PROCEEDINGS OF SCIENCE

Twenty years of Θ^+

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Twenty years ago, in 2003, two experimental groups, LEPS and DIANA, announced the discovery of a light, narrow, exotic baryon with mass within the range of 1540 MeV, which was later dubbed as Θ^+. In this talk we recall the history of this discovery and its theoretical foundations. We also discuss possible future experiments that could determine the existence of Θ^+.

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1. Introduction

Twenty years ago, in 2003, two experimental groups, LEPS [1] and DIANA [2], announced the discovery of a light, narrow, exotic baryon with mass of the order of 1540 MeV, which was later dubbed as Θ^+. Since the decay channel was KN they concluded that the observed resonance was the lightest member of the antidecuplet of exotic pentaquark baryons, namely a $uudd\bar{s}$ state. These searches were motivated by chiral models, in particular by Ref. [3] and references therein, which almost two decades earlier, predicted light pentaquark $\overline{\mathbf{10}}$ flavor multiplet of positive parity.

Soon other experiments presented the results of their analyses in search of Θ^+, some of which confirmed the initial reports of LEPS and DIANA. A summary of the experimental results can be found in Refs. [4–6] and more recently in Ref. [7]. Obviously, none of these experiments, including LEPS and DIANA, has been designed to search for pentaquarks. People used data collected for other purposes. Only later were dedicated experiments conducted, but with mixed results. In 2004 Θ^+ paved its way to the Particle Data Group (PDG) listings [8] as a three star resonance, in 2005 its significance has been reduced to two stars, and in 2007 it has been omitted from the summary table. As of 2008, it is no longer listed by the PDG [9].

One of the peculiar features of Θ^+, if it it exists, is its very small decay width. Indeed, in 2006, the BELLE collaboration reported search results for Θ^+ [10]. No formation signal of the Θ^+ baryon was observed, and an upper limit on the Θ^+ width was estimated: $\Gamma < 0.64$ MeV for $M_{\Theta^+} = 1539$ MeV. Also DIANA in 2006 [11] confirmed their initial observation with the mass of $M_{\Theta^+} = 1537 \pm 2$ MeV estimating the width: $\Gamma = 0.36 \pm 0.11$ MeV. Is such a small width *unnatural*? – a question often raised against Θ^+. In our opinion: not really. Let us recall that recently the LHCb collaboration at CERN reported an excited $\Omega_c(3050)$ state with a reportedly very small width: $\Gamma = 0.8 \pm 0.2 \pm 0.1$ MeV [12]. In a later publication from 2021 [13] the LHCb collaboration concluded that: *The natural width of the* $\Omega_c(3050)$ *is consistent with zero*. $\Omega_c(3050)$ was found in the decay to $\Xi_c K^-$ [12] where the kaon momentum is $p = 275$ MeV. This is approximately ~ 10 MeV above the kaon momentum in the decay of Θ^+. From this perspective the small pentaquark width is not particularly *unnatural*. As a consequence, the small width of $\Omega_c(3050)$ led to its interpretation as a heavy charm pentaquark belonging to the exotic SU(3) $\overline{\mathbf{15}}$ multiplet [14–16].

Clearly, most non-observation experiments do not really exclude the existence of Θ^+, but rather put an upper limit on its production cross-section. The cleanest and decisive experiment would be the so called *formation experiment* where the resonance is directly produced in KN reaction. DIANA is exactly such an experiment where the liquid Xenon bubble chamber was exposed to a separated K^+ beam. On the contrary, LEPS was a photoproduction experiment on carbon nucleus ^{12}C. In the follow-up analyses both DIANA [11, 17, 18], and LEPS in the dedicated photoproduction experiment on deuteron [22, 23], confirmed their initial findings. These results have survived unchallenged to this day.

In this paper in Sect. 2 we briefly describe chiral models, and in Sec. 3 the emergence of baryons including exotica. Then, in Sec. 4, we discuss phenomenology including Θ^+ mass and width, and briefly the properties of other members of $\overline{\mathbf{10}}$. Most important experiments are reviewed in Sec. 5. Summary is given in Sec. 6.

2. Chiral Models and Solitons

Chiral models are effective models for Quantum Chromodynamics (QCD), which explore chiral symmetry and its spontaneous and explicit breaking, and are tractable in the low energy regime. One can imagine that we integrate out from the QCD Lagrangian gluon fields. We are then left with the quark degrees of freedom only, which have the canonical kinetic and possibly mass term, however the interaction Lagrangian consists of an infinite number of nonlocal many-quark vertices. Nevertheless, this effective Lagrangian will be chirally invariant. One can truncate this Lagrangian to local four-quark interaction only, the so called Nambu–Jona-Lasinio model [24, 25].

To ensure chiral invariance it is convenient to introduce eight auxiliary pseudo-Goldstone fields $\boldsymbol{\varphi}$ (pions, kaons and η) in a form of a unitary SU(3) matrix

$$U = \exp\left(i\frac{2\boldsymbol{\lambda}\cdot\boldsymbol{\varphi}}{F}\right), \tag{1}$$

where $\boldsymbol{\lambda}$ are Gell-Mann matrices and F is a pseudoscalar (pion) decay constant that in the present normalization is equal to 186 MeV.

The simplest Lagrangian following from the above procedure, a chiral quark model Lagrangian, is given by

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{\psi}\left(i\partial\!\!\!/ - m - MU^{\gamma_5}\right)\psi\,, \tag{2}$$

where

$$U^{\gamma_5} = U\frac{1+\gamma^5}{2} + U^{\dagger}\frac{1-\gamma^5}{2}\,. \tag{3}$$

This remarkably simple Lagrangian has been in fact derived [26, 27] in the mid eighties from the instanton picture of the QCD vacuum [28, 29]. Here M denotes the *constituent* quark mass of the order of 350 MeV and m is a *current* mass matrix.

