PROCEEDINGS OF SCIENCE

Properties of the Newly Discovered Exotic Particles in Diquark Model

Aparajita Bhattacharya,[a,*] Ballari Chakrabarti,[b] Papiya Dhara,[a] and Shukla Pal [c]

[a] *Department of Physics*
Adamas University, Kolkata, India.
[b] *Department of Physics*
Jogamaya Devi College, Kolkata, India.
[c] *Department of Physics*
Basanti Devi College, Kolkata, India.
E-mail: aparajita.bhattacharya@adamasuniversity.ac.in, ballari_chakrabarti@yahoo.co.in, papiyadharaismpolito@gmail.com, shuklaacharya123@gmail.com

Recently CERN has discovered three new exotic particles in LHCb. The particles are T_{cso} $(2900)^{++}$, a doubly charged tetraquark with quark configuration (csu$\overline{d}$), T_{cso} $(2900)^0$, a neutral tetraquark with quark configuration (cds$\overline{u}$) and a neutral pentaquark $P_{\psi s}$ $(4338)^0$ with quark configuration (c$\overline{c}$ uds). $P_{\psi s}$ $(4338)^0$ is the first observed pentaquark with a strange quark in its configuration. The discovery of these subatomic particles are exciting and the nature of interaction of the constituent quarks are need to be understood for these multiquark states. Diquark is one of the most important candidate for studying the structure and dynamics of exotic states. In the current work we have studied the properties of these particles in the frame work of diquark correlation. Diquark is a hypothetical coloured antisymmetric correlation of two quarks with spin 0 (scalar) or spin 1 (vector). We have suggested two models for diquarks. The composite fermion model and effective mass approximation model for diquark have been used to estimate the masses of T_{cso} $(2900)^{++}$, T_{cso} $(2900)^0$ and $P_{\psi s}$ $(4338)^0$. Results are found to be in good agreement with the experimental masses observed by CERN. We have also studied the higher state masses of the particles in the framework of the flux tube model which may be discovered in future. The Regge trajectories are plotted. The form factor of these three exotics are also studied and plotted with different values of momentum transfer Q^2.

25th International Spin Physics Symposium (SPIN 2023)
24-29 September 2023
Durham, NC, USA

∗ Speaker

PoS(SPIN2023)166

1. Introduction

The aim of particle physics is to search the ultimate structure of nature. Gellman [1] suggested the quark model for hadrons which are colourless bound state of quarks in the form of three quark system qqq which are baryons or quark-antiquark system called mesons $q\overline{q}$. Multiquark states containing more than three quarks are of special importance in the hadron family. Exotic states have been observed by different collaborations [2-11]. In 2022 LHCb has reported [12] a pentaquark which contains a strange quark $[c\bar{c}uds]$ with mass M $= 4338.2 \pm 0.7 \pm 0.4$ MeV and decay width $\Gamma = 7.0 \pm 1.2 \pm 1.3$ MeV and a doubly electrically charged tetraquark which a tetra quark states with four different flavors. It is an open-charm tetraquark $[c\bar{s}u\bar{d}]$ and its partner $[cd\bar{s}\bar{u}]$. Mass is observed as M $= 2.908 \pm 0.011 \pm 0.02$ GeV with its decay width $\Gamma = 0.136 \pm 0.023 \pm 0.013$ GeV. At theoretical level multiquark states are investigated by a number of authors [13-17]. In the current work we have studied the ground state masses of the tetraquarks $T^{++}_{cs}(2900)$, $T^{0}_{cs}(2900)$ and pentaquark $P^{0}_{\psi s}(4338)$ in the framework of diquark-anti diquark configuration and the pentaquark as diquark-diquark-antiquark configuration. We have suggested models of diquarks in our previous works [18,19] to compute diquark masses. We have used the diquark masses computed in effective mass approximation [18] and composite fermion model [19]. The higher state masses are investigated using the flux tube model [20]. The Regge trajectories are studied. The form factors of these exotic states for momentum transfers Q^2 in the range of $0 < Q^2 < 0.1$ GeV^2 have also been studied.

