

Precision Measurements and Prospects with Kaons at CERN

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The NA62 experiment at the CERN SPS collected the world's largest dataset of charged kaon decays during 2016-2018. The dataset was used for the most precise measurement of the branching fraction of the ultra-rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. In this proceedings, the NA62 experiment reports recent results from analyses of decays $K^+ \rightarrow \pi^0 e^+ \nu \gamma$ ($K_{e3\gamma}$), $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ ($K_{\pi\mu\mu}$) and $K^+ \rightarrow \pi^+ \gamma \gamma$ ($K_{\pi\gamma\gamma}$). The radiative $K_{e3\gamma}$ decay has a data sample of $O(100k)$ candidates with sub-percent background contamination. Published results with the most precise measurement of the branching fraction and T-asymmetry are presented. The $K_{\pi\mu\mu}$ sample comprises about 27k signal events with negligible background contamination, and the results include the most precise determination of the branching ratio and the form factor. The $K_{\pi\gamma\gamma}$ sample contains about 4k signal events with 10 % background contamination, and the analysis improves the precision of the branching ratio measurement by a factor of 3 with respect to the previous measurements.

Recent results from the NA48/2 experiment from the analysis of the decay $K^\pm \rightarrow \pi^0 \pi^0 \mu^\pm \nu$ ($K_{\mu 4}^{00}$) are also reported in this proceedings. The sample consists of 2437 events with 15 % background contamination. The result is compared with a newly calculated theoretical prediction to 1-loop level, including previously neglected R_1 form factor.

The NA62 experiment is approved to take data until the CERN Long Shutdown 3. A proposal for next-generation kaon experiments at CERN, called HIKE, was submitted to the SPSC in November 2023 and is also presented in this proceedings. In its first phase, HIKE will continue to study decays of charged kaons at significantly higher beam intensities than NA62 using a new detector system. Neutral kaon decays will be then studied in the second phase of HIKE.

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1. NA62 Experiment

NA62 is a fixed target experiment at CERN specifically designed to study the ultra-rare $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay [1]. The detector's high performance required for this purpose makes the experiment perfectly suited to address a broad physics program with kaons. The physics program of NA62 includes precision measurements of kaon decay observables, analyses of rare decay modes and searches for lepton flavour/number violating (LFV/LNV) processes. Furthermore, NA62 can study the production and decay of feebly interacting particles (FIPs) with data taken in a special beam dump configuration.

The NA62 Experiment is located in the north area of the CERN laboratory. It uses the 400 GeV/c primary proton beam from the Super Proton Synchrotron (SPS) impinging on a 400 mm long beryllium target with 2 mm diameter. The secondary beam consists of pions (70 %), protons (23 %), kaons (6 %) and muons (<1 %) and has a nominal momentum of 75 GeV/c. The NA62 detector (fig. 1) is divided into two parts by evacuated decay region, in which about 6 % of all beam kaons decay. In the upstream part, the beam particles pass through a Cherenkov counter KTAG consisting of a nitrogen-filled chamber encircled at one end by eight segments of optical readout equipment used for time-stamping of the beam kaons with time resolution below 100 ps. The beam particle momenta are measured by a beam spectrometer GTK, consisting of three silicon pixel stations and a pair of dipole magnets. The beam particle momenta are measured by a beam spectrometer GTK, consisting of three silicon pixel stations and a pair of dipole magnets.

