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Production of spin-3/2 hadrons in e^+e^- annihilation and SIDIS

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We investigate the semi-inclusive productions of spin-3/2 hadrons in unpolarized e^+e^- annihilation and deep inelastic lepton-nucleon scattering. The quark transverse momentum dependent (TMD) fragmentation functions (FFs) to spin-3/2 hadrons are defined for the first time from the decomposition of the quark-quark correlator at leading twist, 14 of which are newly defined for rank-3 tensor polarized hadron states. We perform a leading order calculation of the differential cross sections. For two-hadron production in e^+e^- annihilation, half of the 48 structure functions are found nonzero even if the spin of the second hadron is not analyzed, and ten of the rank-3 tensor polarized TMD FFs contribute. We also apply these newly defined FFs in semi-inclusive deep inelastic scattering (SIDIS) for the study of nucleon structures. For a polarized lepton beam, one third of 96 structure functions have nonzero leading order contributions and 42 of rank-3 tensor polarized TMD FFs contribute. For the polarized lepton, half of 192 structure functions are nonzero and 14 of them are from rank-3 tensor polarized hadron states.

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1. Introduction

The quantum chromodynamics (QCD) is the fundamental theory of the strong interaction. Due to the color confinement, quarks and gluons, the fundamental degrees of freedom in QCD, cannot be isolated as free particles at long distance, and are always confined within the hadron by the strong interaction. The fragmentation functions (FFs) are introduced to describe the momentum distribution of final-state hadrons produced by high energy partons. They are defined as functions of the longitudinal momentum fraction z carried by the fragmented hadron. Extending to a three-dimensional description, one can define the transverse momentum dependent (TMD) FFs, which are functions of z and the transverse momentum of the produced hadron.

Taking into account the spin degrees of freedom of quark and produced hadron, one can study the spin effect in hadronization. The spin-dependent TMD FFs encode the correlation of the struck parton momentum distribution and the spin of the parton or the produced hadron. They lead to rich phenomena in high energy reactions, such as the azimuthal asymmetries in semi-inclusive electron-positron annihilations measured by Belle [1–3], BABAR [4], and BESIII [5]. In this talk, we present the study of the TMD FFs for spin-3/2 hadrons, which are still very limited investigated on either theoretical or experimental aspects. While the unpolarized, the vector polarized, and the rank-2 tensor polarized TMD FFs remain the same up to some normalization factors as introducd in previous studies [6–9], the rank-3 tensor polarized ones are newly defined and only exist for hadrons with $s \geq 3/2$.

On the other hand, the TMD FFs are also essential input for the study of three-dimensional structures of the nucleon via the semi-inclusive deep inelastic scattering (SIDIS) process. Applying the TMD factorization [10–12], one can express the cross section for SIDIS process as the convolution of short-distance hard part, quark TMD PDFs, and FFs. Measuring the produced hadron with the strange quantum number can improve the sensitivity to s quark distribution in the nucleon. The Ω baryon, composed of three s quarks, is extremely sensitive to strange sea distribution. Furthermore, the spin state of the produced Ω can be analyzed through its weak decays. Therefore, the production of the Ω in SIDIS has the unique advantage to study the s quark sea distributions in nucleon. This requires us to have a comprehensive theoretical understanding for spin-3/2 hadrons TMD FFs. Among all different high energy reactions, e^+e^- annihilation is the most suitable for studying FFs. The inclusive process $e^+e^- \to \Omega X$ can be used to study the collinear FFs. In order to access transverse momentum dependence, we consider the semi-inclusive hadron production in e^+e^- annihilation, i.e., $e^+e^- \to \Omega h X$. Thus, we focus on the production of spin-3/2 hadrons in e^+e^- annihilation and SIDIS process.

The proceeding is organized as follows. In Sec. 2, we describe the spin information of spin-3/2 hadrons using the spin density matrix. In Sec. 3, we derive a complete set of TMD FFs for spin-3/2 hadrons. We present the calculation of the cross section for the production of spin-3/2 hadrons in e^+e^- annihilation and SIDIS in Sec. 4 and Sec. 5, respectively. We make a summary and outlook in Sec. 6.

