PROCEEDINGS OF SCIENCE

The Study of Pentaquark with Heavy Flavor Quark-Antiquark pair

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In this proceeding, we are presenting new discoveries related heavy particles known as P_c, P_{cs}, and P_b states in the recent years. These heavy baryons have the same quark flavor as the regular three quark baryons, such as nucleons and hyperons, but they also contain additional components involving $c\bar{c}$ and $b\bar{b}$. Understanding these distinct states is crucial because they reveal complexities within Quantum Chromodynamics (QCD). Our objective is to provide a comprehensive understanding of these particles and propose methods for discovering related ones.

16th International Conference on Heavy Quarks and Leptons (HQL2023)
28 November-2 December 2023
TIFR, Mumbai, Maharashtra, India

*Speaker

PoS(HQL2023)036

1. Why study $q_1q_2q_3Q\bar{Q}$?

In the traditional conventional quark model, a baryon is composed of three quarks, and the corresponding excited states are achieved through radial or angular excitations. However, even at the time when the quark model was proposed, Gell-Mann [1] and Zweig [2] suggested the possible existence of multi-quark states. The simplest multi-quark baryon would be a pentaquark state containing an anti-quark, denoted as a five-quark state, i.e., $q_1q_2q_3q_4\bar{q}_5$.

From a quark flavor perspective, pentaquark states can be distinguished into two types. The first type consists of an anti-quark and four distinct quarks, and these five-quark states are unlikely to mix with three-quark states. Until now, this type of pentaquark state has not been discovered. The second type includes an anti-quark and at least one of four quarks of the same flavor. In this case, the five-quark state shares the same quark flavor quantum numbers as the corresponding three-quark state. If other quantum numbers are also consistent, and their masses are similar, there may be significant mixing between these states, making it challenging to demonstrate whether such state is pentaquark or not.

Considerable discussion has centered around the second type of pentaquark states. For instance, the $\Lambda^*(1405)$ and Roper particle ($N^*(1440)$) are believed by many models to contain significant components of pentaquark states, such as meson-baryon molecular picture. However, their specific structures have not been conclusively determined. The main reason for this is that their excited three-quark states and five-quark states have not only consistent quantum numbers, but also similar masses. Therefore, the observed states are likely to consist of both components, making it difficult to definitively identify a particular state as purely a pentaquark state. The main reason for the similar masses is that the additional $q\bar{q}$ pairs in these pentaquark states are light-flavored quark pairs, and their masses are approximately several hundred MeV, which is similar to the excitation energy from radial or angular excitations.

To clarify whether certain particle states are purely pentaquark states, we propose a new approach [3]. This involves using heavy-flavored quark-antiquark pairs instead of light-flavored quark pairs, denoted as $q_1q_2q_3Q\bar{Q}$ where $Q = c, b$. Although the flavor quantum numbers of these pentaquark states still match those of the corresponding three-quark states, the presence of heavy-flavored quark pairs causes the mass of the pentaquark state to be significantly higher than that of the excited three-quark state. For example, $uudc\bar{c}$ state has the same quark flavor quantum numbers as proton and its excitations, but with the heavy charmed quark, the mass of such a pentaquark will be above 4.0 GeV, which is beyond the mass of proton's excitations. If such states are discovered, they would be clear candidates for pentaquark states. This highlights the importance of studying pentaquark states containing heavy-flavored quark pairs in establishing the existence of pentaquark states. Furthermore, not only the mass, but also the width of such state will be very different from the three quark state excitations. Since the annihilation of $c\bar{c}$ or $b\bar{b}$ would be suppressed compare to the decay to two charmed or bottomed hadrons, thus, if such a pentaquark state is below the threshold corresponding the meson-baryon channel, the state will have a very small decay width. Ultimately, $q_1q_2q_3Q\bar{Q}$ states with high mass, small width, but the same quark flavor as three-quark model are definitely pentaquark states.

On the other hand, when considering the inclusion of heavy anti-quark and quark pair, we can utilize heavy quark symmetry [4] in the calculation, which provides a direct interaction between hadrons. Comparatively, in contrast to the light quark pair, the uncertainty of the calculation will be suppressed.

