

Photon energy gain in relativistic Plasma with velocity Shear: A new mechanism for producing power-law spectra at high energies

Mukesh Kumar Vyas^{a,*} and Asaf Pe'er^a

^aBar Ilan University,

Max VeAnna Webb Street, Ramat Gan, Israel, 5290002

E-mail: mukeshkvys@gmail.com, asaf.peer@biu.ac.il

We investigate a novel mechanism of photon energy gain in an optically thick relativistic plasma with velocity shear. It is a Fermi-like process where the photons repeatedly scatter within the regions of different Lorentz factors, thereby gaining energy on average. The resulting spectra have a power-law shape at high energies. Thus, this mechanism is an alternative to the classical emission from power-law accelerated particles that can generate power-law spectra in sources such as Gamma-ray bursts (GRBs) and Active Galactic Nuclei (AGNs). Using both numerical simulations and theoretical calculations, we compute the expected spectra for parameters characterizing GRBs and recover the range of their observed photon indices at high energies.

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*Speaker

1. Introduction

Power law-shaped spectra are ubiquitously observed in sources like gamma-ray bursts (GRBs) [1–6], active galactic nuclei [AGNs] [7–9] and others. The observed power-law spectra are conventionally explained by synchrotron process [1, 2] where a population of accelerated charged particles radiate in the presence of magnetic fields. Alternatively, these spectra can be produced by thermal Comptonization [10–13]. The particles emitting the radiation are believed to be accelerated into a power law distribution by a Fermi process at shock waves produced in the plasma. Indeed, all these objects are characterized by a bipolar relativistic jet, and therefore shock waves, originating from either the interaction of the jetted plasma with the surrounding material or due to internal collisions between propagating shells within the jet itself, are expected [14].

It is seen from numerical simulations of a propagating jet that its Lorentz factor harbours an angular dependence with an inner fast jet surrounded by a slower outer region [15]. Hence the propagating jet has an angle-dependent Lorentz factor along the polar angle. It was speculated that such Lorentz factor profile, $\gamma = \gamma(\theta)$ may be universal in GRB jets [16]. This raises the possibility of an alternative way of obtaining the observed power law spectra, by using the differential kinetic energy of the jet shear layer, rather than shock waves.

Considering that the jets are optically thick at their base (or inner regions), we show that the scattering of the photons with electrons within the shear layers results in a net energy gain for photons. We show that once the photons escape the system, they produce a power-law-shaped spectrum. This underlying mechanism is a photon analogue to Fermi acceleration of charged particles [17], according to which the particles gain energy when scattered within regions of differential motion in a plasma (typically, at the upstream and downstream regions around a shock wave). However, as opposed to the Fermi mechanism, no acceleration of particles is assumed, but rather multiple Compton scattering of photons with electrons in different jet regions.

Hence we provide an alternate mechanism to produce the observed power-law spectra without the requirement to accelerate the charged particles or the presence of magnetic fields [18]. We apply the model to GRB jets to explain their power-law spectra at high energies. The results of the model are simultaneously validated with Monte Carlo simulations. Additionally, the model provides a unique tool to determine the jet structure from their observed spectra. In section 2 below, we explain the assumptions of the model with details of the simulations. We explain the results in section 3 and conclude the work in section 4.

2. Theoretical model and Simulations

The GRB jets are generally relativistic with very high bulk Lorentz factors $\gamma \gg 1$. Within the framework of the GRB "collapsar" model, a jet erupts following the collapse of a massive star [19–21]. Although the emerging jet profile is uncertain, several simulations of erupting jets show a nearly universal jet profile [15, 16, 22, 23]. We therefore consider that the jet has a polar angle (θ) dependent Lorentz factor given by

$$\gamma(\theta) = \gamma_{\min} + \frac{\gamma_0}{\sqrt{\left(\frac{\theta}{\theta_i}\right)^{2p} + 1}}. \quad (1)$$

When considering the jet profile, we use a spherical coordinate system (r, θ, ϕ) with the system possessing an azimuthal symmetry. The origin of the coordinate system lies at the centre of the star. Here, γ_0 and γ_{\min} are the maximum and minimum values of the Lorentz factor: $\gamma = \gamma_0$ at the jet axis and decreases following Equation 1 asymptotically reaching $\gamma = \gamma_{\min}$ at large angles. The angle θ_j has a fixed value: at smaller angles, the jet Lorentz factor is $\gamma(\theta < \theta_j) \approx \gamma_0$, while at larger angles, γ declines as a power law with index p , representing the steepness of the shear.

We assume that the photons are emitted deep inside the jet, where it is optically thick. We further assume that the Lorentz factor remains constant with radial distance. This assumption leads to a decrease in the particle density with distance as an inverse square law (see [18] for details). After their emission, the photons undergo multiple Compton scattering with the electrons in the shearing layers of the jet, before escaping. Following the density decrease with the radial extent, the jetted plasma becomes optically thin beyond a certain distance, known as the photospheric radius, which is defined as the distance from the origin above which the optical depth for photon scattering from this location to infinity equals unity. Once the photons reach the photosphere, they therefore escape. Here, we compute the observed spectra from the escaped photons, both analytically and using numerical simulations, for various assumptions on the uncertain jet profile.