2. The description of spin-3/2 particles

The spin information of particles can be described by the spin density matrix. For a particle with spin-s, the corresponding spin density matrix ρ is a $(2s+1)\times(2s+1)$ matrix. According to the Hermiticity condition $\rho = \rho^\dagger$, we can decompose ρ of spin-3/2 particles on a basis of 16 independent Hermitian matrices,

$$\rho = \frac{1}{4}\left(\mathbf{1} + \frac{4}{5}S^i\Sigma^i + \frac{2}{3}T^{ij}\Sigma^{ij} + \frac{8}{9}R^{ijk}\Sigma^{ijk}\right). \tag{1}$$

Here Σ^i are 4×4 spin matrices for spin-3/2 hadron, Σ^{ij} are the five rank-2 tensor polarization basis defined by $\Sigma^{ij} = 1/2\left(\Sigma^i\Sigma^j + \Sigma^j\Sigma^i\right) - 5/4\delta^{ij}\mathbf{1}$, and Σ^{ijk} are the seven rank-3 tensor polarization basis $\Sigma^{ijk} = 1/6\Sigma^{\{i}\Sigma^j\Sigma^{k\}} - 41/60\left(\delta^{ij}\Sigma^k + \delta^{jk}\Sigma^i + \delta^{ki}\Sigma^j\right)$. Following the common convention, we define the spin vector S^i in the hadron rest frame as

$$S^i = \left(S_T^x, S_T^y, S_L\right), \tag{2}$$

the rank-2 spin tensor T^{ij} as

$$T^{ij} = \frac{1}{2}\begin{pmatrix} -S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & 2S_{LL} \end{pmatrix}, \tag{3}$$

and the rank-3 spin tensor R^{ijk} is defined as

$$R^{ijk} = \frac{1}{4}\left[\begin{array}{c} \begin{pmatrix} -3S_{LLT}^x + S_{TTT}^{xxx} & -S_{LLT}^y + S_{TTT}^{yxx} & -2S_{LLL} + S_{LTT}^{xx} \\ -S_{LLT}^y + S_{TTT}^{yxx} & -S_{LLT}^x - S_{TTT}^{xxx} & S_{LTT}^{xy} \\ -2S_{LLL} + S_{LTT}^{xx} & S_{LTT}^{xy} & 4S_{LLT}^x \end{pmatrix} \\ \begin{pmatrix} -S_{LLT}^y + S_{TTT}^{yxx} & -S_{LLT}^x - S_{TTT}^{xxx} & S_{LTT}^{xy} \\ -S_{LLT}^x - S_{TTT}^{xxx} & -3S_{LLT}^y - S_{TTT}^{yxx} & -2S_{LLL} - S_{LTT}^{xx} \\ S_{LTT}^{xy} & -2S_{LLL} - S_{LTT}^{xx} & 4S_{LLT}^y \end{pmatrix} \\ \begin{pmatrix} -2S_{LLL} + S_{LTT}^{xx} & S_{LTT}^{xy} & 4S_{LLT}^x \\ S_{LTT}^{xy} & -2S_{LLL} - S_{LTT}^{xx} & 4S_{LLT}^y \\ 4S_{LLT}^x & 4S_{LLT}^y & 4S_{LLL} \end{pmatrix} \end{array}\right]. \tag{4}$$

The spin vector and tensors for a moving hadron can be obtained by a Lorentz boost from those defined in the rest frame, and their explicit expressions are given in Eq.(18)–(20) of Ref. [13].

3. TMD fragmentation functions for spin-3/2 hadrons

The unintegrated quark-quark correlation function is defined by

$$\Delta_{\alpha\beta}(k, P, S, T, R) = \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi}\langle 0|\mathcal{L}(\infty,\xi)\psi_\alpha(\xi)|P, S, T, R, X\rangle \\ \times \langle P, S, T, R, X|\bar{\psi}_\beta(0)\mathcal{L}^\dagger(\infty, 0)|0\rangle, \tag{5}$$

where k is the momentum of the quark, α and β are Dirac indices, ψ is the quark field operator, and $\sum_X$ also implies the integration over the momenta of the undetected hadrons labeled by X. The gauge link $\mathcal{L}(y_2, y_1)$ is defined as $\mathcal{L}(y_2, y_1) = \mathcal{P} \exp\left[-ig \int_{y_1}^{y_2} dy \cdot A(y)\right]$.

The correlation function is a 4×4 matrix in the Dirac space, and thus it can be expanded on a basis of 16 Dirac γ matrices. Imposing the constraints of the Hermiticity and parity invariance, one can decompose the quark-quark correlation function (5) for a spin-3/2 hadron as

$$\begin{aligned}\Delta(k, P, S, T, R) = {} & B_1 M \mathbf{1} + B_2 \not{P} + \cdots + \frac{B_{20}}{M^2} \epsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma_5 P^\nu k^\rho k_\tau T^{\tau\sigma} \\ & + i \frac{B_{21}}{M^2} k_\mu k_\nu k_\rho R^{\mu\nu\rho} \gamma_5 + \cdots + \frac{B_{28}}{M^3} \epsilon_{\mu\nu\rho\sigma} \gamma^\mu k^\nu P^\rho k_\tau k_\lambda R^{\tau\lambda\sigma}. \end{aligned} \tag{6}$$

Its complete expression is given in Ref. [13]. Here the B_i's are Lorentz scalar functions of $k \cdot P$ and k^2, and the mass factor M is introduced to balance the dimension. The rank-3 tensor polarized terms, $B_{21} - B_{28}$, are newly defined for spin-3/2 hadrons, while the unpolarized, the vector polarized, and the rank-2 tensor polarized ones also exist in the decomposition of the correlation function of spin-1 hadrons [8].