Chiral symmetry corresponds to the independent global SU(3) rotations of left and right fermions:

$$\psi_L \to L\psi_L, \quad \psi_R \to R\psi_R\,. \tag{4}$$

Transformations (4) leave the interaction term invariant if

$$U \to LUR^{\dagger}\,, \tag{5}$$

which is nothing else but a nonlinear realization of chiral symmetry [35]. Vacum state corresponding to $U = 1$ (or $\boldsymbol{\varphi} = 0$) breaks this $\mathrm{SU_L}(3)\otimes\mathrm{SU_R}(3)$ symmetry to the vector SU(3): $L = R$, and breaks the axial symmetry $L = R^{\dagger}$.

We can further integrate the quark fields (employing suitable regularization). Then the kinetic part for the Goldstone bosons appears [31–34] and we end up with a Lagrangian given in terms of the Goldstone bosons alone. This Lagrangian is organized as a power series in Goldstone boson momenta, *i.e.* in terms of $\partial_\mu U$. Such lagragians are used for precision calculations in chiral perturbation theory [35].

The first term in $\partial_\mu U$ expansion, a quadratic term, is fully dictated by the chiral symmetry, and is known as the Weinberg Lagrangian [36]. Higher order terms of known group structure have, however, free coefficients that are not constrained by any symmetry and have to be extracted from experimental data. Obviously, once we have at our disposal a reliable Lagrangian like (2), we can compute effective Goldstone boson Lagrangian to any order in $\partial_\mu U$.

A simple, truncated Lagrangian with four derivatives only was proposed by Skyrme [37, 38] in 1961 and later generalized by Witten [39, 40]

$$\mathcal{L}_{\rm Sk} = \frac{F^2}{16}{\rm Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}{\rm Tr}([\partial_\mu U\, U^\dagger, \partial_\nu U\, U^\dagger]^2) + \mathcal{L}_{\rm m}\,. \tag{6}$$

The first term in (6) is the Weinberg Lagrangian, the second one is called the *Skyrme term*. Parameter e can be inferred from the pion scattering and is of the order $e = 4 \div 6$. Here $\mathcal{L}_{\rm m}$ stands for an explicit mass Lagrangian for Goldstone bosons, which breaks chiral symmetry.

While both models (2) and (6) can be expanded in powers of $\boldsymbol{\varphi}$ generating *perturbative* Goldstone boson interactions, they also admit nonperturbative solutions of finite mass and size, namely the *solitons*.

Soliton solutions in both models correspond to a specific static form of the chiral field U known as a *hedgehog* Ansatz

$$U(\boldsymbol{r}) = \exp\left(i\, \boldsymbol{n}\cdot\boldsymbol{\tau}\, P(r)\right) \tag{7}$$

where $\boldsymbol{n} = \boldsymbol{r}/r$ and $\boldsymbol{\tau} = \{\lambda_1, \lambda_2, \lambda_3\}$. Function $P(r)$ has to vanish at infinity, so that $U \to 1$. Hedgehog Ansatz (7) has a very special property: any spacial rotation of the unit vector $\boldsymbol{n}$ can be undone by an internal SU(2) (isospin) rotation acting on Pauli matrices $\boldsymbol{\tau}$. This property is called *hedgehog symmetry*.

In a seminal paper from 1979 Witten suggested that that baryons emerge as solitons in the chiral effective theory [41]. While this is true both in the Nambu–Jona-Lasinio (NJL) model (2) and the Skyrme model (6), there are important differences. In the NJL (or Chiral Quark Soliton, χQS for short) model, the specific form of the chiral background field (7) leads to a rearrangement of the energy levels of the Dirac equation corresponding to (2). The lowest positive energy level falls into the mass gap, and the sea levels are distorted, leading a stable static configuration corresponding to a self-consistently determined form of the profile function $P(r)$ in (7). The energy of this configuration is computed as a regularized sum over all energy levels relative to the vacuum (see *e.g.* [42]),

$$M_{\rm sol} = N_c\Big[E_{\rm val} + \sum_{E_n<0}(E_n - E_n^{(0)})\Big]. \tag{8}$$

This is schematically illustrated in Fig. 1. Such configuration carries no quantum numbers other than the baryon number following from the baryon number of the valence quarks.

A convenient way of calculating the soliton mass (8) is variational principle where the soliton size r_0 is treated as a variational parameter. The result is shown in Fig. 2. We see that for large soliton size the valence level sinks into the Dirac sea, whereas for small soliton size entire energy is equal to the energy of the valence quarks. These two limits are called the Skyrme Model (SM) limit and the Nonrelativistic Quark Model (NRQM) limit, respectively. χQS Model interpolates between the two limits.

Therefore, the Skyrme Model can be viewed as a configuration shown in Fig. 1 where the valence level sinked into the negative energy Dirac sea. In this case the soliton energy is given entirely by the energy of the distorted sea levels. An approximate formula for this energy can be

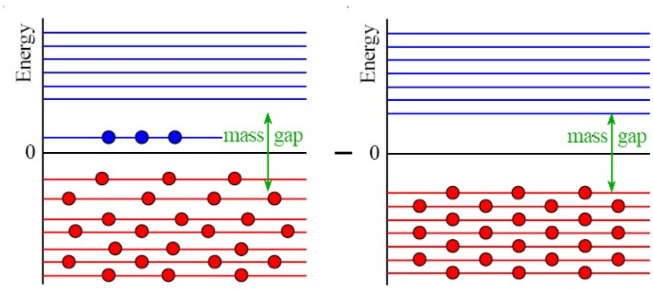

Figure 1: Schematic illustration of the calculation of the soliton mass χQS model, which is the sum over the energies of the valence quarks, and the properly regularized sum over the sea quarks with vacuum contribution subtracted, see Eq. (8).