2. What is $q_1q_2q_3Q\bar{Q}$?

On the experimental side, five pentaquark states have been discovered, with three having statistical significance exceeding 5σ, and the remaining two having statistical significance greater than 3σ but less than 5σ. All of these states are very close the corresponding meson-baryon channels, within 20 MeV difference. The specific results are shown in Table 1. Among these five states, the spin-parity (J^P) of $P_{cs}(4338)$ is measured as $J^P = \frac{1}{2}^-$, while for the others, this information has not been determined. Furthermore, all of these states are discovered from the invariant mass spectra of $J/\psi p$ and $J/\psi\Lambda$ final states for P_c and P_{cs} states, respectively. On the other hand, in the low-statistic data [5], $P_c(4440)$ and $P_c(4457)$ share a broad peak, while $P_c(4312)$ is not observed, highlighting the importance of high-statistic data for the discovery of new states. Lastly, it is important to note that all of these states were discovered by the LHCb collaboration from the decay processes of heavy b-quark hadrons. All of this experimental information indicates that the knowledge of such pentaquark is very limited, but one thing we can confirm that such $q_1q_2q_3Q\bar{Q}$ states do indeed exist.

Table 1: The masses, widths, statistical significance (SS), and the threshold of the closet channels (TCC) of five observed Pentaquarks.

State	Mass (MeV)	Width (MeV)	SS	TCC (MeV)	Ref.
$P_c(4312)$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$> 5\sigma$	$\bar{D}\Sigma_C[4320]$	[6]
$P_c(4440)$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	$> 5\sigma$	$\bar{D}^*\Sigma_C[4460]$	[6]
$P_c(4457)$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	$> 5\sigma$	$\bar{D}^*\Sigma_C[4460]$	[6]
$P_c(4337)$	4337^{+7+2}_{-4-2}	29^{+26+14}_{-12-14}	$\sim 3\sigma$	$\bar{D}\Sigma_C[4320]$	[7]
$P_{cs}(4338)$	$4338.2 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 1.3$	$> 5\sigma$	$\bar{D}\Xi_C[4335]$	[8]
$P_{cs}(4459)$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{8.0}_{-5.7}$	$\sim 3\sigma$	$\bar{D}^*\Xi_C[4475]$	[9]

Now let us turn to theoretical side. Prior to the experimental discovery of the P_c and P_{cs} states in 2015, several studies [3, 10–17] had predicted these states, particularly referred to as the $N_{c\bar{c}}$ and $\Lambda_{c\bar{c}}$ state in Ref. [3, 10] In these early studies, most of them utilized the anti-charmed meson and charmed baryon as building blocks to form pentaquarks in the molecular picture, while only Ref. [17] considered compact quark states. Here we will just focus on the molecular picture. In the studies of light hadrons, such as $\Lambda^*(1405)$, researchers have developed an approach to study them from a meson and baryon [18–20]. The interaction between meson and baryon is constructed based on chiral lagrangian, local hidden symmetry, SU(4) symmetry and so on. Then, they will obtain the T-matrix from the scattering equation, Lippmann-Schwinger equation. Finally, the resonances are located at the pole positions of T-matrix in the complex energy plain. Here, the approach is very similar, focusing on $\bar{D}^{(*)}\Sigma_c$ and $\bar{D}^{(*)}\Lambda_c$ for P_c states and $\bar{D}_s^{(*)}\Lambda_c$ and $\bar{D}^{(*)}\Xi_c^{(\prime)}$ for P_{cs} states, respectively. While the details of the interaction form and scattering equation differ in various papers, the final conclusions are very similar. The bound state of such meson-baryon states with a $c\bar{c}$ component is predicted to exist, with the lowest mass around 4.3 GeV. Notably, the predicted mass for the $\bar{D}\Sigma_c$ bound state in Ref. [14] is 4314 MeV, just 2 MeV higher than that observed in experiments.