By integrating Eq. (5) over k^+, or, equivalently k^2, we obtain the quark-quark correlation function,

$$\Delta(z, k_T) = \frac{1}{4z} \int \mathrm{d}k^+ \Delta(k, P, S, T, R)\Big|_{k^- = \frac{P^-}{z}}, \tag{7}$$

which leads to the definition of quark TMD FFs after the Dirac decomposition as shown in Eq. (6).

Hadron Polarization	Quark Polarization: Unpolarized	Quark Polarization: Longitudinally Polarized	Quark Polarization: Transversely Polarized
U	$D_1(z, k_T^2)$		$H_1^\perp(z, k_T^2)$
L		$G_{1L}(z, k_T^2)$	$H_{1L}^\perp(z, k_T^2)$
T	$D_{1T}^\perp(z, k_T^2)$	$G_{1T}^\perp(z, k_T^2)$	$H_{1T}(z, k_T^2), H_{1T}^\perp(z, k_T^2)$
LL	$D_{1LL}(z, k_T^2)$		$H_{1LL}^\perp(z, k_T^2)$
LT	$D_{1LT}^\perp(z, k_T^2)$	$G_{1LT}^\perp(z, k_T^2)$	$H_{1LT}(z, k_T^2), H_{1LT}^\perp(z, k_T^2)$
TT	$D_{1TT}^\perp(z, k_T^2)$	$G_{1TT}^\perp(z, k_T^2)$	$H_{1TT}^\perp(z, k_T^2), H_{1TT}^{\perp\perp}(z, k_T^2)$
LLL		$G_{1LLL}(z, k_T^2)$	$H_{1LLL}^\perp(z, k_T^2)$
LLT	$D_{1LLT}^\perp(z, k_T^2)$	$G_{1LLT}^\perp(z, k_T^2)$	$H_{1LLT}(z, k_T^2), H_{1LLT}^\perp(z, k_T^2)$
LTT	$D_{1LTT}^\perp(z, k_T^2)$	$G_{1LTT}^\perp(z, k_T^2)$	$H_{1LTT}^\perp(z, k_T^2), H_{1LTT}^{\perp\perp}(z, k_T^2)$
TTT	$D_{1TTT}^\perp(z, k_T^2)$	$G_{1TTT}^\perp(z, k_T^2)$	$H_{1TTT}^\perp(z, k_T^2), H_{1TTT}^{\perp\perp}(z, k_T^2)$

Table 1: The leading-twist TMD FFs for spin-3/2 hadrons.

Here, we consider P^- as a large momentum component and collect the leading-twist TMD FFs, which can be projected out from the correlator $\Delta(z, k_T)$ by the Dirac matrices γ^-, $\gamma^-\gamma_5$, and $i\sigma^{i-}\gamma_5$. For instance, the S_{LLL}-dependent TMD FFs can be parametrized as

$$\Delta_{LLL}(z, k_T) = \frac{1}{4} \left\{ G_{1LLL}\left(z, k_T^2\right) S_{LLL} \gamma_5 \not{n} + H_{1LLL}^\perp \left(z, k_T^2\right) S_{LLL} i \sigma_{\mu\nu} \gamma_5 n^\mu \frac{k_T^\nu}{M} \right\}. \tag{8}$$

Then a complete set of quark TMD FFs for spin-3/2 hadrons at leading twist are shown in Table 1. Among the 32 TMD FFs at leading twist, two are for the unpolarized hadron state, six are for the vector polarized hadron state, ten are for the rank-2 tensor polarized hadron state, and 14 are for the rank-3 tensor polarized hadron state. While the unpolarized, the vector polarized, and the rank-2 tensor polarized ones have been defined in the study of spin-1 hadrons, the rank-3 tensor polarized TMD FFs are newly defined and only exist for hadrons with $s \geq 3/2$.