2. Exotic masses in Effective Mass Approximation of diquark

An elementary particle in vacuum may be suggested to be in a situation exactly resembling that of an electron in a crystal [21]. We have proposed a similar type of picture for the diquark [ud] as a quasi particle inside a hadron. We have considered that the diquark is an independent body which is under the influence of two types of forces like background meson cloud which is represented by potential $V = \frac{2}{3}\frac{\alpha_s}{r}$, ($\alpha_s$ is strong coupling constant) resembling the crystal field whereas for the external force we have considered an average force $F = -ar$,('a' is constant) which is confinement force.

Under the influence of these two types of interactions the diquark behaves like a quasiparticle (color antisymmetric), a low lying elementary excited state simulating many body interaction and its mass gets modified. The ratio of the constituent mass and the effective mass of the diquark m_D can be expressed as [18] :

$$\frac{m_{q_1} + m_{q_2}}{m_D} = 1 + \frac{\alpha}{2ar^3} \tag{1}$$

The diquark has been considered to be independent body behaving as quasiparticle analogous to the many body systems which resembles the low lying excitation and they are non interacting entities and as in the case of ideal gas of quasiparticle, their energies are assumed to be additive [22] and their effective mass characterizes the dynamic properties of the quasi particles. Hence the mass of the tetraquark and pentaquark can be written as:

$$M_x = m_{D_1} + \overline{m}_{D_2} \tag{2}$$

$$M_x = m_{D_1} + m_{D_2} + m_{\overline{q}} \tag{3}$$

The masses of tetra and pentaquarks are estimated and furnished in Table 1 for both scalar and vector diquark constituents.

2.1 Masses of the exotics in Composite Fermion Model of diquark

We have also suggested the composite fermion model of diquark in an analogy with the Composite Fermion (CF) picture in the usual Quantum Hall effect. Diquarks are suggested to behave a like a Composite Fermion in the presence of the chromo-magnetic behaviour of QCD vacuum. The effective mass of the Composite Fermion can be determined in a gauge invariant way and the leading contribution comes in the limit $\frac{\Lambda}{p_F}$, where Λ is cut off parameter and p_F is the Fermi momentum of the Composite Fermion. Chari et al [23] have proposed an effective mass of quasiparticle in a gauge invariant way and expressed it in terms of response function of the system in analogy with Landau's picture of quasiparticle of Fermi liquid. The effective mass m_D^* of the diquark can be expressed as [19]:

$$\frac{1}{m_D^*} = \frac{1}{(m_q + m_{q'})}(1 + \frac{\Lambda^4}{2p_F^4}) \tag{4}$$

The Fermi momentum of mesons of different flavours of diquark have been computed using the work of Bhattacharya et al [24] with the input of diquark radius [25-26] and the masses of diquarks are computed using relation (6).

With $m_u = m_d = 0.360$ GeV, $m_s = 0.540$ GeV, $m_c = 1.71$ GeV, $m_b = 5.05$ GeV , $\Lambda = 0.573$ GeV for light sector [27] and 0.6533 GeV for heavy sectors [28], we have estimated the masses of scalar and vector diquarks of different flavours in our previous work [29]. The mass formula for the tetraquark state with the relevant diquark-antidiquark configuration with binding energy and spin interaction runs as:

$$M = m_D + m_{\bar{D}} + E_{BE} + E_S \tag{5}$$

where m_D and $m_{\bar{D}}$ are the diquark and antidiquark masses respectively. Similarly the mass formula for the pentaquark will be,

$$M = m_D + m_{\bar{D}} + m_{\bar{Q}} + E_{BE} + E_S \tag{6}$$

where, E_{BE} and E_S are the binding energies and their spin contribution respectively [19]. The ground state masses of the tetraquarks $T_{cs}^{++}(2900)$, $T_{cs}^{0}(2900)$ and the pentaquark $P_{\psi s}^{0}(4338)$ have been computed using above equation. The masses in both formalisms have been displayed in Table 1.