2.1 Analytic estimation of the emitted spectrum

The spectrum of the escaping photons from the jet is estimated as follows. Consider an initial injection of N_0 photons at initial energy ε_0 deep inside the jet. After being scattered k times, assume that N photons are left inside the jet while the rest $(N_0 - N)$ photons have escaped. After k scattering, the average photon energy is denoted by ε_k . On the average, a photon gains energy, and therefore $\varepsilon_k > \varepsilon_i$. Denote by \bar{g} the average energy gain per scattering. Then, as long as the photon is in the scattering region, its energy after k scattering is $\varepsilon_k = \varepsilon_i \bar{g}^k$. For estimated average scattering probability \bar{P} , it leads to

$$\frac{N}{N_0} = \left[\frac{\varepsilon_k}{\varepsilon_i} \right]^{\beta'}, \quad (2)$$

where $\beta' = \frac{\ln \bar{P}}{\ln \bar{g}}$. The photon index β is thus,

$$\beta = \beta' - 1 = \frac{\ln \bar{P}}{\ln \bar{g}} - 1. \quad (3)$$

Since the conditions inside the jet vary with location, the average gain and the scattering probabilities are computed by their expectation values over the available scattering volume (V) of the jet,

$$\bar{P} = \frac{1}{V} \int_V P(r, \theta) dV \text{ and } \bar{g} = \frac{1}{V} \int_V g(r, \theta) dV. \quad (4)$$

The escape probability of a photon at a given location is given by $P_e(r, \theta) = \exp[-\tau(r, \theta)]$, with τ being the optical depth along the photon's direction. The requirement that the total probability is equal to unity ensures that the probability for the photon to have the next scattering rather than escaping is $P(r, \theta) = 1 - P_e(r, \theta)$.

Following Vyas et. al. 2023 [18], the energy gain at a given location (r, θ) is estimated as,

$$g(r, \theta) \approx \frac{1}{2} \left[1 + \left[1 + \sum \frac{\partial \log \gamma}{\partial \theta} \delta \theta \right]^2 \frac{1}{(1+a)^2} \right]. \quad (5)$$

Here, $\partial \log \gamma / \partial \theta$ terms express the gain due to repeated scattering (back and forth) between regions of different jet Lorentz factor (jet shear layers); $a = \lambda/r$ is the expansion factor that reduces the gain due to the spherical expansion (adiabatic expansion) of the jet, and λ is the average mean free path at this location. The details of estimating mean free path λ and optical depth τ are given in Ref. [18].

For a given set of jet parameters, \bar{P} and \bar{g} are computed from equation 4 and then the photon indices of the scattered spectra at high energy are calculated following Equation 3.

2.2 Numerical simulations

To confirm the predictions of the analytic model, we perform Monte Carlo simulations of the process described in the previous section. We inject the photons deep inside a cold jet with initial directions being random in the comoving frame of the jet. For an assumed density and jet Lorentz factor profile, each photon goes through multiple scattering within the shear jet layer. The photons are individually tracked for each scattering with electrons simultaneously transforming their four vectors between the fluid frame, electron rest frame and then to the observer frame. After scattering, the distance a photon travels until the next scattering δl along the scattered direction is estimated. The probability for it to travel a distance δl without being scattered is $\exp^{-\tau}$ with τ being the optical depth which is drawn from a randomly selected logarithmic distribution. If τ is greater than the optical depth to escape to infinity, the photon escapes the system towards the same direction without further scattering. Otherwise, the length δl is computed so that the optical depth along this path equals τ . As the photon propagates along the jet extension, the optical depth decreases following the decrease in the density. This process is followed until the photon escapes near the photospheric radius where the optical depth to infinity equals unity.

The escaped photons are distributed over a range of energy, angular direction and time of escape enabling the study of their spectral and temporal properties for a given observer's location. For obtaining the spectrum observed along the observer's angle θ_o , these photons are distributed in several energy bins around θ_o . The spectra are unchanged for the azimuthal location of the observer due to the symmetry along ϕ . The variation of this photon flux with the energy of the photons produces the observed emission spectrum for a given observer. The photon indices of the observed spectrum are compared with the analytically obtained spectra for various jet parameters. Further details of the simulation code are found in [23, 24].

