

## Extraction of the strong coupling with HERA and EIC inclusive data

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We present a study of the investigation into the sensitivity of the strong coupling  $\alpha_S(M_Z^2)$  using existing Deep Inelastic Scattering data from HERA and anticipated future measurements from the Electron Ion Collider (EIC) in a next-to-next-to-leading order QCD analysis. By combining simulated inclusive neutral current EIC data with HERA measurements, including charged and neutral current data, and optionally adding HERA inclusive jet and dijet data, the study aims to achieve a potentially world-leading level of precision. The results, obtainable with less than a year of projected EIC data at the lower end of the energy range, raise questions about uncertainties related to missing higher orders in the theoretical framework.

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## 1. Introduction

The strong coupling constant,  $\alpha_s$ , is a key parameter in the theory of Quantum Chromodynamics (QCD), governing the strength of the strong force between quarks and gluons. Precision determinations of  $\alpha_s$  are essential for testing the consistency of the Standard Model and probing for deviations that may indicate new physics phenomena, yet  $\alpha_s$  stands out as the least precisely determined coupling constant among all the fundamental forces. Recent studies in Deep Inelastic Scattering (DIS) at HERA have shown limited sensitivity to  $\alpha_s$  with only inclusive data but significantly improved precision when incorporating jet production cross sections. Notably, further improvements in QCD theory from next-to-leading order (NLO) to next-to-next-to-leading order (NNLO) have substantially reduced uncertainties in  $\alpha_s$  extractions. However, these uncertainties, often expressed as a QCD scale uncertainty, remain as the predominant source of uncertainty in the most accurate HERA extractions, despite notable progress in recent years. The extensive Electron Ion Collider (EIC) physics program, as detailed in [1], will include high-precision measurements of inclusive DIS cross sections in a phase space region complementary to HERA. This effort aims to enhance sensitivity, particularly in the large Bjorken- $x$  kinematic region. This study estimates the expected experimental uncertainty on the strong coupling at the scale of the Z-pole mass ( $\alpha_s(M_Z^2)$ ) by using simulated inclusive EIC data and HERA data.

This proceeding focuses on recent efforts to precisely determine  $\alpha_s(M_Z^2)$ , based on a published study [2].

## 2. Analysis Method

In this study, the combined H1 and ZEUS inclusive DIS neutral current (NC) and charged current (CC) cross sections, as detailed in [3] are used. Additionally, the H1 and ZEUS inclusive/dijet measurements from a recent study [4] are used. The HERA cross sections are derived from unpolarized beam configurations with proton beam energies at 920, 820, 575, and 460 GeV, alongside an electron beam energy of 27.5 GeV. These data, collected over an integrated luminosity of approximately  $1 \text{ fb}^{-1}$ , cover a broad spectrum, spanning six orders of magnitude in both the four-momentum-transfer squared  $Q^2$ , and in Bjorken  $x$ .

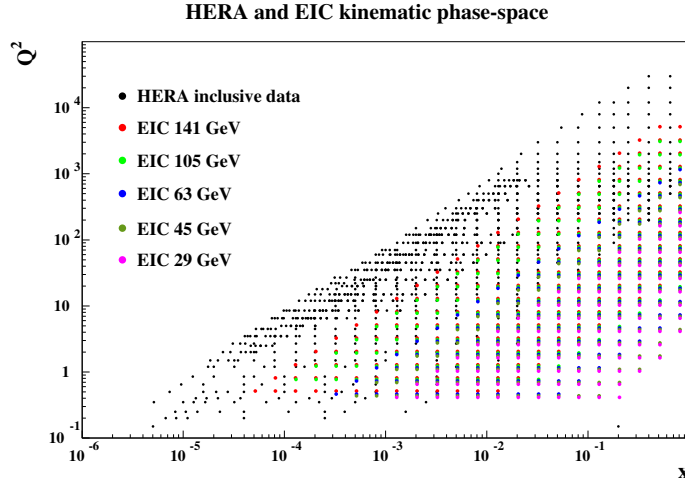
Simulated EIC data used in this study are derived from the ATHENA detector proposal framework. Table 1 summarizes the neutral and charged current EIC pseudodata, reflecting integrated luminosities corresponding to one year of data for five expected beam configurations. The neutral current pseudodata covers a grid of logarithmically-spaced  $x$  and  $Q^2$  values per decade within the range  $0.001 < y < 0.95$ . The inclusive data used in this study is illustrated in Fig. 1, with EIC pseudodata overlapping in their coverage with the HERA data and extending the kinematic reach in the high  $x$ , moderate  $Q^2$  region.

