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Phenomenology with trans-Planckian asymptotic safety

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The framework of trans-Planckian asymptotic safety has been shown to generate phenomenological predictions in the Standard Model and in some of its new physics extensions. As an example, we use this setup to constrain the parameter space of simple models that can accommodate the measured value of the anomalous magnetic moment of the muon and the relic density of dark matter. In these scenarios, the presence of an interactive UV fixed point in the system of gauge and Yukawa couplings imposes a set of boundary conditions at the Planck scale, which allow one to derive unique phenomenological predictions in each case and distinguish the different representations of the gauge group from one another. In this analysis, a heuristic approach is adopted, which bypasses the functional renormalization group by relying on a parametric description of quantum gravity with universal coefficients that are eventually obtained from low-energy observations. Within this approach a few simplifying approximations are typically introduced, including the computation of matter renormalization group equations at one loop, an arbitrary definition of the position of the Planck scale at 10^{19} GeV, and an instantaneous decoupling of gravitational interactions below the Planck scale. We systematically investigate the impact of dropping each of those approximations on the predictions for certain particle physics scenarios and we present numerical and analytical estimates of the uncertainties associated with the predictions from asymptotic safety.

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1. Introduction

Asymptotic safety (AS) is the property of a quantum field theory to develop ultraviolet (UV) fixed points of the renormalization group (RG) flow of the action [1]. Following the development of functional renormalization group (FRG) techniques three decades ago [2, 3], it was shown in numerous papers that AS may arise quite naturally in quantum gravity and provide the key ingredient for the non-perturbative renormalizability of the theory.

From the point of view of particle physics in four space-time dimensions, a particularly exciting possibility is that not only the gravitational action but the full system of gravity and matter may feature UV fixed points in the energy regime where gravitational interactions become strong [4–15]. A trans-Planckian fixed point may provide in that case specific boundary conditions for some of the *a priori* free couplings of the matter Lagrangian, as long as they correspond to *irrelevant* directions in theory space.

In order to properly complete a matter system with trans-Planckian AS, one should consistently calculate gravitational corrections to the matter beta function using the formalism of the FRG. It has been long known, however, that these calculations can be subject to large theoretical uncertainties, stemming from a variety of sources – from the choice of truncation in the gravity action [16–20], to cutoff-scheme dependence [21, 22], to the backreaction of matter [23–25]. On the other hand, because of the universal nature of trans-Planckian interactions, by virtue of which all quantities can be predicted except for a handful of relevant parameters that will have to be determined experimentally, a first-principle calculation of the gravitational contribution to the matter couplings is not necessarily needed to prove the consistency of certain low-energy predictions with quantum gravity. Often one is content with establishing a heuristic framework in which the trans-Planckian interactions are parameterized by coefficients that are eventually obtained from low-energy observations [26–37].

In such an effective trans-Planckian embedding, one generally introduces parametric corrections to the renormalization group equations (RGEs) of the renormalizable matter couplings, which take the form

$$\frac{dg_i}{dt} = \beta_i^{\text{matter}} - f_g\, g_i \tag{1}$$

$$\frac{dy_j}{dt} = \beta_j^{\text{matter}} - f_y\, y_j\,, \tag{2}$$

where $t = \ln\mu$ (renormalization scale), g_i and y_j (with $i, j = 1, 2, \ldots$) indicate, respectively, the set of gauge and Yukawa couplings of the theory, and $\beta_{i,j}^{\text{matter}}$ are the beta functions of the matter theory, which can be evaluated at one loop in dimensional regularization (DREG). The two "gravitational" coefficients f_g and f_y are universal in the sense that gravity is not expected to be affected by the internal degrees of freedom of the matter system. They appear only in the regime where the gravitational action develops an interactive fixed point, at $\mu > M_{\text{Pl}}$, and serve the purpose of inducing trans-Planckian zeros on the matter beta functions. If some of the emerging fixed-point coupling values correspond to irrelevant directions of the RG flow, one can estimate f_g and f_y by requiring that the irrelevant fixed points should be connected, along a unique RG trajectory, to quantities measured in experiments at the low scale.

Such heuristic embedding of a gauge-Yukawa system in trans-Planckian AS has been used in the Standard Model (SM) to attempt a prediction of the top/bottom mass ratio [26], the Cabibbo-Kobayashi-Maskawa [29], and Pontecorvo-Maki-Nakagawa-Sakata [32] matrix elements. New physics (NP) predictions can also be extracted [30–33, 37]. In particular, in Ref. [31] AS was employed to constrain the parameter space of simple NP models that can accommodate the measured value of the anomalous magnetic moment of the muon and the relic density of dark matter (DM). In this setup the trans-Planckian quantum physics was coupled parametrically to a set of $\text{SU}(2)_L \times \text{U}(1)_Y$ invariant extensions of the SM, each comprising an inert scalar field and one pair of colorless fermions that communicate to the muons through Yukawa-type interactions. The presence of an interactive UV fixed point in the system of gauge and Yukawa couplings imposes a set of boundary conditions at the Planck scale, which allow one to derive unique phenomenological predictions in each case and distinguish the different representations of the gauge group from one another.