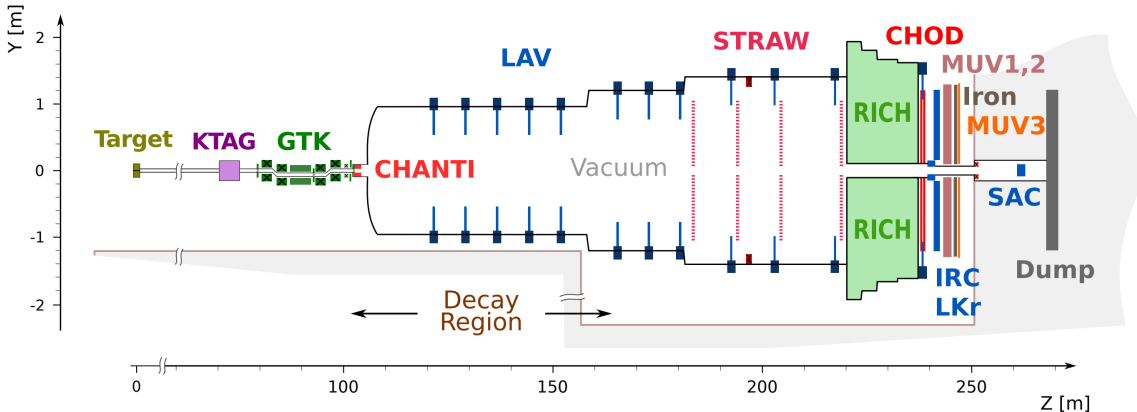


Figure 1: Schematic vertical section of the NA62 experimental setup [1].

The downstream segment of the experiment consists of detectors measuring properties of the decay products. Positioned after the decay region are the four stations of the downstream magnetic spectrometer STRAW. It provides momentum measurement of the charged particles originating from kaon decays. The ring imaging Cherenkov counter RICH is tasked with the identification of these charged daughter particles. It achieves a time resolution better than 100 ps. The RICH is complemented by a quasi-homogeneous liquid krypton calorimeter LKr, two hadron calorimeters (MUV1, MUV2) and by tile scintillator MUV3, which is positioned at the near end of the experimental hall. To detect the photons generated within the decay volume at angles between 0 and 50 mrad, a set of photon-veto detectors (LAV, LKr, IRC, SAC) is installed from the beginning of the decay region to the experiment's conclusion. Two scintillator hodoscopes (CHOD) provide

the time measurements and also provide the trigger signals. A detailed review of the NA62 detector system can be found in [1].

The first run of NA62 took place in 2016–2018. The data taking was resumed after CERN Long Shutdown 2 (LS2) in 2021. The experiment is approved to collect data until CERN Long Shutdown 3, which is scheduled to start at the end of 2025.

2. $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ Decay Analysis

The $K_{\pi\mu\mu}$ decay is a flavour changing neutral current (FCNC) process with dominant contributions from virtual photon exchange

$$K^\pm \rightarrow \pi^\pm \gamma^* \rightarrow \pi^\pm \ell^+ \ell^- \quad (\ell = e, \mu).$$

The transition form factor $W(z)$ is parameterized in Chiral Perturbation Theory (ChPT) at the order $\mathcal{O}(p^6)$ by two real parameters a_+ and b_+ as [2, 3]

$$W(z) = G_F m_K^2 (a_+ + b_+ z) + W^{\pi\pi}(z),$$

where $W^{\pi\pi}(z)$ is a complex function describing the two-pion loop term and $z = m^2(\mu^+ \mu^-)/m_K^2$ is the normalized di-muon mass squared.

The data set used in the $K_{\pi\mu\mu}$ analysis [4] was collected by NA62 in 2017 and 2018. The analysis used the normalisation channel $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ to estimate the effective number of kaon decays in the sample as

$$\mathcal{N}_K = (3.48 \pm 0.09_{\text{sys.}} \pm 0.02_{\text{ext.}}) \times 10^{12}.$$

In this sample 27679 signal events were observed with a sub-percent background contamination of about 8 events.

