimental measurements at the future EIC and EicC of  $\Lambda$  and  $\bar{\Lambda}$  production in both the CFR and TFR, which makes it possible to refine the theoretical framework to enhance our understanding of fracture functions.

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