While in the absence of a fully developed theory of quantum gravity the heuristic approach described above has proven to be extremely fruitful for phenomenological studies in particle physics, it is also important to be aware that it is based on several simplifying approximations: a) the DREG matter beta functions are typically computed at one loop, b) the Planck scale is set arbitrarily at $M_{\text{Pl}} = 10^{19}$ GeV, and c) the scale dependence of f_g and f_y, which should parameterize the cross-over from the interactive to non-interactive regime of quantum gravity, is neglected. Instead, f_g and f_y are treated as constants above the Planck scale and are set to zero below.

The question then naturally arises how robust the predictions derived in this way can be considered, and to what extent dropping any of the approximations listed above may affect a potential observational strategy to test these predictions in the low-scale experiments.

In these proceedings we first review the results of Ref. [31] to illustrate the predictive power of AS in the context of the NP phenomenology. We then report on the results presented in details in Ref. [38] and discuss the effects of discarding one by one the approximations of the minimal parametric setup.

2. Minimal models for the muon $g-2$ confront AS

The discrepancy between the SM prediction [39–60] and the experimental measurement of the anomalous magnetic moment of the muon has been confirmed separately by the Brookhaven National Laboratory [61] and the Fermilab experimental groups [62, 63], giving rise to the combined $5.1\,\sigma$ anomaly:

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = (2.49 \pm 0.48) \times 10^{-9}. \tag{3}$$

In a generic NP model which features heavy scalars ϕ_i and fermions ψ_j coupled to the SM muons via the Yukawa-type interactions $y_L^{ij}\phi_i\,\bar{\psi}_j P_L\,\mu$ and $y_R^{ij}\phi_i\,\bar{\psi}_j P_R\,\mu$ (where $P_{L,R} = (1 \mp \gamma^5)/2$ are the usual projection operators), a well-known one-loop contribution to the muon anomalous magnetic moment reads

$$\Delta a_\mu = \sum_{i,j}\left\{-\frac{m_\mu^2}{16\pi^2 M_{\phi_i}^2}\left(|y_L^{ij}|^2 + |y_R^{ij}|^2\right)\left[Q_j \mathcal{F}_1\left(x_{ij}\right) - Q_i \mathcal{G}_1\left(x_{ij}\right)\right]\right.$$
$$\left. - \frac{m_\mu\, M_{\psi_j}}{16\pi^2 M_{\phi_i}^2}{\rm Re}\left(y_L^{ij} y_R^{ij*}\right)\left[Q_j \mathcal{F}_2\left(x_{ij}\right) - Q_i \mathcal{G}_2\left(x_{ij}\right)\right]\right\}, \tag{4}$$

where M_{ϕ_i} is the physical mass of a heavy scalar, M_{ψ_j} is the physical mass of a heavy fermion, $x_{ij} = M_{\psi_j}^2/M_{\phi_i}^2$, and the electric charges of ϕ_i and ψ_j are related as $Q_i + Q_j = -1$. The loop functions are defined in the following way:

$$\begin{aligned}
\mathcal{F}_1(x) &= \frac{1}{6\,(1-x)^4}\left(2 + 3x - 6x^2 + x^3 + 6x\ln x\right)\\
\mathcal{F}_2(x) &= \frac{1}{(1-x)^3}\left(-3 + 4x - x^2 - 2\ln x\right)\\
\mathcal{G}_1(x) &= \frac{1}{6\,(1-x)^4}\left(1 - 6x + 3x^2 + 2x^3 - 6x^2\ln x\right)\\
\mathcal{G}_2(x) &= \frac{1}{(1-x)^3}\left(1 - x^2 + 2x\ln x\right)\,.
\end{aligned} \tag{5}$$

The first addend in Eq. (4) captures the loop chirality-conserving contributions to Δa_μ. These are known to be generically too small to account for the anomaly (3) when the most recent LHC bounds on the NP masses are taken into account [64, 65]. We will thus focus on the second addend in Eq. (4), which corresponds to the loop chirality-flipping contributions to Δa_μ.

2.1 NP models with vector-like fermions and scalars

We extend the particle content of the SM by a set of heavy scalar and fermion fields. Since the SM fermions are chiral particles, obtaining their mass after the electroweak symmetry breaking (EWSB), one needs either two NP scalar fields or two fermions, belonging to different representations of the $SU(2)_L$ group, to generate both y_L^{ij} and y_R^{ij} in Eq. (4). In this study we focus on the latter case, i.e., we introduce one scalar field (denoted as S) whereas fermions, which can be vector-like (VL) or Majorana, come in pairs whose elements belong to different $SU(2)_L$ representations (denoted E, E' and F, F').

	S	E	F
M_1	$(\mathbf{1},0)$	$(\mathbf{1},1)$	$(\mathbf{2},-\frac{1}{2})$
M_2	$(\mathbf{1},-1)$	$(\mathbf{1},0)$	$(\mathbf{2},\frac{1}{2})$
M_3	$(\mathbf{2},-\frac{1}{2})$	$\left(\mathbf{2},\frac{1}{2}\right)$	$(\mathbf{1},0)$
M_6	$\left(\mathbf{2},-\frac{1}{2}\right)$	$(\mathbf{2},\frac{1}{2})$	$(\mathbf{3},0)$
M_{10}	$(\mathbf{3},-1)$	$(\mathbf{3},0)$	$(\mathbf{2},\frac{1}{2})$

Table 1: $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ quantum numbers of the NP models considered in this work. All models in the table explain Δa_μ and present a phenomenology potentially consistent with AS and DM.