The resulting signal sample was divided into 50 equipopulated bins in the kinematic variable z . The differential decay rate $d\Gamma/dz$ for each bin was evaluated, and a χ^2 fit was performed to extract the a_+ and b_+ parameters. The results are shown in fig. 2a. The theoretically preferred solution with both a_+ and b_+ negative is $a_+ = -0.575 \pm 0.013$, $b_+ = -0.722 \pm 0.043$ with uncertainties dominated by the statistical errors. From the integration of the differential decay rate, the model-independent branching fraction was obtained

$$\mathcal{B}(K^+ \rightarrow \pi^+ \mu^+ \mu^-) = (9.15 \pm 0.08) \times 10^{-8}.$$

A measurement of the forward-backward asymmetry A_{FB} , defined as the asymmetry of $\cos(\theta_{K\mu})$, where $\theta_{K\mu}$ is the angle between K^+ and μ^- in the di-muon rest frame was also performed

$$A_{FB} = (0.0 \pm 0.7_{\text{stat.}} \pm 0.2_{\text{syst.}} \pm 0.2_{\text{ext.}}) \times 10^{-2}.$$

The corresponding upper limit on the asymmetry is [5]

$$|A_{FB}| < 0.9 \times 10^{-2} \quad \text{at 90 \% CL.}$$

In conclusion, a sample size nine times larger than in the previous experiment NA48/2 [6] was collected, improving the measurement precision by a factor of 3. Furthermore, the observed agreement in a_+ , b_+ between $K_{\pi\mu\mu}$ and $K_{\pi ee}$ measurements is in line with the lepton flavour universality (LFU) principle.

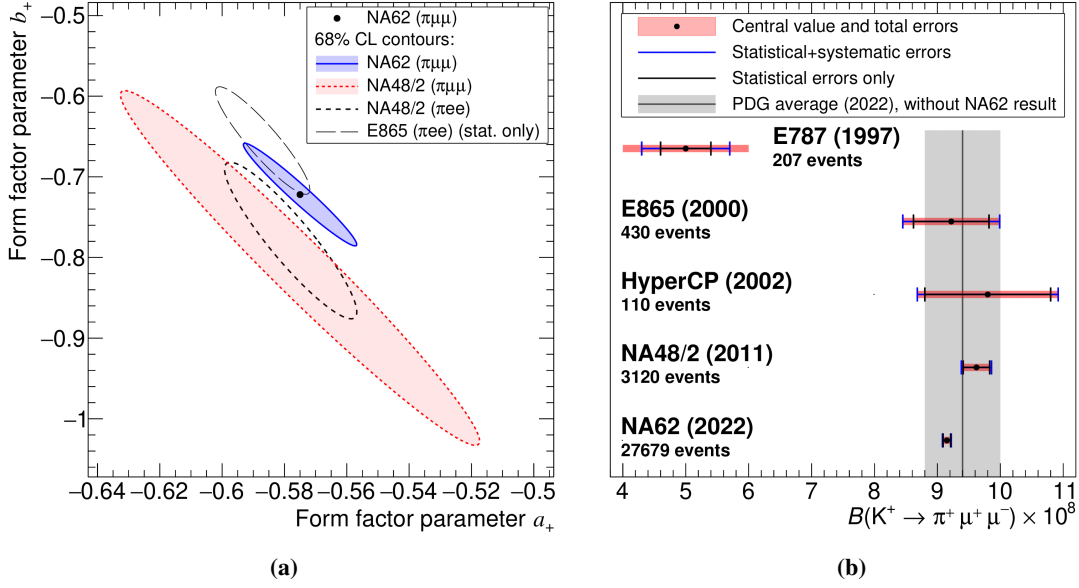


Figure 2: Results from the $K_{\pi\mu\mu}$ analysis and comparison with previous measurements [4].

3. $K^+ \rightarrow \pi^0 e^+ \nu \gamma$ Decay Analysis

In the framework of ChPT, the $K_{e3\gamma}$ decay is described by the inner bremsstrahlung (IB) process, the direct emission (DE) process and their interference [7]. As the process amplitude is divergent for $E_\gamma \rightarrow 0$ or $\theta_{e,\gamma} \rightarrow 0$, where E_γ is the photon energy and $\theta_{e,\gamma}$ is the angle between the positron and photon track, the measurement of the decay is usually studied in 3 kinematic regions S_1 , S_2 and S_3 , defined in Table 1.