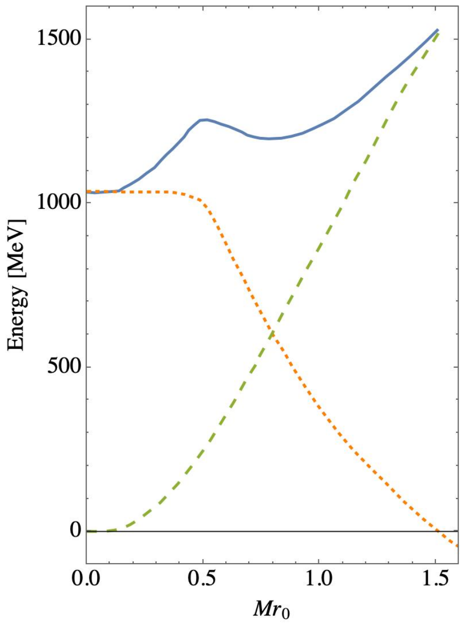

Figure 2: Soliton energy (mass) in MeV for $M = 345$ MeV as a function of a dimensionless variational parameter Mr_0: solid (blue) – total mass, short dash (orange) – energy of valence quarks, long dash (green) – sea contribution. Minimum of $\sim$ 1200 MeV corresponds ro $r_0 \simeq 0.5$ fm. Figure from Ref. [43].

obtained by a heat kernel expansion and can be expressed in terms of functionals of the profile function $P(r)$ with no reference to the underlying Dirac structure. In the simplest case it is just the energy corresponding to Lagrangian (6) with the profile function $P(r)$ determined from the pertinent equations of motion.

Let's recall that the hedgehog Ansatz $U(\boldsymbol{r})$ (7) can be viewed as a mapping of $R^3 \rightarrow \mathrm{SU}(3)$, which is charcterized by a winding number $N_\mathrm{w} = P(0)/\pi$. According to Witten [41] N_w can be interpreted as a baryon number. In the case of the χQS model the condition $P(0)/\pi =$ integer is not necessary [43], but the number of valence levels is given by N_w. Moreover, the requirement that $P(0)/\pi =$ integer ensures that the energy of the soliton in the Skyrme model is finite. Therefore solitons in the Skyrme model are referred to as *topological*, while in the NJL model as *nontopological*.

So both models described above have soliton solutions of finite energy and size (corresponding to the *size* of the profile function $P(r)$), which have baryon number $B = 1$ and no other quantum numbers. We call such a configuration a *classical* baryon. In the next Section we show how other quantum numbers such as isospin or spin are generated and discuss the following mass formulas.

3. Baryons in chiral models

In order to provide the *classical* baryon with specific quantum numbers one has to consider an SU(3)-rotated time-dependent pseudoscalar field

$$U(t, \boldsymbol{r}) = A(t)U(\boldsymbol{r})A^\dagger(t) \tag{9}$$

and derive the pertinent Lagrangian expressed in terms of the collective velocities $da_\alpha(t)/dt$ defined as follows

$$A^\dagger(t)\frac{dA(t)}{dt} = \frac{i}{2}\sum_{\alpha=1}^{8}\lambda_\alpha \frac{da_\alpha(t)}{dt}\,. \tag{10}$$

At this point it is important no note that $A \in$ SU(3)/U(1) rather than full SU(3), since for the hedgehog Ansatz (7) $[\lambda_8, U(\boldsymbol{r})] = 0$. Therefore matrix A is defined up to a *local* U(1) factor $h = \exp(i\lambda_8\phi)$, *i.e.* A and Ah are equivalent. For this reason the eighth coordinate $a_8(t)$ is not dynamical and does not appear in the kinetic energy of the rotating soliton. Instead, it provides a constraint on the allowed states in the collective (*i.e.* corresponding to rotations (9)) Hilbert space.

Standard quantization procedure [44–47] leads to the rotational Hamiltonian, which has a form of a quantum mechanical symmetric top [48]

$$\mathcal{H}_{\rm rot} = M_{\rm sol} + \frac{1}{2I_1}J(J+1) + \frac{1}{2I_2}\left[C_2(\mathcal{R}) - J(J+1) - \frac{3}{4}Y'^2\right] \tag{11}$$

where $C_2(\mathcal{R})$ stands for the SU(3) Casimir operator and J for the soliton angular momentum (spin). Soliton mass $M_{\rm sol}$ and moments of inertia $I_{1,2}$ are calculable within a given model. The constraint mentioned above selects representations $\mathcal{R}$, which contain states of hypercharge Y' equal to

$$Y' = \frac{N_{\rm c}}{3}\,. \tag{12}$$

Soliton spin J is equal to the isospin of states on the Y' rung of the pertinent weight diagram, Fig.3.

In the present case for $N_c = 3$ we have that $Y' = 1$ and the allowed representations are

$$\mathcal{R} = \mathbf{8},\ \mathbf{10},\ \overline{\mathbf{10}},\ \mathbf{27},\ \mathbf{35},\ \overline{\mathbf{35}},\ldots\,. \tag{13}$$

We see that in addition to the octet and decuplet of positive-parity baryons, well known from the quark model, exotic representations, like $\overline{\mathbf{10}}$, emerge, all of positive parity.

Collective Hamiltonian and the constraint (12) are exactly the same in the chiral quark model and in the Skyrme model. The only obvious difference is that the soliton mass and the moments of inertia are in the Skyrme model expressed in terms of space integrals over some functionals of the profile function $P(r)$, while in the case of the quark model they are given as regularized sums over the one particle energy levels of the Dirac Hamiltonian corresponding to Eq. (2).