The Yukawa part of the Lagrangian of such a model can be written as

$$\mathcal{L}_{\mathrm{NP}} \supset -\left(Y_R\, \mu_R E' S + Y_L\, F' S^\dagger l_\mu + Y_1\, E\, h^\dagger F + Y_2\, F' h\, E' + \mathrm{H.c.}\right), \tag{6}$$

where SU(2) and spinor indices are contracted trivially following matrix multiplication and we assume that NP couples only to the second generation of the SM leptons, μ_R and l_μ, and the SM Higgs boson h. We assign $\mathrm{U}(1)_{\mathrm{gl}}$ charge +1 to E, E' and charge -1 to F, F', and S, while the SM fields remain uncharged. In Table 1 we report the gauge quantum numbers of S, E, and F for the models that feature a viable DM candidate, i.e., they admit at least one neutral NP particle and are not currently excluded by DM direct detection constraints.

We work under the assumption that the couplings of Lagrangian (6) to the gravitational field in the trans-Planckian UV give rise to interactive fixed points. By following the system to the infrared (IR) through the RG flow one obtains predictions for the couplings that can be combined with the information from the anomalous magnetic moment and DM to restrict the spectrum and distinguish the models of Table 1 from one another

2.2 Fixed-point analysis

The gauge-Yukawa system under study consists of 10 independent parameters,

$$g_3\,,\ g_2\,,\ g_Y\,,\ y_t\,,\ y_b\,,\ y_\mu\,,\ Y_L,\ Y_R,\ Y_1,\ Y_2, \tag{7}$$

where g_3, g_2, and g_Y are the couplings of the gauge symmetry groups $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)_L$, and $\mathrm{U}(1)_Y$, respectively, while y_t, y_b, and y_μ, denote the Yukawa couplings of the corresponding SM quarks and lepton. Note that y_t and y_b are not decoupled from the leptonic sector, as the chiral enhancement in the second line of Eq. (4) hinges on the coupling of NP to the Higgs boson, and is therefore influenced by the RG evolution of the heaviest SM fermions.

We are now ready to proceed to the fixed-point analysis of the one-loop RGE system given in the Appendix A of Ref. [31]. In what follows, the fixed-point values of dimensionless couplings will be indicated with an asterisk. In agreement with the low-energy phenomenology, the non-abelian gauge couplings remain asymptotically free, $g_3^* = 0$, $g_2^* = 0$. Both g_3 and g_2 correspond to relevant directions in the coupling space and constitute free parameters of the theory. Conversely, g_Y develops an interactive fixed point and corresponds to an irrelevant direction in the coupling space. By matching g_Y onto its phenomenological value in the IR one can uniquely determine the

	f_g	f_y	g_Y^*	y_t^*	Y_L^*	Y_R^*	Y_1^*
M_1	0.016	0.006	0.54	0.41	0.15	1.15	0.78
M_2	0.012	0.007	0.50	0.58	0.54	0.82	0.04
M_3	0.012	0.002	0.50	0.39	0.01	0.72	0.21
M_6	0.012	0.002	0.50	0.38	0.01	0.71	0.27
M_{10}	0.015	0.005	0.52	0.52	0.80	0.67	0.01

Table 2: f_g, f_y and fixed-point values of the irrelevant couplings for the models defined in Table 1.

parameter f_g,

$$g_Y^* = 4\pi\sqrt{\frac{f_g}{B_Y}}\,, \tag{8}$$

where for the different models B_Y takes the values given in Appendix A of Ref. [31].

The second quantum gravity parameter, f_y, can also be fixed if, in addition to g_Y, a UV interactive fixed point is presented by one of the SM Yukawa couplings [26], which we choose to be y_t,

$$y_t^* = F\left(f_g, f_y\right). \tag{9}$$

In this case the freedom of f_y allows one to match the flow of the top Yukawa coupling towards the IR onto the value of the experimentally measured top quark mass. The remaining SM couplings, y_b, and y_μ, will develop non-interactive fixed-points, $y_b^* = 0$, $y_\mu^* = 0$, associated with relevant directions.

In the NP sector, non-multiplicative contributions to the muon Yukawa beta function depend on Y_2. As a consequence, $O(1)$ values of y_μ would be generated radiatively if Y_2 assumed a nonzero fixed-point value. We thus require, for a phenomenologically viable solution, $Y_2^* = 0$. On the other hand, additive terms depending directly on Y_1 do not enter the renormalization of y_μ at one loop. Since at least one among Y_1 and Y_2 is expected to be large in order to generate the chiral enhancement in Eq. (4), we select $Y_1^* \neq 0$. Finally, $Y_L^* \neq 0$, $Y_R^* \neq 0$, as is required for a NP contributions to Δa_μ consistent with the measured value.

In Table 2 we present the numerical fixed-point values of the irrelevant couplings of the system (7), as well as the values of the quantum gravity parameters f_g and f_y, as required by matching onto the SM. Several comments are in order here. Different values of f_g characterizing different models are directly related to the quantum numbers of the heavy fermions and scalars through the one-loop RGE coefficient. Since g_Y^* is proportional to B_Y, f_g increases with the size of the one-loop coefficient. The other gravity-related parameter, f_y, can in principle be fixed by the value of y_t corresponding to the experimentally measured top mass. On the other hand, matching to the top mass is not always consistent with our assumption of real Yukawa couplings.