Considering HERA experience and detailed in [1], systematic precision is conservative for modern detector technologies and larger EIC datasets. Most data points show an uncorrelated systematic uncertainty of 1.9%, extending to 2.75% at the lowest  $y$  values. Additionally, a 3.4% normalization uncertainty is applied, fully correlated within each  $\sqrt{s}$  and fully uncorrelated across different  $\sqrt{s}$ . For QCD fits, point-to-point systematic uncertainties are quadratically added to sta-

tistical uncertainties, and normalization uncertainties are treated as nuisance parameters, consistent with [3].

$e$ -beam energy (GeV)	$p$ -beam energy (GeV)	$\sqrt{s}$ (GeV)	Integrated lumi ( $\text{fb}^{-1}$ )
18	275	141	15.4
10	275	105	100
10	100	63	79.0
5	100	45	61.0
5	41	29	4.4

**Table 1:** Beam energies, centre-of-mass energies, and integrated luminosities of the different configurations for the EIC.



**Figure 1:** The kinematic phase-space ( $x, Q^2$ ) distribution for the HERA and EIC inclusive DIS data points. Different colors for EIC represent different  $\sqrt{s}$  values.

The study uses a QCD fitting approach that adheres to the HERAPDF [3] theoretical framework, including choices for PDF parameterizations and model parameter selections. Within this fitting process, simultaneous constraints are imposed on proton PDFs and  $\alpha_S(M_Z^2)$  through a  $\chi^2$  minimization. The evolution of  $Q^2$  is performed following NNLO DGLAP evolution equations [5–14], using QCDNUM[15] within the xFitter framework [16–18]. The study includes the general-mass variable-flavor-number scheme [19, 20] to handle contributions from heavy quarks. Renormalization and factorization scales are defined as  $\mu_r = \mu_f = \sqrt{Q^2}$  for inclusive DIS data, while  $\mu_r^2 = \mu_f^2 = Q^2 + p_T^2$  is adopted for inclusive jet data and  $\mu_r^2 = \mu_f^2 = Q^2 + \langle p_T \rangle_2^2$  for dijets. Quark masses for charm and beauty quarks ( $M_c, M_b$ ) adhere to the choices in [3]. The minimum  $Q^2$  for inclusive data in the fits is set at  $Q_{\min}^2 = 3.5 \text{ GeV}^2$  to avoid complications related to low  $Q^2$  and mitigate potential influences from  $\ln(1/x)$  resummation[21]. Additionally, a cut is applied to the squared hadronic final state invariant mass,  $W^2 = Q^2(1-x)/x > 10 \text{ GeV}^2$ , removing data points likely affected by power-like higher twist or resummation effects, particularly impacting the EIC data sets at the lowest  $\sqrt{s}$ .

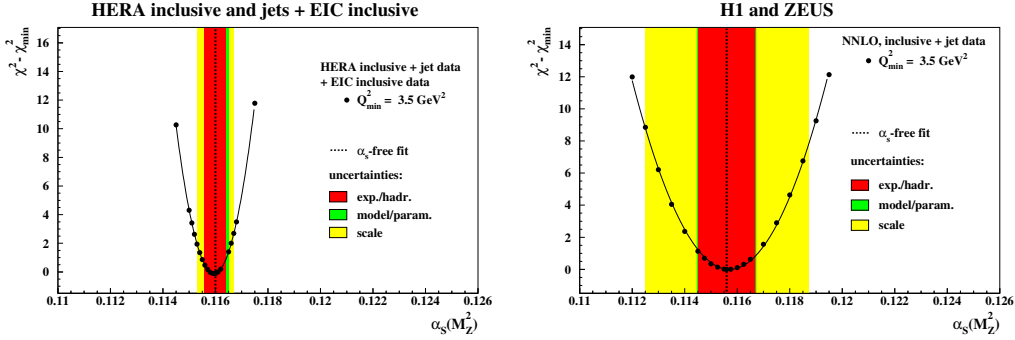
The experimental, model, and parameterisation uncertainties on  $\alpha_s(M_Z^2)$  are evaluated as described in [2]. Fits are iterated with each of the variations, and the largest difference concerning the nominal  $\alpha_s(M_Z^2)$  is considered as the uncertainty. Model and parameterization uncertainties are then combined in quadrature for the final results.