Region	E_γ	$\theta_{e,\gamma}$
S_1	$E_\gamma > 10 \text{ MeV}$	$\theta_{e,\gamma} > 10^\circ$
S_2	$E_\gamma > 30 \text{ MeV}$	$\theta_{e,\gamma} > 20^\circ$
S_3	$E_\gamma > 10 \text{ MeV}$	$0.6 < \cos(\theta_{e,\gamma}) < 0.9$

Table 1: Kinematic regions chosen to study the $K_{e3\gamma}$ decay at NA62 [8].

The measured variable in this process is the ratio of branching fractions of the radiative ($K_{e3\gamma}$) and inclusive (K_{e3}) decays

$$R_j = \frac{\mathcal{B}(K_{e3\gamma j})}{\mathcal{B}(K_{e3})} = \frac{\mathcal{B}(K^+ \rightarrow \pi^0 e^+ \nu \gamma | E_\gamma^j \theta_{e,\gamma}^j)}{\mathcal{B}(K_{e3})},$$

where j corresponds to kinematic region. The radiative decay is also sensitive to T-violating contributions which can be studied with the T-odd observable ξ and the corresponding T-asymmetry A_ξ defined as

$$\xi = \frac{\vec{p}_\gamma \cdot (\vec{p}_e \times \vec{p}_\pi)}{M_K^3}, \quad A_\xi = \frac{N_+ - N_-}{N_+ + N_-},$$

where \vec{p}_i is the three-momentum of the corresponding particle in the kaon rest frame, M_K is the kaon mass, and $N_+(N_-)$ is the number of events with a positive (negative) value of ξ . Previous measurements by the ISTRA+ and OKA collaborations, as well as theoretical calculations in the ChPT framework, are summarised in [7, 9–13].

The data sample used in the NA62 analysis of $K_{e3\gamma}$ [8] was collected in 2017 and 2018. As a normalisation channel the $K^+ \rightarrow \pi^0 e^+ \nu$ (K_{e3}) decay was used. It shares most of the selection criteria with the signal channel ($K_{e3\gamma}$), except for the requirement of an additional photon and the fact that full phase space was used for the evaluation of the acceptance.

The measured values of the ratios of the radiative and inclusive decays in the three kinematic regions as well as the measured T-asymmetry values are presented and compared with previous experiments in Table 2.

Variable wrt. Region	ISTRA+ [12]	OKA [13]	NA62
$R_1(\times 10^2)$	$1.81 \pm 0.03 \pm 0.07$	$1.990 \pm 0.017 \pm 0.021$	$1.715 \pm 0.005 \pm 0.010$
$R_2(\times 10^2)$	$0.63 \pm 0.02 \pm 0.03$	$0.587 \pm 0.010 \pm 0.015$	$0.609 \pm 0.003 \pm 0.006$
$R_3(\times 10^2)$	$0.47 \pm 0.02 \pm 0.03$	$0.532 \pm 0.010 \pm 0.012$	$0.533 \pm 0.003 \pm 0.004$
$A_\xi(S_1)(\times 10^3)$		$-0.1 \pm 3.9 \pm 1.7$	$-1.2 \pm 2.8 \pm 1.9$
$A_\xi(S_2)(\times 10^3)$		$-4.4 \pm 7.9 \pm 1.9$	$-3.4 \pm 4.3 \pm 3.0$
$A_\xi(S_3)(\times 10^3)$	15 ± 21	$7.0 \pm 8.1 \pm 1.5$	$-9.1 \pm 5.1 \pm 3.5$

Table 2: Results of the $K_{e3\gamma}$ analysis of the NA62 collaboration compared with previous measurements [8].