2.3 Low-scale predictions

In Table 3 we show the low-scale values of all the NP Yukawa couplings, as well as the corresponding value for the top Yukawa. All the parameters are evaluated at the reference scale $Q_0 = 2$ TeV. The value of $y_t(Q_0)$ indicates to what extent a given model is able to reproduce the

	$\lvert y_t(Q_0)\rvert$	$\lvert Y_L(Q_0)\rvert$	$\lvert Y_R(Q_0)\rvert$	$\lvert Y_1(Q_0)\rvert$	$\lvert Y_2(Q_0)\rvert$
M_1	0.91	0.21	0.91	0.62	9×10^{-4}
M_2	1.07	0.65	0.59	0.03	6×10^{-4}
M_3	0.95	0.01	0.77	0.18	3×10^{-5}
M_6	0.93	0.04	0.78	0.65	9×10^{-5}
M_{10}	1.03	0.98	0.87	0.03	1×10^{-3}

Table 3: Low-energy value (Q_0 = 2 TeV) of the Yukawa couplings of the models investigated in this work.

prediction of the SM. One can see that in M_1 and M_6 the top mass can be fitted with a very good precision, while in M_2, M_3, and M_{10} it results to be too large by 5-10%.

With the Yukawa couplings fixed, the only remaining free parameters of the models are the relevant ones: fermion masses m_E, m_F and the scalar mass m_S. In order to constrain them, we combine the information extracted from the fixed-point UV analysis with low-energy experimental data to obtain the favored regions of the parameter space.

We present in the left panel of Fig. 1 the summary of experimental constraints for the model M_1, in the plane of fermion mass parameters (m_F, m_E), for fixed value of the scalar mass m_S = 100 GeV. To roughly account for the LEP II limits, we apply a default hard cut on the mass of new charged particles, $m_E, m_F > 100$ GeV. The parameter space allowed at $2\,\sigma$ by the Fermilab+BNL combination measurement of Δa_μ (3) is shown as a red band. The gray shading indicates the 95% C.L. exclusion bound from the $h \to \mu^+\mu^-$ signal strength, $\frac{\sigma(pp\to h\to\mu^+\mu^-)}{\sigma(pp\to h\to\mu^+\mu^-)_{\text{SM}}} = 1.19 \pm 0.41 \pm 0.17$ [66], which is directly imposed on the value of the effective Yukawa coupling of the muon and which proves to be a very strong constraint for models in which $(g-2)_\mu$ is chirally enhanced. We also apply direct LHC searches for electroweak particle production with hard [67] (orange band) and soft [68] (blue band) leptons plus missing energy in the final state. Finally, in green we show the parameter space consistent at $2\,\sigma$ with the determination of the relic abundance of DM by Planck [69], $\Omega h^2 = 0.1188 \pm 0.0010$, to which we add in quadrature a $\sim$ 10% theoretical uncertainty. The only possible DM candidate is in this case the neutral scalar singlet S, as the mixing between E and F splits the masses of the electroweak doublet making the charged component lighter than the neutral one. A combination of low-energy constraints applied to the parameter space of M_1 emerging from the trans-Planckian fixed-point analysis has highlighted a favored region characterized by a mass spectrum of the "split" type: $m_S \approx 100$ GeV, $m_E \approx 160-190$ GeV, $m_F \approx 15-80$ TeV.

Model M_2 presents a radically different parameter space with respect to M_1, as can be seen on the right panel of Fig. 1. The first important distinction pertains to the size of the Yukawa coupling Y_1, which controls the mixing of the Majorana fermions. As Table 3 shows, it is significantly smaller than in M_1, so that we expect the chirality-flip contribution in the second line of Eq. (4) to be suppressed with respect to M_1. A correlated effect is that the mass correction to the effective muon couplings is also smaller than in M_1 or, in other words, the physical muon mass is closer to its running value at the EWSB scale. As a consequence, the $h \to \mu^+\mu^-$ constraint does not effectively reduce the parameter space in model M_2. The second difference pertains to the nature of DM, which is now going to be a neutral fermion with properties not dissimilar from those of a "well-tempered" neutralino [70] in supersymmetry The parameter space shrinks for increasing m_S and there remains no solution for the $(g-2)_\mu$ anomaly at $2\,\sigma$ with scalar mass above $m_S \approx 430$