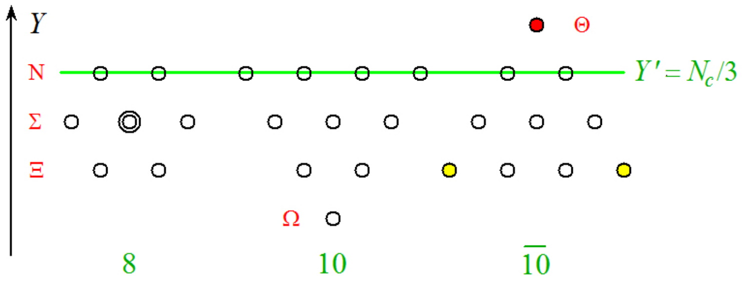

Figure 3: SU(3) representations selected by the constraint (12). Isospin of states on the $Y' = 1$ line is equal to the soliton spin J.

Wave functions for a quantum mechanical symmetric top are given in terms of Wigner $D^{(\mathcal{R})}_{ab}$-functions [48]. Skipping technicalities [49], baryon wave function takes the following form[1]

$$\psi^{(\mathcal{R})}_{(B,J,J_3)}(A) = (-)^{J_3 - Y'/2}\sqrt{\dim(\mathcal{R})}\, D^{(\mathcal{R})*}_{(Y,T,T_3)(Y',J,-J_3)}(A)\,. \tag{14}$$

Here $B = (Y, T, T_3)$ stands for the SU(3) quantum numbers of a baryon in question, and the second index of the D function, $(Y', J, -J_3)$, corresponds to the soliton spin.

We see that the problem of baryon properties has been reduced to the quantum mechanical Hamiltonian with well defined Hilbert space and explicit wave functions. One can therefore compute all matrix elements needed for mass splittings, currents and other quantities.

For $m = 0$ all states in a given representation $\mathcal{R}$ are degenerate. In order to compute the mass splittings we have to express the symmetry breaking Hamiltonian, which is proportional to m, in terms of the collective coordinates. In the χQS Model the result reads

$$\mathcal{H}_{\rm br} = \alpha\, D^{(\mathbf{8})}_{88}(A) + \beta\,\hat{Y} + \frac{\gamma}{\sqrt{3}}\sum_{i=1}^{3} D^{(\mathbf{8})}_{8a}(A)\,\hat{J}_a, \tag{15}$$

where α, β, and γ are proportional to the strange quark mass.[2] Furthermore, α scales as N_c, and β and γ scale as N_c^0. $\hat{Y}$ and $\hat{J}_a$ are hypercharge and soliton spin operators, respectively.

Formally, β and γ are zero in the Skyrme model [50, 51]. In the large N_c limit baryons consist from N_c quarks, and therefore the hypercharge eigenvalue of the physical states is also $Y \sim N_c$. This means that the second term in (15), including $\hat{Y}$, is of the order $O(m_s N_c)$[3] exactly as the first term proportional to α. It was Gudagnini [44] who argued that $\beta\hat{Y}$ should be included in the Skyrme model. In the chiral quark model it arises naturally from $1/N_c$ expansion, similarly to the sub-leading term proportional to γ. Explicit expressions for the coefficients α, β and γ are given *e.g.* in Eq. (4) of Ref [52] and the mass splitting including representation mixing are discussed in Ref. [53].

Since we have identified the symmetries of the soliton, it is straightforward to compute the pertinent currents, in particular the axial current [54]. The axial current is of interest here, since via the Goldberger-Treiman relation it can be related to strong baryon decays.[4] In the non-relativistic

[1] One can find different representations of this wave function in the literature that are equivalent to the one used here.
[2] For simplicity we assume $m_u = m_d = 0$ and then $m = m_s$.
[3] Matrix elements of $D^{(\mathbf{8})}_{88}$ are $O(N_c^0)$.
[4] This approach to the width calculations has been criticised in the literature, see *e.g.* Ref. [55].

limit for the initial and final baryons, B_1 and B_2 respectively, the baryon-baryon-meson coupling can be written in the following form [3]:

$$O_\varphi = 3 \sum_i \left[G_0 D^{(\mathbf{8})}_{\varphi\, i} - G_1\, d_{3bc} D^{(\mathbf{8})}_{\varphi\, b} \hat{J}_c - G_2 \frac{1}{\sqrt{3}} D^{(\mathbf{8})}_{\varphi\, 8} \hat{J}_i \right] \frac{p_i}{M_1 + M_2} \tag{16}$$

where $M_{1,2}$ denote masses of the initial and final baryons and p_i is the c.m. momentum of the outgoing meson, denoted as φ, of mass m:

$$|\boldsymbol{p}\,| = p = \frac{1}{2M_1} \sqrt{(M_1^2 - (M_2 + m)^2)(M_1^2 - (M_2 - m)^2)}\,. \tag{17}$$

The factor of 3 in Eq. (16) is a matter of convenience.

The decay width is related to the matrix element of O_φ squared, summed over the final and averaged over the initial spin and isospin denoted as $\overline{[\ldots]}^2$, see the Appendix of Ref. [3] for details of the corresponding calculations:

$$\Gamma_{B_1 \to B_2 + \varphi} = \frac{1}{2\pi} \overline{\left\langle B_2 \left| O_\varphi \right| B_1 \right\rangle}^2 \frac{M_2}{M_1} p. \tag{18}$$

Factor M_2/M_1, used already in Ref. [14], is the same as in heavy baryon chiral perturbation theory; see *e.g.* Ref. [56, 57].