The results of R_j improved precision by at least a factor of 2 with respect to previous experiments, and the relative uncertainty, being smaller than 1 %, matches the most precise theoretical calculations. Results for A_ξ show no statistically significant asymmetry in the T-odd observable. However, the uncertainty in this measurement remains larger than theoretical expectations.

4. $K^+ \rightarrow \pi^+ \gamma \gamma$ Decay Analysis

The $K^+ \rightarrow \pi^+ \gamma \gamma$ is a radiative non-leptonic decay dominated by long-distance contributions which allows for crucial ChPT tests. The decay is described by two kinematic variables

$$z = \frac{(q_1 + q_2)^2}{M_K^2} = \frac{m_{\gamma\gamma}^2}{M_K^2}, \quad y = \frac{p \cdot (q_1 - q_2)}{M_K^2},$$

where q_1, q_2 are 4-momenta of the photons, M_K is the K^+ mass, $m_{\gamma\gamma}$ is the di-photon invariant mass and p is the K^+ 4-momentum. The differential decay width is described by a single unknown parameter \hat{c} . Theoretical calculations and predictions are described in [14] and [15].

The $K_{\pi\gamma\gamma}$ decay was also analysed with data collected by NA62 in 2017 and 2018 [16]. The $K^+ \rightarrow \pi^+ \pi^0$ decay ($K_{2\pi}$) with the same detector signature as the signal was chosen as the normalisation channel. The difference between them lies in their kinematics. Since the $K_{2\pi}$ is a 2-body decay, it occupies the region around $z = m_{\pi^0}^2/m_K^2$ while the signal process is a 3-body decay with z widely distributed. After applying the selection criteria to the data, 3984 signal events were observed in the kinematic region $z = (P_K - P_\pi)^2/M_K^2 > 0.2$ with 291 ± 14 background events

expected. The main source of background is from merging γ clusters in the LKr from $K^+ \rightarrow \pi^+\pi^0\pi^0$ and $K^+ \rightarrow \pi^+\pi^0\gamma$ decays.

The analysis results for the \hat{c} parameter in the ChPT at order $O(p^6)$ (\hat{c}_6), and model-dependent and model-independent branching fractions are

$$\hat{c}_6 = 1.144 \pm 0.069_{stat.} \pm 0.034_{syst.},$$

$$\mathcal{B}(K^+ \rightarrow \pi^+\gamma\gamma) = (9.61 \pm 0.15_{stat} \pm 0.07_{syst}) \times 10^{-7},$$

$$\mathcal{B}_{MI}(z > 0.2) = (9.46 \pm 0.19_{stat} \pm 0.07_{syst}) \times 10^{-7}.$$

The results are compared with previous measurements in figure 3. In the analysis, external ChPT parameters were fixed to the values estimated in [17]. If the parameters had been fixed at the values used by the E787 and NA48/2 experiments the measured values of the \hat{c}_6 parameter would have been as indicated by the orange triangle and orange square respectively.