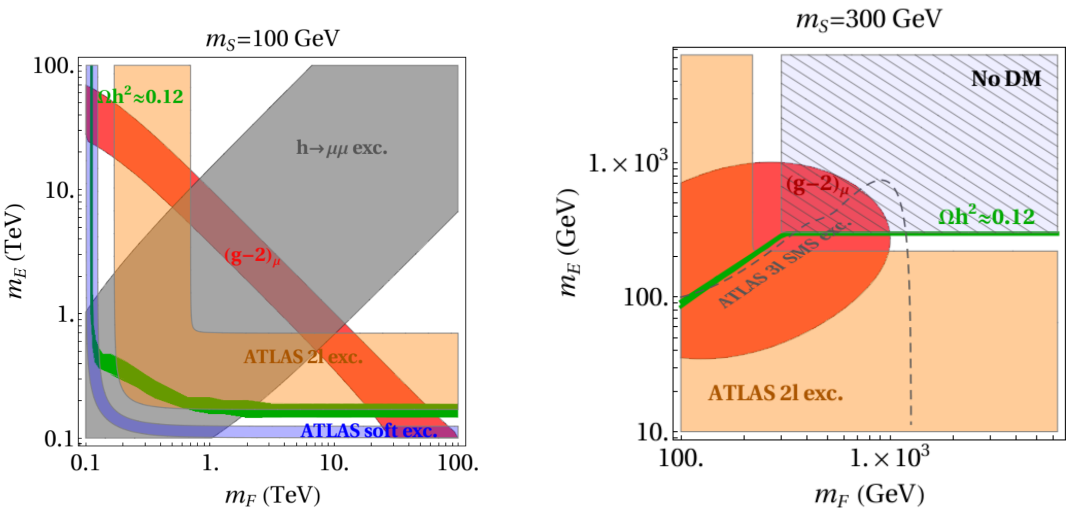

Figure 1: Experimental constraints on the parameter space $(m_F\,, m_E)$ in model M_1 (left) and M_2 (right) for selected values of the scalar mass m_S. In red the 2σ region allowed by Δa_μ is shown. In gray, the 95% C.L. exclusion limit from the $h \to \mu^+\mu^-$ signal strength is indicated [66]. $\Omega h^2 \approx 0.12$ is obtained in the part of the parameter space marked in green. Orange band is excluded at the 95% C.L. by the 13 TeV ATLAS 2 hard leptons search [67], whereas a blue band shows the exclusion by the ATLAS compressed spectra search [68].

GeV. The part of the parameter space not featuring a scalar DM candidate is marked in Fig. 1 as a striped light-blue shading

The comparison of phenomenological predictions for models M_1 and M_2 demonstrates that the Planck-scale boundary conditions imposed by AS lead to fundamentally different and testable signatures in different NP scenarios. Moreover, relevant mass parameters, which in the framework of AS remain free at the UV scale, can now be determined more precisely by a combination of experimental constraints. All in all, our results provide an instructive illustration of how the framework of AS can be adopted to derive specific predictions about the scale of NP. Such information could prove to be useful for the experimental collaborations as an indication and guideline for future search strategies. The construction presented in this study could easily be extended to alternative NP models and observational phenomena [30–33, 37].

3. Robustness of predictions from AS

As we mentioned in the Introduction, the phenomenological predictions discussed in Sec. 2 are based on several simplifying approximations. We will now discuss how stable the predictions are if these assumptions are relaxed. Specifically, we consider a) the inclusion of higher-order corrections in the matter sector, b) changing the position of the Planck scale by a few orders of magnitude, and c) the non-trivial functional dependence of the running gravitational couplings, $f_{g,y}(t)$, resulting in the non-instantaneous decoupling of the trans-Planckian UV completion.

3.1 Gauge sector

As an example, let us consider the $B-L$ symmetry, i.e. the gauge group $\mathrm{U}(1)_Y{\times}\mathrm{U}(1)_{B-L}$. It is straightforward to extend our conclusions to any pair of abelian symmetries. The pertinent RGEs take the following parametric form, common to all models with abelian mixing

$$\frac{dg_Y}{dt} = \frac{1}{16\pi^2}\left(b_Y + \Pi^{(Y)}_{n\geq 2}\right) g_Y^3 - f_g(t)\, g_Y \tag{10}$$

$$\frac{dg_d}{dt} = \frac{1}{16\pi^2}\left[\left(b_Y + \Pi^{(Y)}_{n\geq 2}\right) g_d g_\epsilon^2 + \left(b_d + \Pi^{(d)}_{n\geq 2}\right) g_d^3 + \left(b_\epsilon + \Pi^{(\epsilon)}_{n\geq 2}\right) g_d^2 g_\epsilon\right] - f_g(t)\, g_d \tag{11}$$

$$\begin{aligned}\frac{dg_\epsilon}{dt} &= \frac{1}{16\pi^2}\left[\left(b_Y + \Pi^{(Y)}_{n\geq 2}\right)\left(g_\epsilon^3 + 2 g_Y^2 g_\epsilon\right) + \left(b_d + \Pi^{(d)}_{n\geq 2}\right) g_d^2 g_\epsilon\right.\\ &\quad \left. + \left(b_\epsilon + \Pi^{(\epsilon)}_{n\geq 2}\right)\left(g_Y^2 g_d + g_d g_\epsilon^2\right)\right] - f_g(t)\, g_\epsilon\,. \end{aligned} \tag{12}$$

With respect to the simplified analysis of Sec. 2, the RGEs of the gauge couplings are now extended by the higher-order corrections $\Pi^{(Y)}_{n\geq 2}$, $\Pi^{(d)}_{n\geq 2}$ and $\Pi^{(\epsilon)}_{n\geq 2}$. We also allow for explicit dependence of the gravity parameters f_g and f_y on the renormalization scale t.

Assuming for the moment that the fixed point for the gauge couplings is developed sharply at $M_{\rm Pl} = 10^{19}$ GeV, one can express the value of f_g in terms of the (known) $\mathrm{U}(1)_Y$ trans-Planckian fixed point g_Y^*, with an accuracy that increases at each successive order,

$$f_g(n \text{ loops}) \approx \frac{g_Y^{*2}(n \text{ loops})}{16\pi^2}\left(b_Y + \Pi^{(Y)*}_{n\geq 2}\right), \tag{13}$$

where the asterisk refers to all couplings being set at their UV fixed-point value. The ratios of the gauge couplings, on the other hand, do not depend explicitly on the value of f_g. One can define

$$r^*_{g,d}(n \text{ loops}) \equiv \frac{g_d^*}{g_Y^*}(n \text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}, \tag{14}$$