After 2015, over a thousand papers have been published to study such pentaquark states. There are mainly three descriptions for these states: meson-baryon molecular state, the compact pentaquark [21–24] and kinematics triangle singularity structures [25–27]. More references can be found in these review papers, [28–31]. Currently, the debate over pentaquark states continues, and even within the framework of the molecular state, there is ongoing controversy over which coupled channel is the primary pentaquark state. For example, $P_{cs}(4338)$ state recognized as $\bar{D}^*\Xi_c$ [32], while most works believe it is a molecular state of $\bar{D}\Xi_c$. Furthermore, various pentaquark states are predicted, yet most of them are still missing in the observation of experiments. For example, seven P_c states are predicted around $4.3-4.6$ GeV [33, 34]. To clarify all these puzzles, we need more data about these pentaquark states.

3. Where is $q_1q_2q_3Q\bar{Q}$?

In the previous discoveries, all pentaquark states with heavy quark pairs were observed in b-quark hadron decays, with only LHCb collaboration detecting them. In such decay processes, complex hadronic loop exist, generating pure kinematic loop singularities that can produce peaks unrecognizable as resonances, known as triangle singularities. Therefore, it is crucial to discover $q_1q_2q_3Q\bar{Q}$ states through two-body collision reactions, where the triangle loop will disappear. Here we want to emphasize the importance of J/ψ photo-production process. If these states are observed in the photo-production process, the J^P quantum number can be measured. The current molecular picture favors the negative parity of P_c state, and confirming the J^P quantum number of these states will help verify the correctness of the molecular picture.

Many references have studied $\gamma p \to J/\psi p$ process [35–44]. The recent review paper is [45]. The main idea is vector meson dominate mechanism, where the γ couples directly with J/ψ, followed by the reaction $J/\psi p \to P_c \to J/\psi p$. However, in our paper [43], we found that the direct coupling of J/ψ to real photons will result in a deep off-shell effect, which suppresses the strength of this reaction. Until now, accurately estimating this effect has been a chanllenge, leading to a large uncertainty in the prediction of $\gamma p \to P_c \to J/\psi p$ with a factor of 10^3. In this process, the only model-independent conclusion is the angular distribution. If the J/ψ originates from Pomeron exchange, it primarily contributes to the cross sections at forward angles, as illustrated by the red dotted line in Fig. 1(a). This makes it easier to observe the resonance peaks at large angles. Subsequently, in Fig. 1(b,c,d), when considering the large $\theta_{J/\psi}$, the peaks related to the P_c state become more and more clearly visible. However, the magnitudes of the differential cross sections decrease rather rapidly with angles. Therefore, we recommend choosing an angle around 30^o, as it may be optimal for examining the existence P_c. As of the very recent total cross section of $\gamma p \to J/\psi p$ from the GlueX collaboration [46], there has been no signal for the P_c state.

On the other hand, in Ref. [47], we also predicted the existence of the P_b state with $qqqb\bar{b}$ components. This state has not been found yet, since it is too heavy and almost impossible to detect from the decay reactions. Therefore, the process $\gamma p \to \Upsilon p$ may be the ideal place to search for this state. Additionally, in Ref. [48], predictions were made for future electron-ion colliders (EICs) in the USA and the electron-Ion collider in China (EicC).