	EMA Model		CF Model		Expt. Mass in GeV
Particle	Scalar mass in GeV	Vector mass in GeV	Scalar mass in GeV	Vector mass in GeV	
$T_{cso}(2900)^{++}$ $[c\bar{s}][u\bar{d}]$	3.0397	3.1407	2.0379	2.5565	2.908 ±0.011 [11]
$T_{cso}(2900)^{0}$ $[c\bar{u}][d\bar{s}]$	3.0261	3.0845	2.0379	2.5565	2.908 ±0.011 [11]
$P_{\psi s}(4338)^{0}$					
$[cs][ud]\bar{c}$	4.749	4.8507	3.5311	4.3014	4.338 ±0.0007[11]
$[uc][ds]\bar{c}$	4.7361	4.7945	3.3545	4.0065	

Table 1: Ground state masses of exotic hadrons computed in Effective Approximation Model and Composite Fermion Model.

2.2 Higher state masses of the exotic hadrons in the context of the Flux-Tube Model

Mass Loaded flux Tube model has been suggested by Selem and Wilczek [20] where two masses are connected by a relativistic string of string tension σ, rotating with angular momentum L. Considering the exotic tetraquark in the diquark-antidiquark picture in the context of flux tube model, the system can be represented as the heavy diquark at rest and the other diquark connected to it by flux tube which is in constant rotation with the heavy diquark. The mass formula for the higher states of the multiquak states can be written as:

$$E = m_D + \sqrt{\frac{\sigma L}{2}} + 2^{\frac{1}{4}} K L^{-\frac{5}{4}} \mu_D^{\frac{3}{2}} + a\vec{L} \cdot \vec{S} \tag{7}$$

where μ_D is the mass of the diquark which is connected by the flux tube and $K = \frac{2\pi^{\frac{1}{2}}}{3\sigma^{\frac{1}{4}}}$, L represents the orbital angular momentum of the diquark and S is the total spin. $\vec{L} \cdot \vec{S}$ is the spin-orbit interaction. The mass formula for the pentaquark can be expressed as,

$$E = M_Q + \sqrt{\frac{\sigma L}{2}} + 2^{\frac{1}{4}} K L^{-\frac{5}{4}} \mu_D^{\frac{3}{2}} + a\vec{L} \cdot \vec{S} \tag{8}$$

where M_Q is the heavy antiquark mass, $\mu_D^{\frac{3}{2}} = m_{D_1}^{\frac{3}{2}} + m_{D_2}^{\frac{3}{2}}$, m_{D_1}, m_{D_2} are the effective masses of the two diquarks estimated in the framework of the CF model. The orbital excitations of the particle can be described by the Regge trajectories and has been expressed by:

$$E^2 = \alpha J \tag{9}$$

where E is the energy at the higher state for different values of L and J is the total angular momentum and Regge slope is (α). The energies of the heavy tetraquark $T_{cs}^{++}(2900)$ and the pentaquark $P_{\psi s}^0(4338)$ have been estimated for different values of L. The input values are $\sigma = 0.999 GeV^2$ and $a = 0.04 GeV$. The higher state masses of the tetraquark $T_{cs}^{++}(2900)$ and the pentaquark $P_{\psi s}^0(4338)$ have been displayed in Table 2 and Table 3 respectively. The variations of E^2 versus L for the tetraquark is displayed in Figure 1(i) and the pentaquark in Figure 1(ii) for quark combinations of $[cs][ud]\bar{c}$ and $[uc][ds]\bar{c}$ respectively. The value of the Regge slopes α have been extracted from the graphs. The ground state masses of the tetraqurks and pentaquarks are also extracted from the Regge trajectories of the particles at $L = 0$.