The terms corresponding to different spin components of the hadron have been separated and labeled by U for unpolarized hadron, by L and T for vector polarized hadron, by LL, LT, and TT for rank-2 tensor polarized hadron, and by LLL, LLT, LTT, and TTT for rank-3 tensor polarized hadron. The D, G, and H are used to represent unpolarized, longitudinally polarized, and transversely polarized quarks. A superscript "$\perp$" is assigned if there is inhomogeneous dependence on the quark transverse momentum. The superscript "$\perp\perp$" in $H_{1TT}^{\perp\perp}$, $H_{1LTT}^{\perp\perp}$, and $H_{1TTT}^{\perp\perp}$ is introduced to differentiate them from $H_{1TT}^{\perp}$, $H_{1LTT}^{\perp}$, and $H_{1TTT}^{\perp}$.

4. Production of spin-3/2 hadrons in e^+e^- annihilation

To access TMD FFs, we consider the process

$$e^-(l_1) + e^+(l_2) \longrightarrow \Omega(P_1) + h(P_2) + X(P_X), \tag{9}$$

where the variables in parentheses are the four momenta of the corresponding particles and for simplicity the spin of the second hadron h is not taken into account. With one-photon-exchange approximation, the differential cross section can be expressed as

$$\frac{P_1^0 P_2^0 d\sigma}{d^3\boldsymbol{P}_1 d^3\boldsymbol{P}_2} = \frac{\alpha^2}{4Q^6} L_{\mu\nu} W^{\mu\nu}, \tag{10}$$

where the leptonic tensor $L_{\mu\nu}$ is

$$L_{\mu\nu} = 2\left(l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu}(l \cdot l')\right), \tag{11}$$

and the hadronic tensor $W^{\mu\nu}$ is given by

$$\begin{aligned} W^{\mu\nu}(q; P_1, S, T, R; P_2) = {} & \frac{1}{(2\pi)^4} \sum_X (2\pi)^4 \delta^4 (q - P_X - P_1 - P_2) \\ & \times \langle 0 | J^\mu(0) | P_X; P_1, S, T, R; P_2 \rangle \langle P_X; P_1, S, T, R; P_2 | J^\nu(0) | 0 \rangle, \end{aligned} \tag{12}$$

where S, T, and R represent the spin states of the first hadron, while the second hadron is unpolarized. The hadronic tensor satisfies the Hermiticity, the parity invariance, and the gauge invariance relations. We first construct the *basic Lorentz tensors*,

$$t_U^{\mu\nu} = \left\{ \widetilde{g}^{\mu\nu}, \widetilde{P}_1^\mu \widetilde{P}_1^\nu, \widetilde{P}_2^\mu \widetilde{P}_2^\nu, \widetilde{P}_1^{\{\mu} \widetilde{P}_2^{\nu\}} \right\}, \tag{13}$$

$$t_U^{\mathcal{P},\mu\nu} = \left\{ \widetilde{P}_1^{\{\mu} \epsilon^{\nu\} q P_1 P_2}, \widetilde{P}_2^{\{\mu} \epsilon^{\nu\} q P_1 P_2} \right\}, \tag{14}$$

which correspond to parity conserving and parity nonconserving ones, respectively. Here $\widetilde{g}^{\mu\nu}$ and $\widetilde{P}^\mu$ are so-called conserved vectors, which are defined as $g^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2$ and $\widetilde{P}^\mu =$

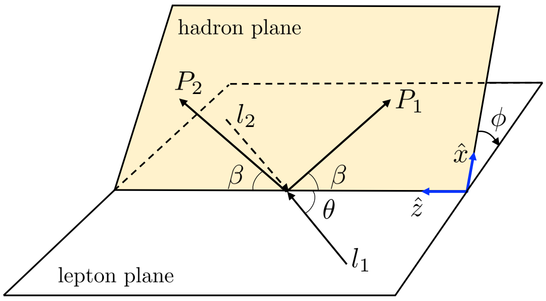

Figure 1: The Collins-Soper frame for $e^+e^- \to \Omega h X$.

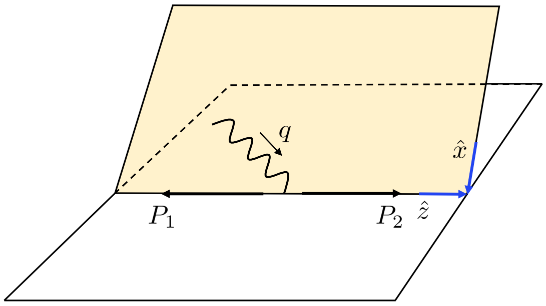

Figure 2: The two-hadron center-of-mass frame for $e^+e^- \to \Omega h X$.