Here some remarks are in order. While the mass spectra are given as systematic expansions in both N_c and m_s, the decay widths cannot be organized in a similar way. They depend on modelling and 'educated' guesses, and hence are subject to additional uncertainties [58]. Most important uncertainty comes from the fact that the baryon masses M_1 and M_2 are formally infinite series in N_c and m_s. The same holds for the momentum of the outgoing meson. It is a common practice to treat the phase factor exactly, rather than expand it up to a given order in N_c and m_s, despite the fact that in O_φ only a few first terms in $1/N_c$ and m_s are included. Here we have adopted a convention with $M_1 + M_2$ in (16) and M_2/M_1 in (18), for other choice see *e.g.* [58]. Formally, in the large N_c limit and small m_s limit $M_1 = M_2$ and both conventions are identical. Nevertheless, if we use physical masses for $M_{1,2}$, different conventions will result in different numerical results.

The leading term proportional to $G_0 \sim N_c$ has been introduced already in the Skyrme model in Ref. [50], whereas the subleading terms $G_{1,2} \sim N_c^0$ have been derived in the chiral quark model [3, 54].

Since we know the collective wave functions (14), it is relatively straightforward to compute the matrix elements for the mass splittings and decay widths. They are simply given in terms of the SU(3) Clebsch-Gordan coefficients [59].

4. Antidecuplet phenomenology

4.1 Θ^+ mass

In principle we can compute the classical mass, moments of inertia (11) and splitting parameters (15) in either model (SM or χQS), however – following Ref. [50] – we adopt here a *model-independent* approach and try to constrain these parameters from the data. Mass splittings between different multiplets are related to the moments of inertia $I_{1,2}$ of the rotating soliton:

$$\Delta_{\mathbf{10}-\mathbf{8}} = \frac{3}{2}\frac{1}{I_1}, \quad \Delta_{\overline{\mathbf{10}}-\mathbf{8}} = \frac{3}{2}\frac{1}{I_2}. \tag{19}$$

Therefore, we have no handle on the strange moment of inertia I_2, and it is impossible to predict the mean antidecuplet mass from the non-exotic baryon masses alone.

Chiral symmetry breaking terms following from the fact that $m_s > m_{u,d} \simeq 0$ generate mass splittings within the $\mathrm{SU}(3)_{\text{flavor}}$ multiplets [3]:

$$\begin{aligned}
\Delta M_{\mathbf{8}} =& \frac{1}{20}(2\alpha+3\gamma) + \frac{1}{8}\Big[(2\alpha+3\gamma) + 4(2\beta-\gamma)\Big]Y \\
&- \frac{1}{20}(2\alpha+3\gamma)\left[T(T+1) - \frac{1}{4}Y^2\right], \\
\Delta M_{\mathbf{10}} =& \frac{1}{16}\Big[(2\alpha+3\gamma) + 8(2\beta-\gamma)\Big]Y, \\
\Delta M_{\overline{\mathbf{10}}} =& \frac{1}{16}\Big[(2\alpha+3\gamma) + 8(2\beta-\gamma) + 4\gamma\Big]Y,
\end{aligned} \tag{20}$$

where parameters α, β and γ are proportional to m_s. Note that in the Skyrme model $\gamma = 0$, and $\beta = 0$ if we do not take into account the Guadagnini term [44]. Equations (20) are written in a form, from which one can immediately see that mass splittings of non-exotic baryons depend in fact only on two combinations of parameters α, β and γ, namely on $2\alpha + 3\gamma$ and $2\beta - \gamma$, whereas mass splittings in exotic $\overline{\mathbf{10}}$ depend additionally on γ. Again, this means that we cannot predict mass splittings within $\overline{\mathbf{10}}$ from the spectrum of non-exotic baryons.

Resorting to the model calculations one obtains a relatively small $\overline{\mathbf{10}} - \mathbf{8}$ splitting, namely $\Delta_{\overline{\mathbf{10}}-\mathbf{8}} \simeq 600$ MeV [60, 61]. One can now make a rough estimate of the Θ^+ mass accepting the Skyrme model $\Delta_{\overline{\mathbf{10}}-\mathbf{8}}$ given above and assuming that the mass splittings in $\overline{\mathbf{10}}$ are approximately equal to the ones in the decuplet, $140 \div 150$ MeV. One obtains then that Θ^+ mass is as low as ~ 1460 MeV [62]. This is much lower than any quark model expectations. More detailed analyses in the Skyrme model [62, 63] and in the quark-soliton model [3] led to the mass $1530 \div 1540$ MeV, which has been reinforced by the experimental results of LEPS [1] and DIANA [2].

4.2 Θ^+ width

There are two possible strategies to constrain the decay parameters $G_{0,1,2}$ (16): one can either try to use directly data on strong decays, or use the Goldberger-Treiman relation:

$$\{G_0, G_1, G_2\} = \frac{M_1 + M_2}{2F_\varphi}\frac{1}{3}\{a_0, -a_1, -a_2\} \tag{21}$$

where constants $a_{0,1,2}$ enter the definition of the axial-vector current [54, 64, 65] and can be extracted from the semileptonic decays of the baryon octet [66].

Here, rather than discussing decay phenomenology in detail, we shall concentrate on a very peculiar feature of the antidecupet decay constant (18)

$$\overline{\langle N | O_K | \Theta^+ \rangle^2} \sim G^2_{\overline{10}\to 8}, \qquad G_{\overline{10}\to 8} = -a_0 - \frac{N_c+1}{4}a_1 - \frac{1}{2}a_2, \tag{22}$$

where we have explicitly displayed the N_c dependence following from the pertinent N_c dependence of the flavor SU(3) Clebsch-Gordan coefficients [67].