$$r^*_{g,\epsilon}(n \text{ loops}) \equiv \frac{g_\epsilon^*}{g_Y^*}(n \text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}, \tag{15}$$

where we have adopted a simplified notation,

$$\tilde{b}_i \equiv b_i + \Pi^{(i)*}_{n\geq 2}\,. \tag{16}$$

In order to obtain some quantitative estimates of the higher-order effects, let us retain for simplicity only the two-loop corrections, $\Pi_2^{(i)}$, and quantify the uncertainty on the ratios $r^*_{g,i(=d,\epsilon)}$ by defining

$$\frac{\delta r^*_{g,i}}{r^*_{g,i}} = \frac{r^*_{g,i}(2 \text{ loops}) - r^*_{g,i}(1 \text{ loop})}{r^*_{g,i}(1 \text{ loop})}\,. \tag{17}$$

The precise numerical calculation for the $B-L$ model reveals that the shift in the fixed-point values of the NP gauge couplings is very small, $\delta r^*_{g,d}/r^*_{g,d} = -0.41\%$ and $\delta r^*_{g,\epsilon}/r^*_{g,\epsilon} = -0.44\%$. As a consequence, uncertainty of the low scale predictions is also tiny, $\delta g_d/g_d(M_t) = -0.4\%$ and $\delta g_\epsilon/g_\epsilon(M_t) = -0.5\%$.

When assessing the impact of the Planck-scale position on the predictions for gauge couplings, one should note that this uncertainty is effectively equivalent to an uncertainty on the fixed-point value of the hypercharge gauge coupling g_Y^*, hence on f_g. On the other hand, since the f_g dependence cancels out from Eqs. (14) and (15), moving the Planck scale back and forth does not affect the predicted ratios $r^*_{g,i}$ at the one-loop order. This feature is not preserved at higher orders in the perturbative expansion. However, the impact of the Planck scale position remains negligibly small, at about 0.01%.

To evaluate the impact of running gravity parameter f_g, let us first notice that the explicit gravitational contribution to the matter beta function f_g entirely factors out of the running ratios:

$$\frac{d}{dt}\left(\frac{g_d}{g_Y}\right) = \frac{1}{g_Y}\left(\beta_d^{\text{matter}} - \frac{g_d}{g_Y}\beta_Y^{\text{matter}}\right)[t] \equiv F_d(g_Y(t), g_d(t), g_\epsilon(t), ...)\,, \tag{18}$$

$$\frac{d}{dt}\left(\frac{g_\epsilon}{g_Y}\right) = \frac{1}{g_Y}\left(\beta_\epsilon^{\text{matter}} - \frac{g_\epsilon}{g_Y}\beta_Y^{\text{matter}}\right)[t] \equiv F_\epsilon(g_Y(t), g_d(t), g_\epsilon(t), ...)\,, \tag{19}$$

where the matter beta functions β_i^{matter} are given in parametric form in Eqs. (10)–(12) and we have formally defined slope functions F_d and F_ϵ which do not depend on f_g. Let us now apply the definition of total derivative to Eq. (18) and focus on a sequence of infinitesimal scale intervals, ... $t_2 < t_1 < t_0$, with the system lying at the fixed point at t_0. Moving backwards in t, one gets

$$\frac{g_d(t_1)}{g_Y(t_1)} = r^*_{g,d} + (t_1 - t_0)F_d(g_Y^*, g_d^*, g_\epsilon^*, ...) \tag{20}$$

$$\frac{g_d(t_2)}{g_Y(t_2)} = \frac{g_d(t_1)}{g_Y(t_1)} + (t_2 - t_1)F_d(g_Y(t_1), g_d(t_1), g_\epsilon(t_1), ...) \tag{21}$$

$$\frac{g_d(t_3)}{g_Y(t_3)} \quad \ldots$$

where $r^*_{g,d}$ was defined in Eq. (14) for REGs at arbitrary loop order. The same steps can be repeated for Eq. (19). One can check, by directly imposing boundary condition (14) into Eq. (18) at t_0, that F_d vanishes at the fixed point: $F_d(g_Y^*, ...) = 0$. Equivalently, one can show that $F_\epsilon(g_Y^*, ...) = 0$. Thus, $g_d(t_1)/g_Y(t_1) = r^*_{g,d}$ and $g_\epsilon(t_1)/g_Y(t_1) = r^*_{g,\epsilon}$. For the next time interval one can plug Eqs. (14), (15) into Eqs. (10)–(12) and express the slope functions in terms of fixed-point ratios:

$$F_d(g_Y(t_1), g_d(t_1), ...) = \frac{g_Y^2(t_1)}{16\pi^2}\left[\tilde{b}_Y(t_1)r^{*2}_{g,\epsilon} + \tilde{b}_d(t_1)r^{*2}_{g,d} + \tilde{b}_\epsilon(t_1)r^*_{g,\epsilon}r^*_{g,d} - \tilde{b}_Y(t_1)\right]r^*_{g,d}\,, \tag{22}$$

$$F_\epsilon(g_Y(t_1), g_d(t_1), ...) = \frac{g_Y^2(t_1)}{16\pi^2}\left[\left(\tilde{b}_Y(t_1)r^*_{g,\epsilon} + \tilde{b}_\epsilon(t_1)r^*_{g,d}\right)\left(1 + r^{*2}_{g,\epsilon}\right) + \tilde{b}_d(t_1)\, r^{*2}_{g,d}\, r^*_{g,\epsilon}\right], \tag{23}$$

where, in agreement with Eq. (16), we have defined $\tilde{b}_i(t) \equiv b_i + \Pi^{(i)}_{n\geq 2}(t)$ away from the fixed point.