$P^\mu - P \cdot q/q^2 q^\mu$, respectively. The four vectors $\widetilde{P}_1^\mu$ and $\widetilde{P}_2^\mu$ have the similar definitions to $\widetilde{P}$. The unpolarized basis tensors are directly given by those in $t_U^{\mu\nu}$. The polarized basis tensors can be obtained by multiplying the basic Lorentz tensors by a spin-dependent scalar or pseudoscalar. Then the eight vector polarized basis tensors, 16 rank-2 tensor polarized basis tensors, and 20 rank-3 tensor polarized basis tensors are constructed from those in $t_U^{\mu\nu}$ and $t_U^{\mathcal{P},\mu\nu}$ as

$$t_V^{\mu\nu} = \left\{\epsilon^{SqP_1P_2}\right\} t_U^{\mu\nu}, \left\{S\cdot q, S\cdot P_2\right\} t_U^{\mathcal{P},\mu\nu}, \tag{15}$$

$$t_T^{\mu\nu} = \left\{T^{P_2P_2}, T^{P_2q}, T^{qq}\right\} t_U^{\mu\nu}, \left\{\epsilon^{T^{P_2}P_1P_2q}, \epsilon^{T^q P_1P_2q}\right\} t_U^{\mathcal{P},\mu\nu}, \tag{16}$$

$$t_R^{\mu\nu} = \left\{\epsilon^{R^{P_2P_2}P_1P_2q}, \epsilon^{R^{P_2q}P_1P_2q}, \epsilon^{R^{qq}P_1P_2q}\right\} t_U^{\mu\nu}, \left\{R^{P_2P_2P_2}, R^{qqq}, R^{P_2P_2q}, R^{P_2qq}\right\} t_U^{\mathcal{P},\mu\nu}. \tag{17}$$

With the 48 basis tensors above, we can expand the hadronic tensor $W^{\mu\nu}$ as

$$W^{\mu\nu} = \sum_{i=1}^{4} V_{U,i} t_{U,i}^{\mu\nu} + \sum_{i=1}^{8} V_{V,i} t_{V,i}^{\mu\nu} + \sum_{i=1}^{16} V_{T,i} t_{T,i}^{\mu\nu} + \sum_{i=1}^{20} V_{R,i} t_{R,i}^{\mu\nu}, \tag{18}$$

where the coefficients V's are scalar functions of q^2, $P_1 \cdot q$, $P_2 \cdot q$, and $P_1 \cdot P_2$.

Contracting the hadronic tensor with the leptonic tensor, one can derive the general form of the differential cross section. It is convenient to specify a reference frame to obtain a general angular distribution of this cross section. There are two commonly used frames, one is the Collins-Soper (CS) frame as illustrated in Fig. 1, the other is the hadronic center-of-mass (c.m.) frame as illustrated in Fig. 2. In this calculation, the CS frame is more convenient to describe the angular distributions of the produced hadrons, but the spin components are easier to be defined in the c.m. frame. Therefore, we introduce θ and ϕ in the CS frame and define the spin components in the c.m. frame, S_L, $|S_T|$, ϕ_T, S_{LL}, $|S_{LT}|$, ϕ_{LT}, $|S_{TT}|$, ϕ_{TT}, S_{LLL}, $|S_{LLT}|$, ϕ_{LLT}, $|S_{LTT}|$, ϕ_{LTT}, $|S_{TTT}|$, and ϕ_{TTT}.

After contracting the hadronic tensor and the leptonic tensor, we can express the differential cross section in terms of 48 structure functions. The explicit expression of the cross section has been presented in Ref. [13] and here we only show the rank-3 tensor polarized part below

$$\frac{P_1^0 P_2^0 d\sigma}{d^3\boldsymbol{P}_1 d^3\boldsymbol{P}_2} = \frac{\alpha^2}{4Q^4}\Bigg\{\cdots + S_{LLL}\left[\left(\sin^2\theta \sin 2\phi\right) F_{LLL,U}^{\sin 2\phi} + (\sin 2\theta \sin\phi) F_{LLL,U}^{\sin\phi}\right]$$
$$+ |S_{LLT}|\left[\sin\phi_{LLT}\left(\left(1+\cos^2\theta\right) F_{LLT,U}^{T} + \left(1-\cos^2\theta\right) F_{LLT,U}^{L} + (\sin 2\theta\cos\phi) F_{LLT,U}^{\cos\phi}\right.\right.$$