PoS(SPIN2023)166

Particle /Quark content	Value of L	Mass in GeV for S=0 (J^P)	Mass in GeV for S=1 (J^P)	Mass in GeV for S=2 (J^P)	Mass in GeV from graph at L=0	α in GeV^2
T_{cs}^{++}	1	2.0102 (1^-)	2.5396 (2^-)	2.8310 (3^-)		
	2	2.2471 (2^+)	2.8165 (3^+)	3.1021 (4^+)		
$[uc][\bar{s}\bar{d}]$	3	2.4397 (3^-)	3.0491 (4^-)	3.3549 (5^-)	2.5534	1.73
	4	2.6083 (4^+)	3.2577 (5^+)	3.5906 (6^+)		

Table 2: Exotic tetraquark higher state masses with scalar and vector diquark-antidiquark.

2.3 Form factors of the exotic hadrons

In estimating the form factor of the multiquark state we have used the wave function for hadrons derived in the context of Statistical Model [30]. The expression for the form factor is obtained as:

$$F(Q^2) = 48(Qr_{12})^{-3}J_3(Qr_{12}) \tag{10}$$

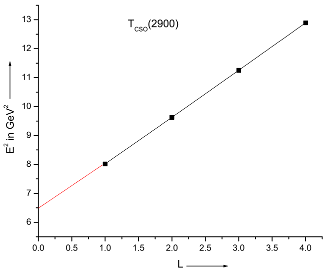
where $J_3(Qr_{12})$ is the Bessel function of the first kind. The radii of the scalar and vector diquarks depending on their spins have been given input from the existing literature [25,26]. The form factors of the tetraquark $T_{cs}^{++}(2900)$ and the pentaquark $P_{\psi s}^0(4338)$ comprising the scalar and vector diquark-antidiquark for different values of Q^2 are estimated in the range $0.01 < Q^2 < 0.1\ GeV^2$ and displayed in Figure 1(iii), and 1(iv) respectively.

3. Conclusion

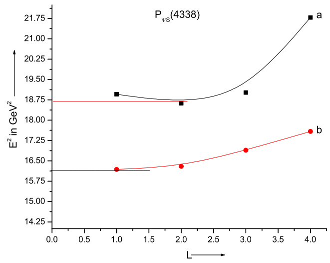
In the present work we have invsestigated the masses of the recently discovered exotic tetraquarks $T_{cs}^{++}(2900)$, $T_{cs}^{0}(2900)$ and the pentaquark $P_{\psi s}^0(4338)$ in diquark framework with quasi-particle model and composite fermion model of diquarks. The results have been displayed in Table 1. The results are reasonably in good agreement with the experimental values with a variation of approximately $300 MeV$. Ground state mass of pentaquarks are extracted as 4.318 GeV and slope as 0.54 from graphs. Regge slopes show a non-linear behaviour and found to be less than the universal accepted value ~ 1 GeV^2 [31]. The form factor variations of $T_{cs}^{++}(2900)$ and $P_{\psi s}^0(4338)$ have been displayed in the range of $0.01 < Q^2 < 0.1$ GeV^2 in the Figures 1(iii) and 1(iv). The form factors are found to be falling sharply in the low range indicating a clear violation of scaling at lower momentum transfers. The effect may be attributed to the interaction of quarks by the exchange of gluons. From the current investigation it is observed that the diquark combination reproduces the masses of these states very well and diqurk is a potential candidate to describe multi-quark states. However, more experimental attempts will enlighten us to understand the spectroscopy, the structure and dynamics of these exotic systems.