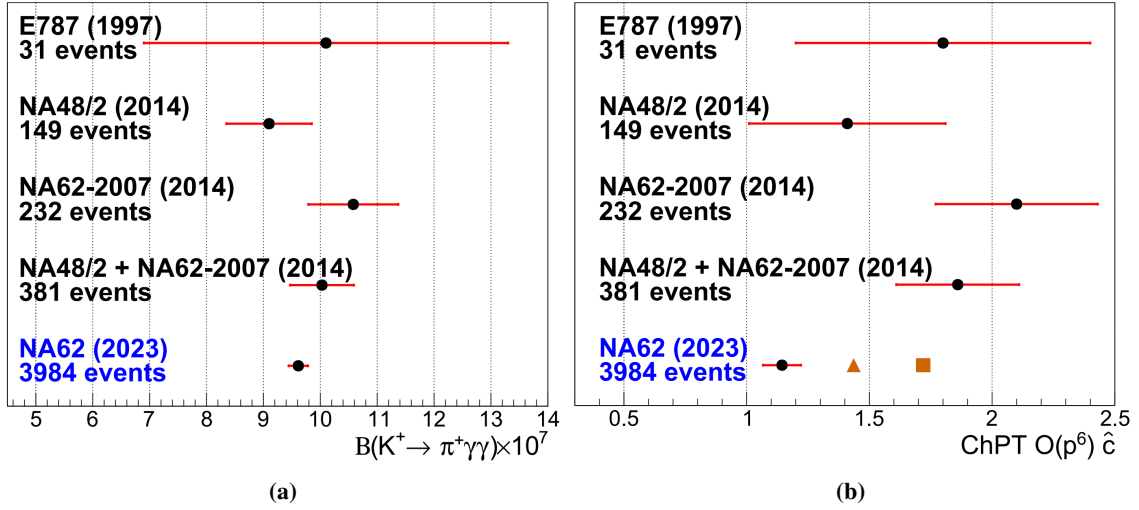


Figure 3: Results from the $K_{\pi\gamma\gamma}$ analysis and comparison with previous experiments [16].

The analysis also showed that in order to describe the observed di-photon mass spectrum, it is necessary to include the next-to-leading order contribution in chiral perturbation theory. The comparison between ChPT at orders $O(p^4)$ and $O(p^6)$ is shown in figure 4.

5. The NA48/2 $K^\pm \rightarrow \pi^0\pi^0\mu^\pm\nu$ Decay Analysis

The semileptonic $K^\pm \rightarrow \pi\pi\ell^\pm\nu$ ($K_{\ell 4}$) decay where $\ell = (e, \mu)$ depends on 4 form factors (F, G, H and R) described by 5 kinematic (Cabibbo-Maksymowicz) variables. The variables are the di-pion invariant mass squared S_π , the di-lepton invariant mass squared S_ℓ , and three angles θ_π , θ_ℓ and ϕ ; their definitions can be found in [18]. Specifically for the $K_{\mu 4}^{00}$ decay, the amplitude depends only on form factors F and R, as there is no dependence on $\cos(\theta_\pi)$ and ϕ in $\pi^0\pi^0$ s-wave. The situation simplifies even further in the $K_{e 4}^{00}$ decay where dependence on R drops out as a consequence of the very small positron mass. In previous experiments [19–21], the $K_{e 4}^\pm$, $K_{e 4}^{00}$ and $K_{\mu 4}^\pm$ decays were observed. In the following section, the analysis leading to the first observation of the $K_{\mu 4}^{00}$