$$+\left(\sin^2\theta\cos 2\phi\right)F^{\cos 2\phi}_{LLT,U}\Big)+\cos\phi_{LLT}\left(\left(\sin^2\theta\sin 2\phi\right)F^{\sin 2\phi}_{LLT,U}+(\sin 2\theta\sin\phi)F^{\sin\phi}_{LLT,U}\right)\Big]$$
$$+|S_{LTT}|\Big[\sin 2\phi_{LTT}\Big(\left(1+\cos^2\theta\right)F^{T}_{LTT,U}+\left(1-\cos^2\theta\right)F^{L}_{LTT,U}+(\sin 2\theta\cos\phi)F^{\cos\phi}_{LTT,U}$$
$$+\left(\sin^2\theta\cos 2\phi\right)F^{\cos 2\phi}_{LTT,U}\Big)+\cos 2\phi_{LTT}\left(\left(\sin^2\theta\sin 2\phi\right)F^{\sin 2\phi}_{LTT,U}+(\sin 2\theta\sin\phi)F^{\sin\phi}_{LTT,U}\right)\Big]$$
$$+|S_{TTT}|\Big[\sin 3\phi_{TTT}\Big(\left(1+\cos^2\theta\right)F^{T}_{TTT,U}+\left(1-\cos^2\theta\right)F^{L}_{TTT,U}+(\sin 2\theta\cos\phi)F^{\cos\phi}_{TTT,U}$$
$$+\left(\sin^2\theta\cos 2\phi\right)F^{\cos 2\phi}_{TTT,U}\Big)+\cos 3\phi_{TTT}\left(\left(\sin^2\theta\sin 2\phi\right)F^{\sin 2\phi}_{TTT,U}+(\sin 2\theta\sin\phi)F^{\sin\phi}_{TTT,U}\right)\Big]\Bigg\}, \quad (19)$$

where the two subscripts for each structure function represent the polarization states of the two hadrons, Ω and h, respectively. The superscripts either label the azimuthal modulation or represent the virtual photon polarization. The F's, the scalar functions, are linear combinations of the V's in Eq. (18). Among them, the 20 rank-3 tensor polarized ones are newly defined and only exist when the spin of the detected hadron is no less than $s \geq 3/2$.

Then we calculate the structure functions in the parton model and consider the kinematic region $\boldsymbol{q}_T^2 \ll Q^2$, where one can apply the TMD factorization. At the leading order, the hadronic tensor can be expressed as a convolution of two correlation functions,

$$W^{\mu\nu}=N_c z_1 z_2\sum_q e_q^2\int d^2\boldsymbol{k}_{1T}d^2\boldsymbol{k}_{2T}\delta^{(2)}(\boldsymbol{k}_{1T}+\boldsymbol{k}_{2T}-\boldsymbol{q}_T)\mathrm{Tr}[\Delta^{\Omega/q}\gamma^\mu\Delta^{h/\bar{q}}\gamma^\nu], \quad (20)$$

where $\boldsymbol{k}_{1T}$ and $\boldsymbol{k}_{2T}$ are defined in the c.m. frame and directly correspond to the quark transverse momentum in the definition of the correlation function.

After contracting the hadronic tensor and the leptonic tensor, one can obtain the structure functions in terms of the convolutions of two TMD FFs. For conciseness, we introduce the transverse momentum convolution notation

$$C\left[w_a(k_{1T},k_{2T})D(z_1,k_{1T}^2)D(z_2,k_{2T}^2)\right]$$
$$\equiv\frac{1}{4}N_c z_1 z_2\sum_q e_q^2\int d^2\boldsymbol{k}_{1T}d^2\boldsymbol{k}_{2T}\delta^{(2)}(\boldsymbol{k}_{1T}+\boldsymbol{k}_{2T}-\boldsymbol{q}_T)w_a(k_{1T},k_{2T})D_q(z_1,k_{1T}^2)D_{\bar{q}}(z_2,k_{2T}^2), \quad (21)$$

where $D_q(z_1,k_{1T}^2)$ is a TMD FF for the first hadron Ω, $D_{\bar{q}}(z_2,k_{2T}^2)$ is a TMD FF for the second hadron h, and $w_a(k_{1T},k_{2T})$, with $a=1,\cdots,10$, is one of the dimensionless scalar functions as provided below,