For small soliton size (the NRQM limit) one can compute constants $a_{0,1,2}$ analytically [64]

$$a_0 \to -(N_c+2),\ a_1 \to 4,\ a_2 \to 2\,. \tag{23}$$

which gives

$$G_{\overline{10}\to 8} = 0\,. \tag{24}$$

We see that the decay constant of antidecuplet is zero! The cancellation takes place for any N_c [67]. This explains the smallness of the Θ^+ width, which for realistic soliton size is not equal to zero, but still very small. In contrast, the decuplet decay constant is large, $G_{10\to 8} = N_c + 4$, explaining the large width of the Δ resonance.

This is not the only reason why Θ^+ width may be very small. The symmetry breaking Hamiltonian (15) inevitably introduces representation mixing [3, 68], which in the case of the nucleon takes the following form

$$|\, N^{\text{phys}}\rangle = \cos\alpha\ |\, N_{\mathbf{8}}\rangle + \sin\alpha\ |\, N_{\overline{\mathbf{10}}}\rangle\,, \tag{25}$$

where $\sin\alpha > 0$ is small and therefore $\cos\alpha \simeq 1$. Note that Θ^+ does not mix, and $|\,\Theta^{\text{phys}}\rangle = |\,\Theta_{\overline{\mathbf{10}}}\rangle$. Therefore, the decay of Θ^+ to KN proceeds either directly to $|\,N_{\mathbf{8}}\rangle$ or through mixing with $|\,N_{\overline{\mathbf{10}}}\rangle$

$$g_{\Theta NK} \simeq G_{\overline{10}\to 8} + \sin\alpha\, H_{\overline{10}\to\overline{10}} \tag{26}$$

leading to a new decay constant $H_{\overline{10}\to\overline{10}}$

$$H_{\overline{10}\to\overline{10}} = -a_0 - \frac{5}{2}a_1 + \frac{1}{2}a_2\,. \tag{27}$$

From the model calculations and phenomenological studies one finds that $H_{\overline{10}\to\overline{10}}$ is large and negative [68–70]. Indeed, in the NRQM limit (23) $H_{\overline{10}\to\overline{10}} = -4$. This leads to a strong cancellation in Eq. (26), yielding the decay width of Θ^+ very small.

The observation that the width of Θ^+ may be quite small was perhaps the most important result of Ref. [3] since the smallness of its mass was anticipated a decade earlier [60, 61, 63] (see the discussion of the Θ^+ width in Refs. [71–73]).

4.3 Other members of antidecuplet

As explained after Eq. (20), even if we anchor the exotic antidecuplet taking for the Θ^+ mass the experimental value from LEPS and DIANA, we still cannot predict the masses of other members of $\overline{\mathbf{10}}$. Here we have two truly exotic pentaquarks corresponding to the corners of antidecuplet (marked as yellow circles in Fig. 3), namely Ξ^+ and Ξ^{--}, whose quantum numbers cannot be constructed from 3 quarks, and the remaining cryptoexotic states whose quantum numbers can be constructed both from 3 or 5 quarks.

In 2003 the NA49 Collaboration at CERN announced the observation of an exotic Ξ^{--} pentaquark (lower left vertex in Fig. 3) at 1.862 GeV [74]. If confirmed, it would be the second input besides Θ^+ to anchor the exotic antidecuplet. Unfortunately, 17 years later the successor of NA49

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the NA61/SHINE Collaboration did not confirm the Ξ^{--} peak around 1.8 GeV with 10 times greater statistics [75]. One possible reason of this non-observation might be the extremely small width of Ξ^{--}. Indeed, in Ref. [58] it was argued that in the SU(3) symmetry limit (*i.e.* without mixing effects) this width is up to a factor of ~ 2 equal to the width of Θ^+, *i.e* of the order of 1 MeV. Original analysis of NA49 [74] reported the width Ξ^{--} below detector resolution of 18 MeV, while NA61/SHINE [75] does not discuss their sensitivity to the width of Ξ^{--}.

There exists, however, a potential candidate for a cryptoexotic pentaquark, namely the nucleon resonance $N(1685)$ [76], which has been initially announced by GRAAL Collaboration at NSTAR Conference in 2004 [77]. $N(1685)$ has been observed in the quasi-free neutron cross-section and in the ηn invariant mass spectrum [78, 79] and was later confirmed by CBELSA/TAPS [80] and LNS-Sendai [81]. The observed structure can be interpreted as a narrow nucleon resonance with the mass 1685 MeV, total width $\leq$ 25 MeV and the photocoupling to the proton much smaller than to the neutron. Especially the latter property is easily understood assuming that $N(1985)$ is a cryptoexotic member of $\overline{\mathbf{10}}$ [82, 83].

The argument for small proton coupling is based on an approximate U-spin sub-symmetry of flavor SU(3). Both η and photon are U-spin singlets and neutron and proton are U-spin triplet and doublet, respectively. The neutron- and proton-like members of $\overline{\mathbf{10}}$ are U-spin triplet and 3/2 multiplet, respectively. Therefore in the SU(3) symmetry limit proton photo-excitation to $p_{\overline{\mathbf{10}}}+\eta$ is forbidden, while neutron transition to $n_{\overline{\mathbf{10}}}+\eta$ is allowed. For alternative explanations see Refs. [84–86].