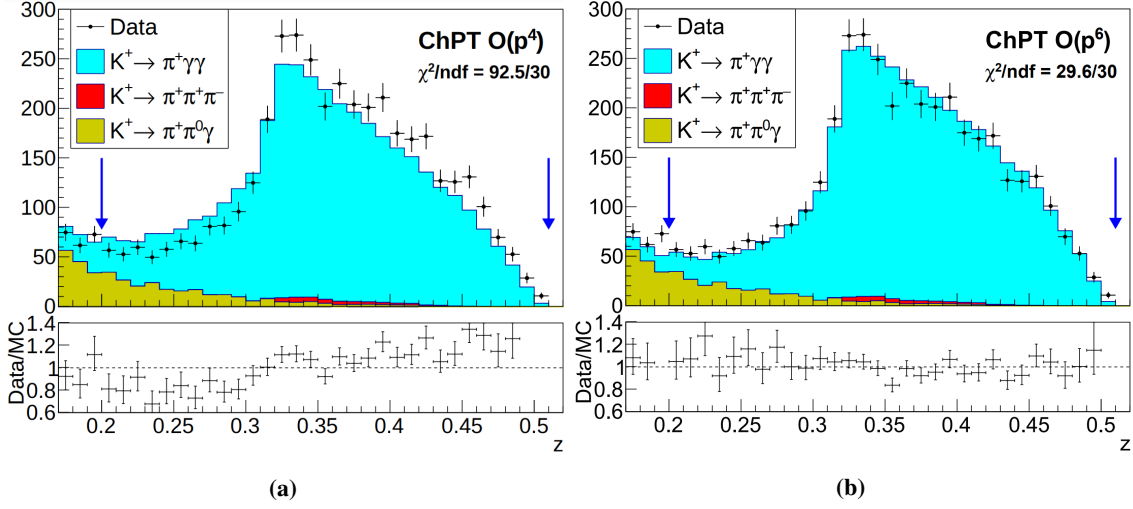


Figure 4: Comparison between NA62 data and ChPT at orders $O(p^4)$ (left) and $O(p^6)$ (right) from the $K_{\pi\gamma\gamma}$ analysis [16].

decay at NA48/2 is described. Furthermore, the presence of the contribution from the form factor R is reported.

The NA48/2 experiment is a predecessor of the NA62 experiment. It collected data in 2003 and 2004 with simultaneous K^+/K^- beams. The signal selection algorithm looked for 1 charged muon track and 4 photons consistent with $2\pi^0$ energies. As a normalisation channel the $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ ($K_{3\pi}$) decay was used as it generates the same detector signature with a difference in charged track identification. However, the biggest source of background is also from the normalisation channel as most of the charged pions decay into charged muon and neutrino(antineutrino) pair ($\pi^\pm \rightarrow \mu^\pm \nu(\bar{\nu})$). To suppress background the phase-space was restricted to events with $S_\ell > 0.03 \text{ GeV}/c^2$.

In the resulting dataset 2437 events were observed with expected $354 \pm 33_{stat.} \pm 62_{syst.}$ background events [22]. The branching fraction for the $K_{\mu 4}^{00}$ decay was analysed with and without the phase space restriction. The results are

$$\mathcal{B}(K_{\mu 4}^{00}, S_\ell > 0.03) = (0.65 \pm 0.02_{stat} \pm 0.02_{syst} \pm 0.01_{ext}) \times 10^{-6},$$

$$\mathcal{B}(K_{\mu 4}^{00}) = (3.45 \pm 0.10_{stat} \pm 0.11_{syst} \pm 0.05_{ext}) \times 10^{-6}.$$

NA48/2 results and theoretical predictions ([18], [23]) to different orders of precision are compared in figure 5b. The measurement results are compatible with theory calculations to 1-loop level including the form factor R_1 .

6. Future Prospects

The NA62 experiment is approved to take data until CERN Accelerator Long Shutdown 3 (LS3). The main objective of the remaining period is to improve the precision of the branching fraction measurement of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay. For this purpose, multiple new detectors and upgrades were installed in the experiment to suppress background sources and increase signal