Value of L	J^P	Mass in GeV $P_{\psi s}^0$ $[cs][ud]\bar{c}$	Mass in GeV $P_{\psi s}^0$ $[uc][ds]\bar{c}$
1	$\frac{1}{2}^-$	4.3539	4.0231
	$\frac{3}{2}^-$	4.5657	4.3076
		6.015	5.1725
2	$\frac{3}{2}^+$	4.3153	4.0371
	$\frac{5}{2}^+$	4.5397	4.3227
		5.7592	5.0499
3	$\frac{5}{2}^-$	4.3612	4.1098
	$\frac{7}{2}^-$	4.6118	4.4156
		5.7129	5.0728
4	$\frac{7}{2}^+$	4.6674	4.1934
	$\frac{9}{2}^+$	4.7089	4.5264
		5.7737	5.1379

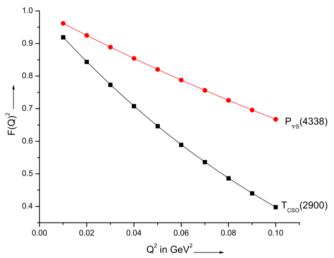
Table 3: Higher state masses of exotic pentaquark.



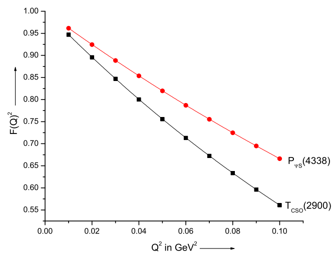
(i)



(ii)



(iii)



(iv)

Figure 1: (i) Variation of E^2 versus L for the exotic tetraquark T_{cs}^{++}. (ii)(a) Variation of E^2 versus L for the exotic pentaquark $P_{\psi s}^0$ $[cs][ud]\bar{c}$, (b) Variation of E^2 versus L for the exotic pentaquark $P_{\psi s}^0$ $[uc][ds]\bar{c}$ (iii) Variation of the Form factor with Q^2 in the range $0.01 < Q^2 < 0.1\ GeV^2$ for the exotics with scalar diquarks. (iv) Variation of the Form factor with Q^2 in the range $0.01 < Q^2 < 0.1\ GeV^2$ for the exotics with vector diquarks.

References

[1] M. Gellman, "A Schematic model of Baryons and Mesons",Phys. Letts. **8** (1964) 214.

[2] S. K. Choi et al. [Belle Collaboration], "Observation of a Narrow Charmonium like State in Exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ Decays", Phys. Rev. Lett. **91** (2003) 262001.

[3] K. Abe et al. [Belle Collaboration], "Observation of a Near-Threshold $\omega J/\psi$ Mass Enhancement in Exclusive B $\to$ K$\omega J/\psi$ Decays", Phys. Rev. Lett. **94** (2005) 182002.