It was found that the width of $N(1685)$ is in the range of tens of MeV with a very small πN partial width of $\Gamma_{\pi N} \leq 0.5$ MeV [87]. One should stress that the decay to πN is not suppressed in the SU(3) limit and it can be made small only if the symmetry violation is taken into account. Therefore in Ref. [69] masses and widths of exotic $\overline{\mathbf{10}}$ were reanalyzed taking into account mixing of the ground state octet with antidecupelt, already discussed in Sect. 4.2, and anidecuplet mixing with the excited Roper resonance octet. Taking into account all the available data on different branching ratios and some model input it was possible to constrain the mixing angles[5] leading to

$$\begin{aligned} 1795 \text{ MeV} &< M_{\Sigma_{\overline{10}}} < 1830 \text{ MeV}\,, \\ 1900 \text{ MeV} &< M_{\Xi_{\overline{10}}} < 1970 \text{ MeV} \end{aligned} \tag{28}$$

with decay widths

$$\begin{aligned} 9.7 \text{ MeV} &< \Gamma_{\Sigma_{\overline{10}}} < 26.9 \text{ MeV}\,, \\ 7.7 \text{ MeV} &< \Gamma_{\Xi_{\overline{10}}} < 11.7 \text{ MeV}\,. \end{aligned} \tag{29}$$

These limits follow from the assumptions that Θ^+ mass is 1540 MeV and its width is 1 MeV, and that the decay width of $N(1685)$ is smaller than 25 MeV. One sees that the decay width of $\Xi_{\overline{10}}$ is still small, but larger than in the SU(3) limit. Its mass is still in the range scanned by NA61/SHINE.

[5] Due to the accidental equality of the SU(3) Clebsch-Gordan coefficients mixing angles of Σ and N states in ocet and decuplet are equal, so only two mixing angles were necessary for the discussed mixing pattern.

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5. Experiments

The positive evidence for Θ^+ by LEPS [1] and DIANA [2] has prompted a number of searches by other experimental groups. At that time, only data collected originally for searches other than Θ^+ was available. Only later were dedicated experiments designed and conducted. For a complete list of experiments we refer the reader to reviews from 2008 [4], from 2014 [5] and to a more recent review from 2022 [7]. Below we will briefly recall only a few experiments, mainly those that have so far uphold their initial positive results.

5.1 Photoproduction

Photoproduction on a nucleon is not the best experiment to discover Θ^+. Indeed, photon has to dissociate into an $s\bar{s}$ pair, and the antistrange quark has to be injected into a nucleon. In practice γ can dissociate in K^+K^- but not into $K^0\bar{K}^0$. In the first case K^+ may excite neutron to Θ^+, but for Θ^+ to be produced on a proton, K^0 must be replaced by $K^{0\,*}$. Furthermore, since $g_{\Theta NK}$ is very small the cross-section will be small as well. The situation is even worse for the photoproduction on a proton, since $g_{\Theta NK^*}$ is not known. Estimates based on the SU(3) symmetry and experimental results on η photoproducion off the neutron show that the cross-section for the reaction $\gamma p \to \bar{K}^0\Theta^+$ is of the order of 1 nb [19]. For this reason, the negative result of the CLAS experiment with the proton target in 2006 [20] was not surprising (superseding earlier positive report [21] from 2003).

Nevertheless, at LEPS (Laser-Electron Photon facility at SPring-8 in Japan) Θ^+ peak was observed just in photoproduction on a neutron inside a carbon nucleus: $\gamma n \to K^-\Theta^+$ and subsequently $\Theta^+ \to K^+n'$. Only kaons were detected, and the peak was observed in the missing mass $M_{\gamma K^-} = E_\gamma - E_{K^-}$ distribution. The main problem was, however, that since the target neutron was inside the carbon nucleus, its momentum was smeared by Fermi motion. After applying the Fermi motion correction, the Θ^+ peak was clearly visible at $M_{\Theta^+} = 1.54 \pm 0.01$ GeV with 4.6 σ Gaussian significance. The width has been estimated to be smaller than 25 MeV.

Five years later, in 2008, LEPS published results from a dedicated photo-production experiment, this time on a deuteron target [22]. Although the measurement strategy was basically the same as in the case of carbon, the deuteron setup offered a possibility to cross-check the pentaquark production in a reaction $\gamma n \to K^-\Theta^+$ with $\Lambda(1520)$ production in $\gamma p \to K^+\Lambda^0(1520)$. This was possible because the LEPS detector has a symmetric acceptance for positive and negative particles. The analysis confirmed the existence of a narrow Θ^+ signal at $M_{\Theta^+} = 1.524 \pm 0.002 \pm 0.003$ GeV. The significance has been estimated to be 5.1 σ and the width much smaller than 30 MeV.

In the meantime a number of experiments reported negative results, and skepticism about the existence of Θ^+ was growing. Most importantly, in the analogous experiment carried out by the CLAS (CEBAF Large Acceptance Spectrometer[6]) collaboration, no narrow peak corresponding to Θ^+ was observed [89], contradicting the earlier CLAS report from 2003 [90].

The CLAS experiment is analogous to LEPS, but not identical. In γd reaction CLAS observed all charged particles in the final state, including the spectator proton. This required an elastic rescattering of K^- off the proton from a deuteron, so that the proton could acquire sufficient momentum to enable detection. The probability of such a rescattering was an essential factor in the

[6]CEBAF stands for Continuous Electron Beam Accelerator Facility at Jefferson Laboratory located in Newport News, VA, USA.

CLAS analysis. Since LEPS assumed the proton to be a spectator, the kinematic conditions of the two experiments were different. Moreover, the angular coverage of both detectors is also different: less than 20 degrees for LEPS and greater than 20 degrees for CLAS in the LAB system [91].

To clarify the situation, the LEPS collaboration performed the search for Θ^+ in $\gamma d \to K^+K^-np$ reaction with 2.6 times higher statistics. The peak was still there. In 2013–2014 a new measurement was performed with the improved proton acceptance. Partial results have been published in different conference proceedings [91–93] but to the best of our knowledge, a full fledged journal article has not yet been released.