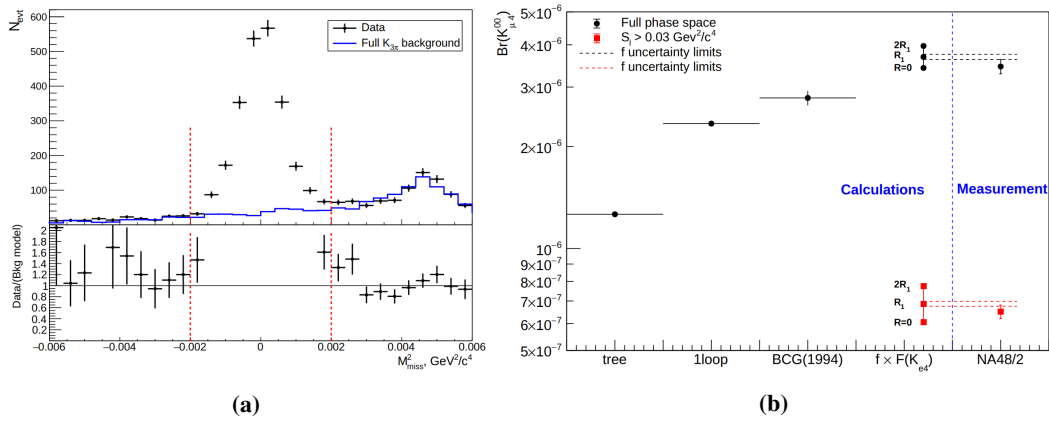


Figure 5: Missing mass square of signal events analysed by NA48/2 and comparison of branching fraction between NA48/2 measurement and different theoretical predictions [22].

sensitivity. To ensure the continuation of the kaon physics at CERN after LS3, a long-term project called HIKE has been proposed [24]. The HIKE program consists of two phases.

The first phase (fig. 6) represents a continuation of the NA62 experiment with an improved experimental layout. It allows operation with four times higher beam intensity to collect about eight times more events than NA62.

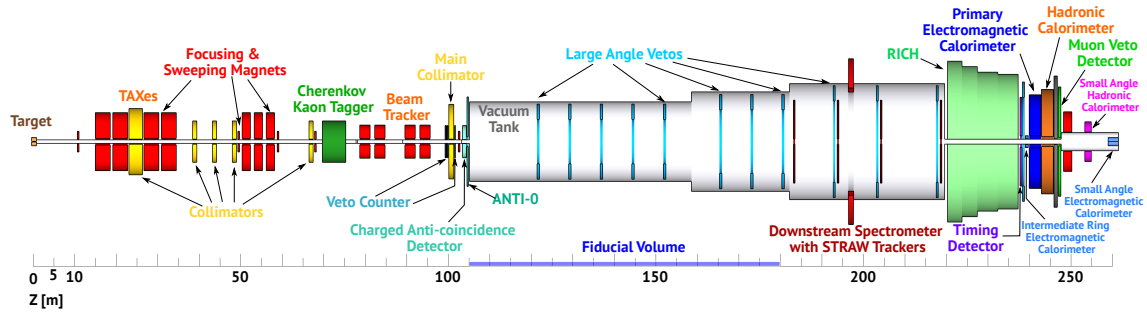


Figure 6: The schematic layout of the HIKE experiment during Phase 1 [24].

In the second phase (fig. 7), HIKE will study neutral kaon (K_L) decays. The main goal is the first observation of decays $K_L \rightarrow \pi^0 \ell^+ \ell^-$ ($\ell = e, \mu$).

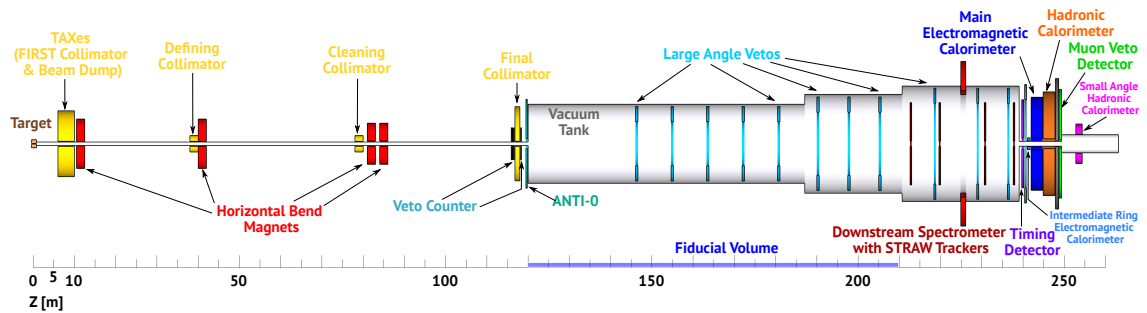


Figure 7: The schematic layout of the HIKE experiment during Phase 2 [24].

Acknowledgements

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