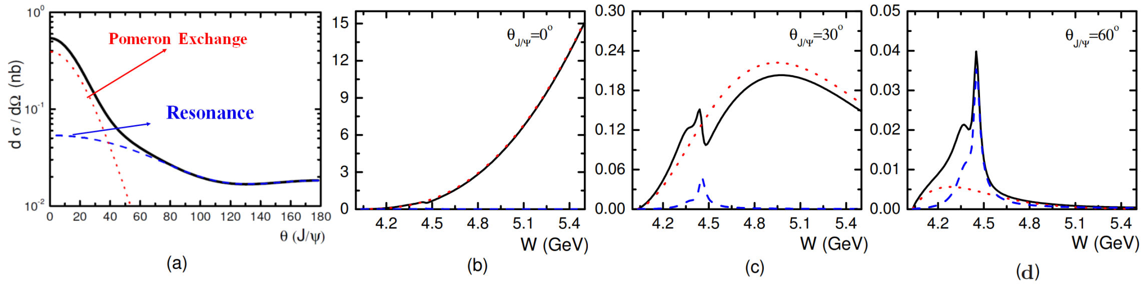

Figure 1: The differential cross sections of $\gamma p \to J/\psi p$ diagram with the angular distribution of outgoing J/ψ at invariant mass of γp W = 4.45 GeV. The red dotted, blue dashed lines are the contribution purely from the Pomeron and P_c with $\Lambda = 0.55$ GeV, respectively. The black solid line is for the coherent summation of all contributions. Figures taken from Ref. [43]. Copyright(2019) APS.

4. How to extract $q_1 q_2 q_3 Q\bar{Q}$?

In this section, we will discuss how to extract the resonance information from experimental data. In current analyses, the Breit-Wigner (BW) form is commonly used to parameterize the resonance, even when a large amount of statistical data is available, as seen in Refs. [6, 8]. For a single narrow resonance far away from the thresholds of coupled channels, the BW form is a reasonable assumption. However, when two resonances interfere in the considered energy regime, the BW form breaks the unitarity of the system. Additionally, near the threshold of coupled channels, the BW form cannot accurately describe the lineshape due to a cusp correction resulting from the opening of a new channel. These corrections can significantly impact the extraction of resonance state information. Furthermore, for resonances with relatively large widths around several hundreds of MeV, non-resonant contributions strongly influence them, while the BW form is only a reasonable approximation near the peak of the resonance. Therefore, the simple BW form is not a suitable choice for broad resonance structures. In such cases, experimental data analysis often includes an additional non-resonant background fit, and the selection of this non-resonant background component is highly arbitrary, leading to increased uncertainty in extracting resonance information. Thus, we want to emphasize the use of the coupled channel model for analyzing experimental data.

An example of this approach is the analysis of the data of invariant mass distribution of $J/\psi p$ in the reaction $\Lambda_b \to K^- J/\psi p$ from Ref. [6]. In Ref. [49], several different methods were used to analyze the data while maintaining the unitarity of $J/\psi p$ channel. As show in Fig. 2, it is evident that when only $J/\psi p$ rescattering is considered, the experimental data cannot be accurately described. However, when $\bar{D}^{(*)}\Sigma_c$ channels are included, several peaks are well explained. This comparison clearly demonstrates the importance of including all coupled channels and using a coupled channel model to fit the experimental data.

The second example involves the analysis of the $P_{cs}(4338)$. In Ref. [50], we built a data-driven Coupled-Channel Model to study the differential cross section of the reaction $B^- \to \bar{p} J/\psi \Lambda$. The model incorporates four diagrams, as illustrated in Fig. 3. Figs. 3(a,b) depict the contribution of the coupled channel system between $\bar{D}\Xi_c$ and $\bar{D}_s \Lambda_c$, where the $J/\psi\Lambda$ is directly generated from these two channels. Additionally, Fig. 3(c) shows the contribution of the intermediate channel $\bar{\Lambda}_c D$,

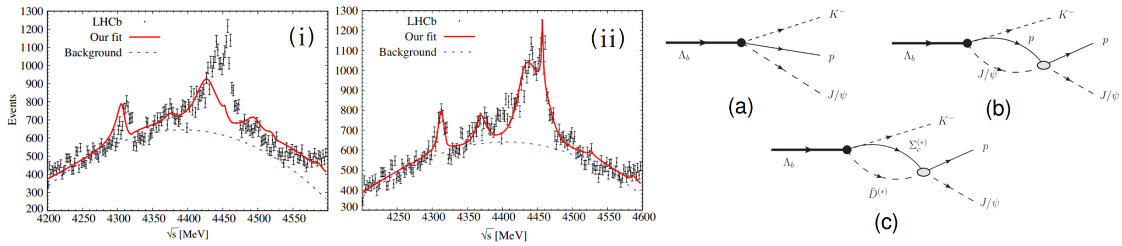

Figure 2: The differential cross sections of the invariant mass spectrum of $J/\psi p$ from $\Lambda_b \to K^- J/\psi p$. The (i) just includes the mechanisms as shown in Fig.(a, b), while (ii) includes an additional mechanism as shown in Fig.(c). The blue dotted curves are the contribution from the polynomial backgrounds and red solid lines are for the coherent summation of all contributions. Figures taken from Ref. [49]. Copyright(2019) APS.

which couples to $J/\psi \bar{p}$. Finally, Fig. 3(d) represents the background contribution. The first vertex v_1 serves as an interaction kernel, while the second vertex includes the re-scattering contribution. Further details and equations can be found in Ref. [50].