### 3. Results and Conclusions

A simultaneous fitting procedure at NNLO is used to extract the PDFs and  $\alpha_s(M_Z^2)$  from HERA inclusive and jet data, along with the simulated EIC inclusive data at all  $\sqrt{s}$  values. Detailed information about this process is provided in Section 3.2 of [2]. The result of this QCD fitting is

$$\alpha_s(M_Z^2) = 0.1160 \pm 0.0004 \text{ (exp)} \begin{matrix} +0.0003 \\ -0.0002 \end{matrix} \text{ (model + parameterisation)} \pm 0.0005 \text{ (scale)}.$$

The parameters of the PDFs derived from the fitting process align closely with those present in the HERAPDF2.0 set. The results and associated uncertainties, both including and excluding EIC data, are illustrated in a  $\chi^2$  scan of Fig. 2. In this figure, in each point, 14 PDF parameters are fitted for a different  $\alpha_s$  value which is a fixed parameter in the QCD fit. Both experimental and scale uncertainties are reduced when we add the simulated EIC inclusive data to HERA data. The substantial influence of the EIC inclusive data on the precision of  $\alpha_s(M_Z^2)$  naturally leads to the inquiry of whether a comparable result can be achieved in the absence of HERA jet data.



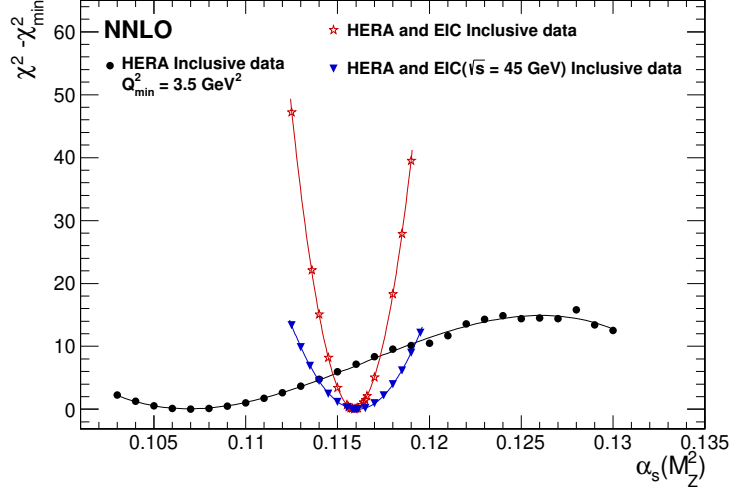
**Figure 2:**  $\Delta\chi^2 = \chi^2 - \chi_{min}^2$  value as a function of  $\alpha_s(M_Z^2)$  for the NNLO fits to HERA inclusive and jets data in addition to the simulated EIC inclusive data (left) and without the EIC data as published in [4] (right). The experimental, model, parameterisation, and scale uncertainties are displayed with different colored bands.

Further, QCD fittings are performed by using HERA inclusive and the simulated EIC inclusive data only, the result is

$$\alpha_s(M_Z^2) = 0.1159 \pm 0.0004 \text{ (exp)} \begin{matrix} +0.0002 \\ -0.0001 \end{matrix} \text{ (model + parameterisation)}. \quad (1)$$