At order $n = 1$, $\tilde{b}_i(t) = b_i$. Boundary conditions (14) and (15) ensure that $F_d^{(n=1)}(t_1) = 0$ and $F_\epsilon^{(n=1)}(t_1) = 0$, independently of the actual values of the running gauge couplings. As an immediate consequence, $g_d(t)/g_Y(t) = r^*_{g,d}$ and $g_\epsilon(t)/g_Y(t) = r^*_{g,\epsilon}$ along the entire RG flow. Given that the ratios $r^*_{g,d}$ and $r^*_{g,\epsilon}$ are, at one loop, uniquely determined by the gauge quantum numbers, we can conclude that the one-loop ratios g_d/g_Y and g_ϵ/g_Y are exact invariants of the RG flow, whereas, at order $n \geq 2$, RG invariance is respected up to a very good approximation.

For illustration, we show in the left panel of Fig. 2 the RG flow of the three gauge couplings of the $B - L$ model: g_Y (red), g_d (blue), and $-g_\epsilon$ (green). Dotted lines correspond to the benchmark scenario with f_g and f_y constant above the Planck scale. Darker solid lines indicate two random parametrizations of the functional dependence $f_g(t)$, where the gravity parameter is allow to vary by a factor 10 in the range between 10^{16} GeV and 10^{20} GeV. One can see that, in spite of different

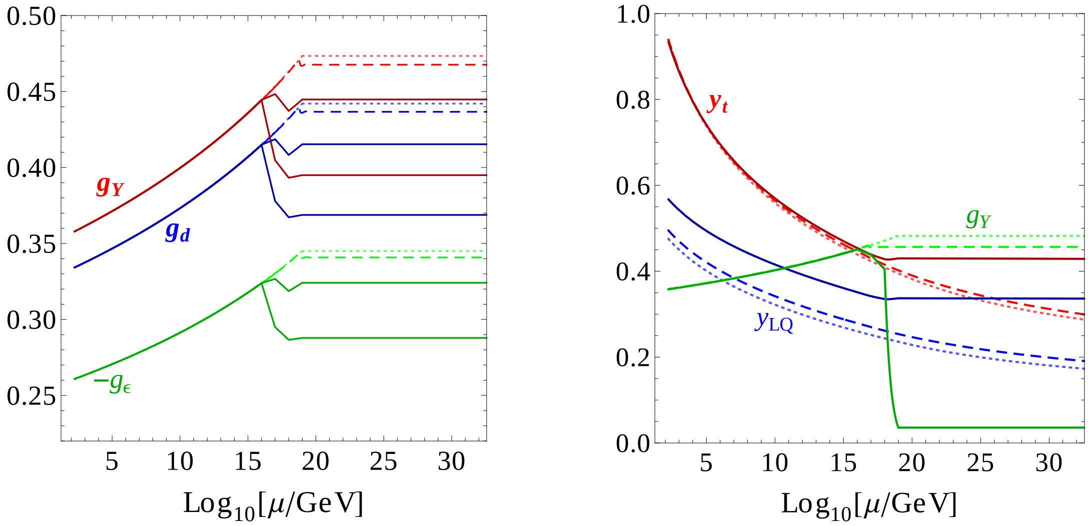

Figure 2: Left: RG flow of the hypercharge (red), dark gauge (blue), and kinetic mixing (green) couplings of the $B-L$ model at one loop. Dotted lines correspond to the benchmark scenario with f_g constant above the Planck scale. Darker solid lines indicate two arbitrary parametrizations of $f_g(t)$. Dashed lines correspond to the FRG results of Ref. [71]. Right: Same for the top Yukawa (red), LQ Yukawa (blue) and hypercharge gauge (green) couplings of the S_3 model for different parametrizations of $f_g(t)$ and $f_y(t)$. Solid lines show a parametrization resulting in $f_g(t) \ll |f_y(t)|$ in the trans-Planckian regime.

fixed-point values in each case, the RG invariance of the coupling ratios leads to unchanged low-scale predictions. Finally, the dashed lines correspond to the $f_g(t)$ parametrization based on the FRG results of Ref. [71].

3.2 Yukawa sector

In the Yukawa sector, the situation is slightly more involved. Let us consider a theory with two different Yukawa couplings, y_2 and y_1. We assume that the latter plays the role of a reference coupling, whose low-scale value is determined by an experiment and which must be matched to the fixed-point prediction through the RG flow. The theory also features a gauge coupling g_1.