[4] T. Aaltoneon et al.[CDF Collaboration], "Evidence for a Narrow Near-Threshold Structure in the $J/\psi\phi$ Mass Spectrum in $B^+ \to J/\psi\phi K^+$ Decays", Phys. Rev. Lett. **102** (2009) 242002.
[5] S. K. Choi et al. [Belle Collaboration], "Observation of a Resonance like Structure in the $\pi^{\pm}\psi'$ Mass Distribution in Exclusive $B \to K\pi^{\pm}\psi'$ Decays",Phys. Rev. Lett. **100** (2008) 142001.
[6] K. Chilikin et al. [Belle Collaboration], "Observation of a new charged charmonium like state in $\bar{B}^0 \to J/\psi K^-\pi^+$ decays", Phys. Rev. D **90** (2014) 112009.
[7] R. Aaij et al. [LHCb collaboration], "Observation of $J/\psi\phi$ Structures Consistent with Exotic States from Amplitude Analysis of $B^+ \to J/\psi\phi K^+$ Decays", Phys. Rev. Lett. **118** (2017) 022003. [8] R. Aaij et al. [LHCb Collaboration], "Observation of structure in the J/ψ-pair mass spectrum", Science Bulletin **65** (2020) 1983.
[9] A. Bonder et al. [Belle Collaboration], "Observation of Two Charged Bottomonium like Resonances in $\Upsilon(5S)$ Decays", Phys. Rev. Lett. **108** (2012) 122001.
[10] R. Aaij et al. [LHCb collaboration], "Observation of $J/\psi\phi$ Structures Consistent with Exotic States from Amplitude Analysis of $B^+ \to J/\psi\phi K^+$ Decays", Phys. Rev. Lett. **118** (2017) 022003.
[11] R. Aaij et al. [LHCb collaboration],' Observation of narrow pentaquark states Pc$(4312)^+$ and of the two peak structure of Pc$(4450)^+$',Phys. Rev. Lett. **122** (2019) 222001.
[12] C. Chen., 'Particle Zoo 2.0: New tetra- and pentaquarks at LHCb', LHCb Report no.LHCb-PAPER-2022-031 (2022),
https://indico.cern.ch/event/1176505.
[13] M. Karliner and J. L. Rosner, 'New strange pentaquarks'., Phys. Rev. D **106** (2022) 036024.
[14] W. Wilcox and S. Baral., 'Multiquark states in the Thomas Fermi quark model and on the lattice' Int. J. Mod. Phys. A **34** (2019) 1941008.
[15] U. Ozdem et al., 'Investigation of magnetic moment of Pcs(4338) and Pcs(4459) pentaquark states' Phys. Lett. B **836** (2023) 137635.
[16] S. Pal, B. chakrabarti and A. Bhattacharya, 'Mass of the charm tetraquark in diquark antidiquark and di-hadronic states approach', Eur. Phys. J. Plus **136** (2021) 625.
[17] P. Bicudo,'Tetraquarks and pentaquarks in lattice QCD with light and heavy quarks', Phys. Rep **1039** (2023) 1.
[18] R.Ghosh, A.Bhattacharya,B.Chakrabarti,"A study on $P_c^*(4380)$ and P_c^* in the quasi particle diquark model", Phys. Part. Nuclei Lett.**14** (2017)

550552.
[19] S. Pal, B. Chakrabarti, A. Bhattacharya., "A theoretical investigation on the spectroscopy and structure of the exotic tetraquark states", Nucl. Phys. A **1029** (2023) 122559.
[20] A. Selem and F. Wilczek, 'Hadron Systematic and Emergent Diquarks', New Trends in HERA Physics, (2006) 337.
[21] A. Haug, "Theoretical Solid State Physics", Pergamon Press, Oxford, p-100 (1975).
[22] E. M. Lifshitz and L. P. Pitaevskii, 'Statistical Physics', Vol 9, Butterworth-Heinemann (2005).
[23] A. R. Chari et al., 'A Gauge Invariant Way to Evaluate Quasi particle Effective Mass in Fermionic Systems with Gauge Interactions', arxiv:cond-mat/9707055V1 (1997).
[24] A. Bhattacharya et al.,'On some properties of the ratio of matrix elements related to the Fermi momentum', Eur. Phys. J. C **2** (1998) 671.
[25] A. S. de Castro et al., 'A diquark model for baryon spectroscopy' Z. Phys. C **57** (1993) 315.
[26] K. Nagata and A. Hosaka, 'Structure of the Nucleon and Roper Resonance with Diquark Correlations', Mod. Phys. Lett. A **23** (2008) 2397.
[27] S. Pepin et al., 'Nucleons or diquarks: Competition between clustering and color superconductivity in quark matter.', Phys. Rev. D **61** (2000) 055209.
[28] M. Huang et al., 'Charge neutrality effects on two-flavor color superconductivity', Phys. Rev. D **67** (2003) 065015.
[29] S. Pal et al.,'A theoretical investigation on the spectroscopy and structure of the exotic tetraquark states', Nucl. Phys. A **1029** (2023) 122559.
[30] S. Bisht, N. Hothi and G. Bhakuni, 'Phenomenological Analysis of Hadronic Regge Trajectories' Elect. J. Theo. Phys. **7** (2010) 299.

PoS(SPIN2023)166