At the end of 2022, the LEPS2 detector started to collect new data in the search for Θ^+ [94]. LEPS2 detector has better angular coverage than LEPS and will look for Θ^+ in the following reactions [95]: (1) $\gamma n \to K^-\Theta^+$ and (2) $\gamma p \to \bar{K}^{0*}\Theta^+$, where Θ^+ will be reconstructed from the following reactions $\Theta^+ \to pK^0_S \to p\pi^+\pi^-$ and in the second case additionally $\bar{K}^{0*} \to K^-\pi^+$. Apparently, all four or five particles in the final state will be identified, which means that the uncertainty of the previous measurements due the Fermi motion of the target neutron or proton will be removed. We therefore look forward to future results.

To circumvent the problem of smal $g_{\Theta NK}$ coupling the authors of Ref. [96] proposed in 2006 to look for Θ^+ at CLAS in the interference with ϕ meson, which is copiously produced. The interference cross-section is linear in the Θ^+ coupling and hence can be substantially larger than the production cross-section where ϕ contribution is removed by kinematical cuts. Such analysis was published six years later [97] with positive result. Nevertheless, this paper has not been formally approved be the entire CLAS Collaboration, which criticised kinematical cuts applied in [97] and published an official disclaimer [98].

5.2 Resonance formation

Unlike photoproduction, resonance formation in KN scattering is the cleanest experiment possible in the search for Θ^+. The Breit-Wigner cross-section for a production of a resonance of spin J and mass M in the scattering of two hadrons of spin s_1 and s_2 takes the following form (see *e.g.* Eq.(51.1) in Ref. [99])

$$\sigma_{\rm BW}(E) = \frac{2J+1}{(2s_1+1)(2s_2+1)}\frac{\pi}{k^2}B_{\rm in}B_{\rm out}\frac{\Gamma^2}{(E-M)^2+\Gamma^2/4}, \tag{30}$$

where E is the c.m. energy, k is the c.m. momentum of initial state and Γ is the full width at half maximum height of the resonance. The branching fraction for the resonance into the initial-state channel is $B_{\rm in}$ and into the final-state channel is $B_{\rm out}$ – in the present case for KN scattering and one of the possible final states K^+n or K^0p we have $B_{\rm in} = B_{\rm out} = 1/2$. Substituting the Θ^+ mass one gets that the cross-section at the peak $\sigma_{\rm BW}(M_{\Theta^+}) \sim 15 \div 20$ mb. This is a model independent prediction, and we see that the cross-section for Θ^+ production in KN scattering is large. More detailed study of Θ^+ production in $K^+d \to K^0pp$ reaction shows that the production cross-section is in this case of the order of 5 mb [100]. The feasibility study of searching for Θ^+ in this channel at J-PARC has been recently performed in Ref. [95].

The formation process was used in the DIANA experiment where the bubble chamber DIANA filled with liquid Xenon has been exposed to a K^+ beam from the ITEP proton synchrotron. In Ref. [2] the authors analyzed the K^0p effective mass spectrum in the reaction $K^+n \to K^0p$ on a nucleon bound in a Xenon nucleus. A resonant enhancement with $M = 1539 \pm 2$ MeV and $\Gamma \leq 9$ MeV was observed. The statistical significance of the enhancement was estimated to be $4.4\,\sigma$.

The DIANA collaboration continued analysis of the bubble chamber films and in 2006 published new results from the larger statistics sample [11]. They confirmed their initial observation with the mass of $M = 1537 \pm 2$ MeV with, however, much smaller estimate of the width: $\Gamma = 0.36 \pm 0.11$ MeV. Depending on the significance estimator they obtained statistical significance of 4.3, 5.3 or 7.3 σ. Three years later they increased again statistics confirming the existence of Θ^+ with approximately the same mass and width, but higher statistical significance reaching 8 σ [17]. These results were confirmed in their last publication from 2014 [18].

As already mentioned the formation experiment with K^+ beam could be easily performed at the J-PARC facility in Japan looking at the three body final state K^0pp [95]. Another very promising search for Θ^+ will be possible at already approved program at K_L facility at JLab [7, 101, 102]. Here with a secondary beam of kaons, one may look at a two-body reaction $K^0_L p \to K^+ n$ on the hydrogen target. Here the plan is to measure the initial energy benefiting from the design momentum resolution below 1 MeV, rather than the invariant mass of K^+n system. According to the current schedule data collection will start in 2026 [102]. Note that the two-body final state is much cleaner than the three-body one, which is proposed to be studied at J-PARC. Finally, at the K_L facility one will also be able to look for other members of antidecuplet, like Ξ^+.

6. Summary

Although pentaquarks with heavy quarks are by now well established exotic baryons, Θ^+ still remains elusive. Its story is full of unexpected twists, emotions and lost hopes. The purpose of this review was to recall the theoretical basis of Θ^+ and some experimental evidence. While the small mass of exotic antidecuplet can be easily justified in the chiral models, where the $q\bar{s}$ pair is injected into a nucleon not as two independent quarks, but rather as an almost massless Goldstone boson, its very small width has often been considered *unnatural*. In the Chiral Quark Soliton model the small width of Θ^+ is *natural* due to the cancellation of the corresponding couplings in the non-relativistic limit. Unfortunately, since there is no intuitive argument as to why this is the case, many authors did not take it seriously. Moreover, the chiral models discussed here are largely considered only qualitative, although they are able to describe non-exotic baryons quite well. However, this is also true of other models that have trouble accommodating light exotic baryons.

The fate of Θ^+ can only be decided experimentally. In our opinion the most promising are formation experiments that can be conducted at the JLab and J-PARC facilities. The time scale is here of the order of a few years. Even earlier one may expect results from the photoporduction experiment at LEPS2. Whatever the results, we will eventually get a clear answer to the main question that has been bothering us in this article, whether Θ^+ exists or not.

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