3. Results

We choose a shearing jet with the following parameters: luminosity $L = 10^{50}$ erg s⁻¹, $\gamma_0 = 100$, $\theta_j = 0.01$ rad, $\gamma_{\min} = 1.1$, and $p = 3$. The photons are injected deep inside the jet with energy $\varepsilon_0 = 10^{-7}$ in the units of the electron's rest energy. The full spectrum over the range of observed energies ε_1 is obtained by numerical simulations whereas the value of the high energy slope is computed analytically. The binned spectrum from our simulations is plotted in Figure 1 for an on-axis observer situated along the jet axis (black solid curve). The spectrum shows a power law shape at high energies. The analytically obtained photon index (Equation 3) for the given parameters is $\beta = -2.1$. It is over-plotted with the red dashed curve over the simulated spectra. The analytic

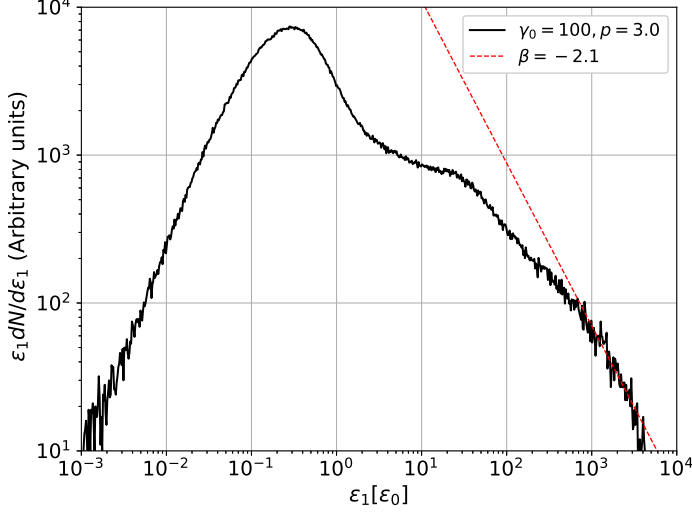


Figure 1: Spectrum obtained by numerical simulations for photons with observed energy ε_1 (black solid) and theoretically obtained spectral slope (red dashed) for chosen parameters $L = 10^{50}$ erg s $^{-1}$, $\gamma_0 = 100$, $p = 3.0$, $\theta_j = 0.01$ rad. The observer is along the jet axis.

prediction of the photon index has a good agreement with the photon index obtained numerically, using the simulations.

As p characterizes the steepness of the shear layer in the jet due to which the power law tail appears in the spectrum at high energies, we calculate β by varying p keeping other parameters the same as in Figure 1. The distribution of photon indices β over varying jet profile index p is shown in Figure 2. The simulated photon indices (blue spheres) are in agreement with the analytic curve (black solid curve). The spectra become harder for higher values of p which makes the shear stronger. However, at very sharp shear (as $p \rightarrow \infty$), the photon indices asymptote to $\beta \rightarrow -1.5$ because the effective region of shear scattering gets narrower (Equation 1). For $p < 2.3$, the power law shape vanishes and β approaches $-\infty$ as the photons' energy loss due to the jet expansion dominates over the energy gain by shear Comptonization. For a jet with a cylindrical cross-section (*i.e.*, without geometric expansion), we would expect $\beta \rightarrow -\infty$ only when $p \rightarrow 0$.

4. Conclusions

In this proceeding, we have considered a shearing jet with variable Lorentz factor along its polar angle. We showed that a photon population injected deep inside the flow and that scatters freely within an optically thick shearing jet gains energy on average, producing a power-law-shaped spectrum at high energies. The developed theoretical model is in good agreement with the simulation results. The adopted parameters for a typical gamma-ray burst jet lead to a power law index in the range $\beta \rightarrow -\infty$ to -1.5 . This index is in agreement with the obtained range of high energy spectral slopes (β) detected in prompt phase observations of GRBs [2, 4, 5]. Thus we conclude that the alternate mechanism presented here can explain the observed GRB spectra at high energies.

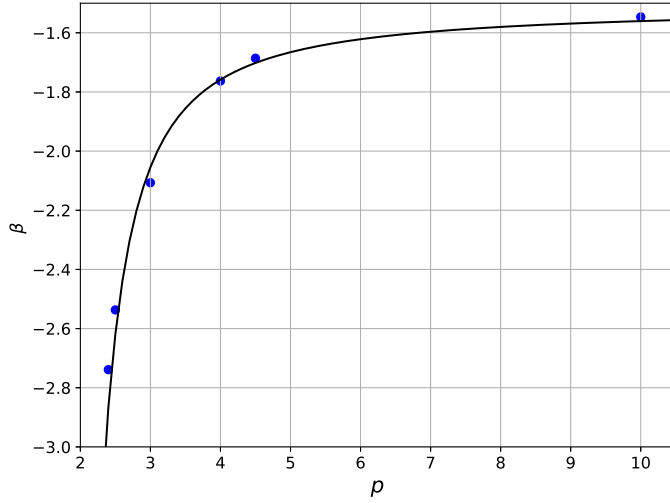


Figure 2: Range of photon indices obtained for various values of jet profile index p . Greater p signifies sharper shear in the jet. $p \rightarrow \infty$ denotes a step function in γ from γ_0 to γ_{\min} at $\theta = \theta_j$ while $p = 0$ means jet with constant (θ -independent) Lorentz factor, γ_0 . Other parameters are the same as in Figure 1

The model has applications beyond the spectra of GRBs such as in AGNs [25]. The further applications of the model in astrophysical relativistic flows with shear in accretion discs around black holes as well neutron stars will be explored and published elsewhere.

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