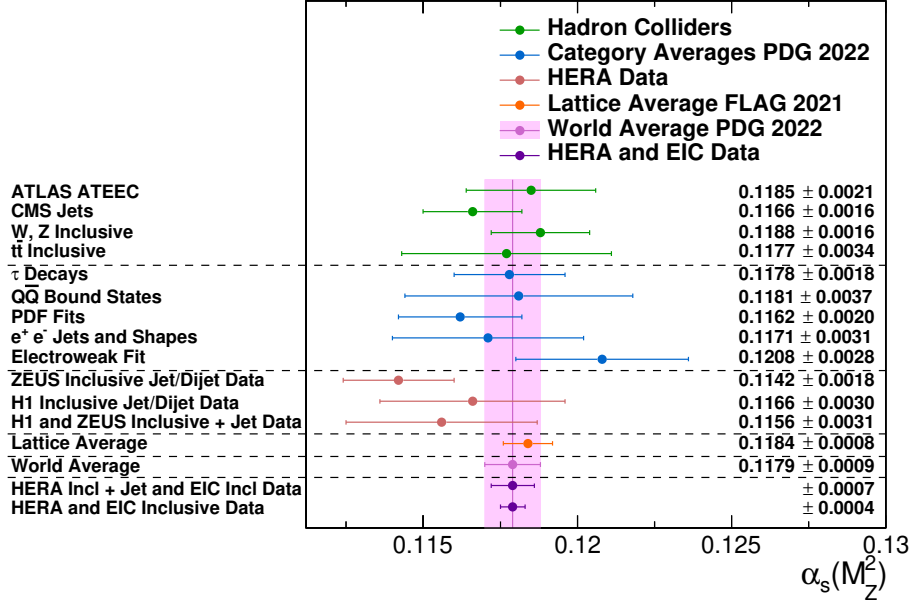
corresponding to an overall accuracy exceeding 0.4%. Figure 3 presents the results of the dependence of the fit  $\chi^2$  on the strong coupling  $\alpha_s(M_Z^2)$ . All the  $\chi^2$  scans were performed at NNLO. HERA inclusive data only shows a limited dependence of the fit  $\chi^2$  on the strong coupling  $\alpha_s(M_Z^2)$ . When we add the simulated EIC inclusive data, the  $\chi^2$  minimum around  $\alpha_s(M_Z^2) = 0.116$  becomes

highly noticeable in fits. The best result is achieved by including all EIC  $\sqrt{s}$  values, as shown in Fig.3. All these results are compared with other determinations of  $\alpha_s(M_Z^2)$ , as shown in Fig.4. The figure also includes the world average of experimental determinations by the Particle Data Group (PDG)[22] and an average from QCD lattice calculations of the strong coupling constant [23]. Notably, this study shows much more precise results with respect to previous determinations. However, this encouraging result has no scale uncertainty, which has been considered for missing higher orders beyond NNLO in the QCD analysis. The high precision observed in this study is attributed to the additional phase space coverage that EIC pseudodata provide in the large  $x$ , moderate  $Q^2$  region. This extends and complements the kinematic coverage provided by HERA data. The overlap phase space coverage results in improved precision on the logarithmic derivative of the inclusive structure function  $dF_2/d \ln Q^2$ . Specifically, at higher  $x$  values, this quantity is primarily influenced by the  $q \rightarrow qg$  splitting, providing insights into the product of  $\alpha_s$  and the large  $x$  quark densities.



**Figure 3:**  $\Delta\chi^2 = \chi^2 - \chi_{min}^2$  as a function of  $\alpha_s(M_Z^2)$  for the NNLO fits. Black points show the HERA inclusive  $ep$  data only. Red points show the HERA inclusive data and the simulated EIC inclusive data with all five  $\sqrt{s}$  values together. Blue points show the HERA inclusive data and the simulated EIC inclusive data with only  $\sqrt{s} = 45$  GeV. The black points are taken from [3].

This study shows the potential for achieving a world-leading precision in determining the strong coupling constant at the scale of the Z-boson mass. The data used in this study include inclusive DIS data from the HERA experiment and simulated data from the EIC. The total uncertainties in  $\alpha_s$  improve significantly, surpassing the precision of current global experimental and lattice averages. The improved precision is attributed to the large  $x$ , intermediate  $Q^2$  region, accessible at the EIC but not covered by HERA. Assigning a meaningful scale uncertainty related to potential higher-order contributions beyond NNLO remains challenging when we use inclusive DIS data only. Addressing the scale uncertainties becomes a focal point, aiming for a consensus in  $\alpha_s(M_Z^2)$  determinations reliant on EIC data.



**Figure 4:** Projected total uncertainties on the strong coupling constant  $\alpha_s(M_Z^2)$  estimated using HERA and simulated EIC data, compared with extractions using other data sets and methods [4, 24–30], with the world average according to the PDG [22] and with an average from lattice QCD calculations [23]. Scale uncertainties are not yet included in the treatment of inclusive DIS data for any of the results shown. The plotting style follows [24].

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