$$w_1=-\frac{\hat{q}_T\cdot k_{1T}}{M_1},\quad w_2=-\frac{\hat{q}_T\cdot k_{2T}}{M_2},\quad w_3=\frac{2(\hat{q}_T\cdot k_{1T})(\hat{q}_T\cdot k_{2T})+k_{1T}\cdot k_{2T}}{M_1M_2},$$
$$w_4=\frac{k_{1T}^{ij}\hat{q}_{Ti}k_{2Tj}+2k_{1T}^{ij}\hat{q}_{Ti}\hat{q}_{Tj}(\hat{q}_T\cdot k_{2T})}{M_1^2M_2},\quad w_5=\frac{2k_{1T}^{ij}\hat{q}_{Ti}\hat{q}_{Tj}}{M_1^2},$$
$$w_6=\frac{2\left[k_{1T}^{ijl}\hat{q}_{Ti}\hat{q}_{Tj}k_{2Tl}+2k_{1T}^{ijl}\hat{q}_{Ti}\hat{q}_{Tj}\hat{q}_{Tl}(k_{2T}\cdot\hat{q}_T)\right]}{M_1^3M_2},\quad w_7=-\frac{k_{1T}\cdot k_{2T}}{M_1M_2},\quad w_8=\frac{4k_{1T}^{ijl}\hat{q}_{Ti}\hat{q}_{Tj}\hat{q}_{Tl}}{M_1^3},$$
$$w_9=\frac{4\left[k_{1T}^{ijlm}\hat{q}_{Ti}\hat{q}_{Tj}\hat{q}_{Tl}k_{2Tm}+2k_{1T}^{ijlm}\hat{q}_{Ti}\hat{q}_{Tj}\hat{q}_{Tl}\hat{q}_{Tm}(k_{2T}\cdot\hat{q}_T)\right]}{M_1^4M_2},\quad w_{10}=\frac{2k_{1T}^{ij}\hat{q}_{Ti}k_{2Tj}}{M_1^2M_2}, \quad (22)$$

where $\hat{q}_T^\mu \equiv q_T^\mu/\sqrt{\boldsymbol{q}_T^2}$ is the direction of the virtual photon transverse momentum in the c.m. frame.

At leading twist, 24 structure functions have nontrivial expressions. Here we only list the nonzero structure functions for rank-3 tensor polarized hadron states,

$$F_{LLL,U}^{\sin 2\phi} = -C\left[w_3 H_{1LLL}^{\perp}(z_1,k_{1T}^2)H_1^{\perp}(z_2,k_{2T}^2)\right], \tag{23}$$

$$F_{LLT,U}^{T} = C\left[w_1 D_{1LLT}^{\perp}(z_1,k_{1T}^2)D_1(z_2,k_{2T}^2)\right], \tag{24}$$

$$F_{LLT,U}^{\sin(2\phi+\phi_{LLT})} = \frac{1}{2}\left(F_{LLT,U}^{\cos 2\phi}+F_{LLT,U}^{\sin 2\phi}\right) = C\left[w_4 H_{1LLT}^{\perp}(z_1,k_{1T}^2)H_1^{\perp}(z_2,k_{2T}^2)\right], \tag{25}$$

$$F_{LLT,U}^{\sin(2\phi-\phi_{LLT})} = \frac{1}{2}\left(F_{LLT,U}^{\sin 2\phi}-F_{LLT,U}^{\cos 2\phi}\right) = -C\left[w_2 H_{1LLT}(z_1,k_{1T}^2)H_1^{\perp}(z_2,k_{2T}^2)\right], \tag{26}$$

$$F_{LTT,U}^{T} = -C\left[w_5 D_{1LTT}^{\perp}(z_1,k_{1T}^2)D_1(z_2,k_{2T}^2)\right], \tag{27}$$

$$F_{LTT,U}^{\sin(2\phi+2\phi_{LTT})} = \frac{1}{2}\left(F_{LTT,U}^{\cos 2\phi}+F_{LTT,U}^{\sin 2\phi}\right) = -C\left[w_6 H_{1LTT}^{\perp\perp}(z_1,k_{1T}^2)H_1^{\perp}(z_2,k_{2T}^2)\right], \tag{28}$$

$$F_{LTT,U}^{\sin(2\phi-2\phi_{LTT})} = \frac{1}{2}\left(F_{LTT,U}^{\sin 2\phi}-F_{LTT,U}^{\cos 2\phi}\right) = C\left[w_7 H_{1LTT}^{\perp}(z_1,k_{1T}^2)H_1^{\perp}(z_2,k_{2T}^2)\right], \tag{29}$$

$$F_{TTT,U}^{T} = -C\left[w_8 D_{1TTT}^{\perp}(z_1,k_{1T}^2)D_1(z_2,k_{2T}^2)\right], \tag{30}$$

$$F_{TTT,U}^{\sin(2\phi+3\phi_{TTT})} = \frac{1}{2}\left(F_{TTT,U}^{\cos 2\phi}+F_{TTT,U}^{\sin 2\phi}\right) = -C\left[w_9 H_{1TTT}^{\perp\perp}(z_1,k_{1T}^2)H_1^{\perp}(z_2,k_{2T}^2)\right], \tag{31}$$

$$F_{TTT,U}^{\sin(2\phi-3\phi_{TTT})} = \frac{1}{2}\left(F_{TTT,U}^{\sin 2\phi}-F_{TTT,U}^{\cos 2\phi}\right) = -C\left[w_{10} H_{1TTT}^{\perp}(z_1,k_{1T}^2)H_1^{\perp}(z_2,k_{2T}^2)\right], \tag{32}$$

which can be utilized to study the TMD FFs for a spin-3/2 hadron, although the other 24 structure functions in the unpolarized differential cross section only arise at high twist or high order.