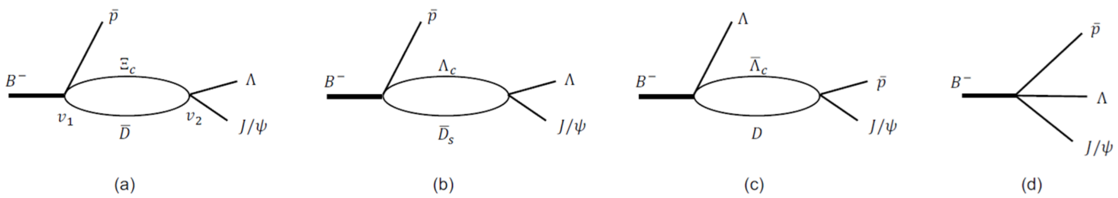

Figure 3: The diagrams of $B^- \to \bar{p} J/\psi \Lambda$. Figures taken from Ref. [50]. Copyright(2023) APS.

In Fig. 4, we present the fitting results for the four differential branching fractions of $B^- \to \bar{p} J/\psi \Lambda$, along with the contributions of different diagrams shown in Fig. 3. It is evident that the primary contributions stem from diagrams in Fig. 3(c,d), while the two peaks in the invariant mass spectrum of $J/\psi \Lambda$ arise from the contributions of Figs. 3(a,b). Comparison to the analysis model (referred to as the LHCb model) in Ref. [8] reveals unique attributes of our model. Firstly, our model includes coupled channel contribution, whereas the LHCb model solely utilizes the BW form. Given that the peak of $P_{cs}(4338)$ is in close proximity to the $\bar{D}\Xi_c$ threshold, and another peak is situated on the $\bar{D}_s\Lambda_c$ threshold at 4255 MeV, it is apparent that threshold effects are of significance, rendering the BW form less appreciate, as discussed previously. Despite the increased complexity of our model, we utilize only 9 free parameters, whereas the LHCb model employs 16 free parameters. Additionally, in our model, the non-resonance $J/\psi \bar{p}$ contribution is predominatly characterized by S-wave, unlike the P-wave dominance in the LHCb model. Given that the momenta of particles in this reaction are primarily around 130 MeV, the prevalence of S-wave contribution is more fitting.

Our model also enables the determination of the pole positions of these two peaks, allowing for an examination of the pure kinematic contribution. In addition to the pole for $P_{cs}(4338)$ at $4338.2 - i1.9$ MeV, we have identified another virtual state at 4254.7 ± 0.4 MeV, attributed to the relatively large number of events in the data at the threshold of $\bar{D}_s\Lambda_c$. While this may potentially be a statistical fluctuation, its occurrence at the threshold suggests a high likelihood that it is due to the threshold effect. Our detailed analysis further indicates that the existence of this virtual state

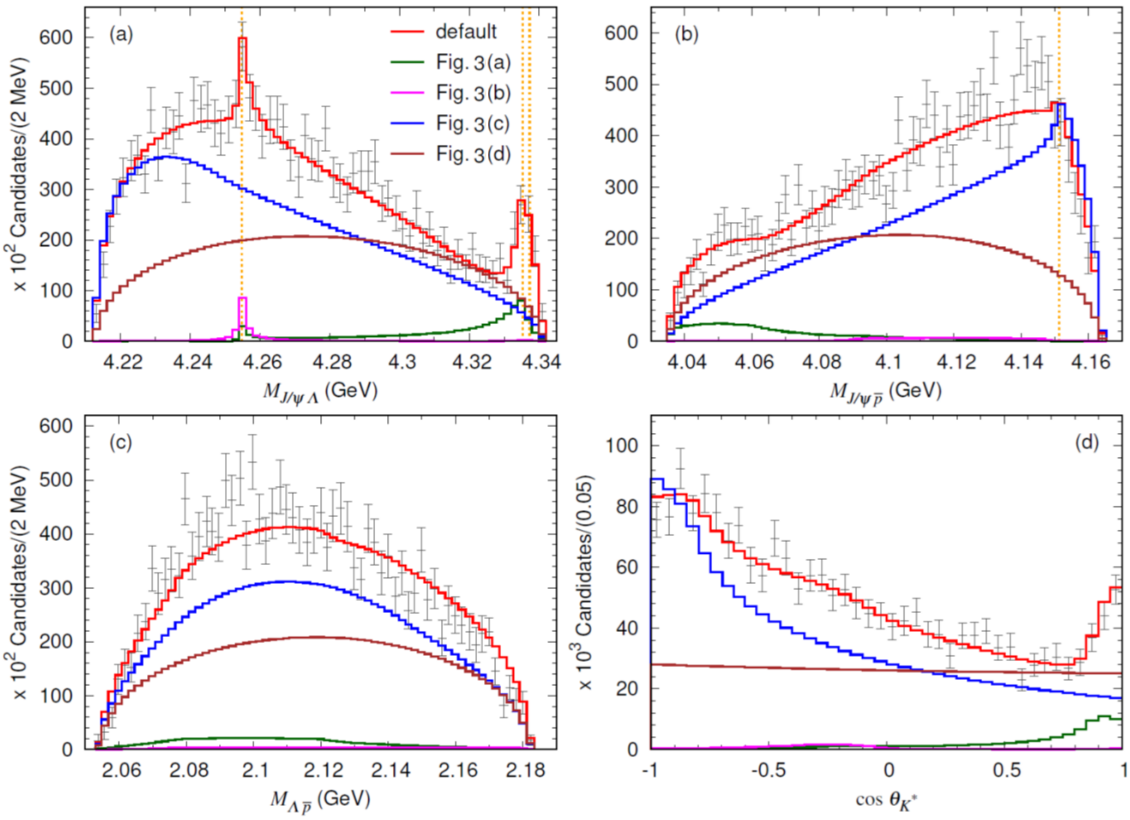

Figure 4: The differential cross section of $B^- \to \bar{p}J/\psi\Lambda$. Figures taken from Ref. [50]. Copyright(2023) APS.

impacts the unknown singularity of the P_{cs} state, underscoring the need for additional experimental data to confirm this finding. Another significant outcome of our coupled channel analysis is the demonstration that pure threshold effects are inadequate to explain the peak structure of $P_{cs}(4338)$. In our calculations, closing the coupled-channel and calculating the single loop fails to describe the peak structure of $P_{cs}(4338)$ after fitting.

From these two examples, we can conclude that a coupled channel model is essential for extracting the details of $q_1q_2q_3Q\bar{Q}$.

5. Summary

In this proceeding, we present an introduction to the pentaquark states featuring heavy antiquark and quark pairs. These states exhibit high masses and narrow widths, and can be identified as pure pentaquarks without mixing with three-quark states. While several states have been observed in experimental data, crucial information such as spin-parity is still lacking. Consequently, a confident conclusion regarding the internal structures of the existing P_c and P_{cs} states remains elusive. While many experts speculate that these states are molecular states formed with anti-charmed meson and charmed baryon, further evidence is required to support this hypothesis. The photo-production reaction may provide a promising avenue for the search of these states, but making precise predictions on the theoretical side presents challenges. To extract the information about these states, the use of coupled channel model is imperative, given their proximity to the threshold of coupled channels.

6. Acknowledgments

This work is in part supported by National Natural Science Foundation of China (NSFC) under contracts 12175239, and 12221005, and also by National Key Research and Development Program of China under Contracts 2020YFA0406400, and also by Chinese Academy of Sciences under Grant No. YSBR-101.

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