5. Production of spin-3/2 hadrons in SIDIS

We denote the production of Ω in SIDIS,

$$e^-(l) + N(P) \rightarrow e^-(l') + \Omega(P_h) + X(P_X), \tag{33}$$

where the variables in parentheses indicate the four momenta of the corresponding particles. For this process, we follow the similar procedure in the calculation of e^+e^- annihilation. Here we take into account all possible combinations of the polarization states of the nucleon, the lepton, and the produced spin-3/2 hadron. The cross section for the SIDIS process can also be written as the contraction of leptonic tensor and hadronic tensor. The symmetric part of the hadronic tensor contributes to the cross section for the unpolarized lepton beam, whereas the antisymmetric part of the hadronic tensor is also necessary to be taken into account for the polarized lepton beam. With the constraints of the properties of the hadronic tensor, we can construct six symmetric and three antisymmetric basic Lorentz tensors,

$$t_U^{S\mu\nu} = \left\{\tilde{g}^{\mu\nu}, \tilde{P}^\mu\tilde{P}^\nu, \tilde{P}^{\{\mu}\tilde{P}_h^{\nu\}}, \tilde{P}_h^\mu\tilde{P}_h^\nu\right\}, \tag{34}$$

$$\tilde{t}_U^{S\mathcal{P},\mu\nu} = \left\{\epsilon^{\{\mu qPP_h}\tilde{P}^{\nu\}}, \epsilon^{\{\mu qPP_h}\tilde{P}_h^{\nu\}}\right\}, \tag{35}$$

$$t_U^{A\mu\nu} = \left\{\tilde{P}^{[\mu}\tilde{P}_h^{\nu]}\right\}, \tag{36}$$

$$\tilde{t}_U^{A\mathcal{P},\mu\nu} = \left\{\epsilon^{\mu\nu qP}, \epsilon^{\mu\nu qP_h}\right\}, \tag{37}$$

where the superscripts S and A represent the symmetric terms and antisymmetric terms, respectively. To construct the polarized basis tensors, we still use these basic Lorentz tensors multiplied by spin-dependent scalars or pseudoscalars. Summing over all symmetric and antisymmetric terms, one can obtain the complete expression of hadronic tensor

$$W^{\mu\nu} = \sum_{i=1}^{192} V_i^S t_i^{S\mu\nu} + i \sum_{i=1}^{96} V_i^A t_i^{A\mu\nu}. \tag{38}$$

After contracting the hadronic tensor and leptonic tensor, we can express the differential cross section in terms of 288 structure functions in accordance to the azimuthal distributions and all polarization configurations. We then perform a leading order calculation in the parton model. For an unpolarized lepton beam, half of the 192 structure functions have nonzero leading order contributions in the parton model, among which 42 are from rank-3 tensor polarized fragmentation functions of the hadron. For a polarized lepton beam, one third of the 96 structure functions contribute at the leading order and 14 of them are from rank-3 tensor polarized fragmentation functions. The complete results have been given in Ref. [14].

The measurement of these nonzero structure functions, particularly those for rank-3 tensor polarized states, can be utilized to study the TMD FFs for a spin-3/2 hadron, although the other structure functions in the differential cross section only arise at high twist or high order.

6. Summary and outlook

We use the spin density matrix to describe the spin states of spin-3/2 hadrons. At leading twist, there are 32 TMD FFs to spin-3/2 hadrons defined by the parametrization of the quark-quark correlation function. Through the kinematic analysis, one can obtain the general expression of differential cross section in terms of structure functions. Applying the TMD factorization, we perform the leading order calculations to express the structure functions in terms of the TMD PDFs and TMD FFs in the parton model. For two-hadron production in e^+e^- annihilation, half of 48 structure functions contribute at leading twist and ten of them are from rank-3 tensor polarized FFs. For semi-inclusive production of spin-3/2 hadrons in DIS, the complete differential cross section is expressed in terms of 288 structure functions. For an unpolarized lepton beam, half of the 192 structure functions have nontrivial expressions in the parton model, among which 42 are for rank-3 tensor polarized states. For a polarized lepton beam, one third of the 96 structure functions contribute at the leading order and 14 of them are for rank-3 tensor polarized states. In the future, the Belle II experiment with 40 times higher luminosity than the Belle experiment is expected to produce enough Ω events for polarization analysis and makes it possible to extract the rank-3 tensor polarized FFs. In addition, the production of spin-3/2 hadrons can be measured in future experiments, such as EIC and EicC, and will provide valuable insights into the study of nucleon structures.

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