Unlike for the system of pure gauge couplings, the fixed-point value of the to-be-predicted Yukawa coupling *does depend* on the fixed-point values of all the other couplings in the theory. At the two-loop order it reads

$$y_2^*(2\text{ loops}) \approx \left[\frac{a_1^{(2)} - a_1^{(1)}}{a_2^{(1)} - a_2^{(2)}} \left(y_1^{*2}(1\text{ loop}) + \delta y_1^{*2}\right) + \frac{\left(a_{11}'^{(1)} - a_{11}'^{(2)}\right)\left(g_1^{*2}(1\text{ loop}) + \delta g_1^{*2}\right) + \widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}}{a_2^{(1)} - a_2^{(2)}}\right]^{1/2} . \tag{24}$$

Thus, y_2^* can be modified both by the two-loop corrections in its own RG running, $\widetilde{\Pi}_2^{(2)*}$ (which are perturbatively small), and by a shift in the fixed-point values of the reference couplings, δg_1^{*2} and δy_1^{*2} (which in turn can be large). Note that the impact of the latter is more significant the smaller

y_2 is in comparison to the other couplings. To illustrate it with a numerical example, let us consider a simple extension of the SM by a complex scalar S_3 that carries both lepton and baryon number (hence called a leptoquark), with the $(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$ quantum numbers under the $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ gauge group. Following the notation of Ref. [30], we introduce the Yukawa-type interaction of S_3 with SM fermions as follows: $\mathcal{L} \supset -Y_{\mathrm{LQ}}\, Q^T \tilde{\epsilon} S_3\, L + \mathrm{H.c.}$, where Q and L are the SM quark and lepton doublets, respectively, and Y_{LQ} is a 3-by-3 matrix in flavor space. For simplicity we shall only focus on the 3rd generation, denoting $y_{\mathrm{LQ}} \equiv (Y_{\mathrm{LQ}})_{33}$. For the one-loop fixed-point values $y_t^* = 0.21$ and $y_{\mathrm{LQ}}^* = 0.08$, the shift in y_{LQ}^* due to inclusion of two-loop corrections amounts to 24%. On the other hand, in a BSM model featuring, beside the SM fermions, a right-handed neutrino with the Yukawa coupling $\sim Y_\nu N\, (\tilde{\epsilon} H^*)^\dagger\, L$ and the one-loop fixed-points $y_t^* = 0.27$ and $y_\nu^* = 0.52$, the corresponding two-loop the shift is only 3%.

The shifts δg_1^{*2} and δy_1^{*2} in Eq. (24) can also arise from the trans-Planckian RG running of the gravity parameters f_g and f_y. Opposite to what was observed in the case of the gauge couplings, the running ratio $y_2/y_1(t)$ is not an RG-invariant. Thus, even at one loop, if any of the couplings deviates from its fixed point (due to the change of the gravity parameters or the presence of relevant directions in the coupling space) the ratio $y_2(t)/y_1(t)$ starts to flow.

As a matter of fact, the dominant source of uncertainty for the prediction of a Yukawa coupling is not given by the changing of $y_2(t)/y_1(t)$ along the RG flow, but rather by the fact that the fixed-point ratio itself becomes unknown once we factor in the t-dependence of f_g and f_y. This is illustrated on the right panel of Fig. 2, where we show the RG flow of the top Yukawa (red), LQ Yukawa (blue) and hypercharge (green) gauge coupling of the S_3 model with different functional forms of $f_g(t)$ and $f_y(t)$. The color code is the same as in the left panel of Fig. 2. While it was shown above that even extreme fluctuations do not modify the low-scale predictions for the gauge couplings, we can see in Fig. 2 that the trans-Planckian behavior of the running y_{LQ} (blue) and y_t (red) couplings is drastically different in the cases in solid vs. those in dotted/dashed. For the case in solid, which corresponds to $f_g(t) \ll |f_y(t)|$, the low-scale uncertainty on y_{LQ} reads $\sim 19\%$.

Finally, let us notice that the low-scale uncertainties of the NP Yukawa couplings are smaller than those at the fixed-point. This is not a coincidence and it is intrinsically related to modifications of the RG flow due to higher-order corrections. Due to the negative contribution of the gauge coupling g_3 to the beta functions of the Yukawa couplings of particles carrying the color charge, the running value of y_t at two loops is generically smaller than at one loop. It follows that the fixed-point values of the NP Yukawa couplings also tend to be shifted downwards. On the other hand, since the same reference low-scale value $y_t(M_t)$ is used in the fixed-point analysis at any loop order, the RG trajectories of the NP Yukawa couplings have the tendency to focus towards their one-loop value in the infrared, which results in a reduction of the uncertainty with respect to the prediction at the fixed point. A generic rule of thumb thus applies: the uncertainty in the determination of the fixed-point value of a NP Yukawa coupling provides an upper bound on the uncertainty of the same prediction at the low scale.

4. Summary

In these proceedings we illustrated how the trans-Planckian AS can be adopted to derive specific predictions about the scale of NP. First, we used this framework to boost the predictivity of a class of simple NP models known to produce at one loop the observed deviation in the anomalous magnetic moment of the muon through a chiral enhancement. All models contain, besides the SM particles, an inert scalar field and two colorless fermions that transform according to different representations of $SU(2)_L$. We have completed the models in the UV by parametrically coupling their fields to the trans-Planckian asymptotically safe quantum gravity. In the presence of a gravity-induced UV fixed point, the values of the Yukawa couplings between the SM leptons and the NP sector are fixed, as they correspond to irrelevant directions in the coupling space. Their RG flow towards the low energies is exclusively determined by the relevant couplings of the SM, whose IR values are set by the experiment. As a consequence, the NP fermion and scalar masses remain as the only free parameters of the models, which are then determined by low-energy constraints.

We have also quantified the impact of relaxing several approximations commonly used in the literature to extract phenomenological predictions. In particular we have considered: the effect of including higher-order corrections in the matter RGEs of the gauge-Yukawa system; the impact of selecting a value different from 10^{19} GeV for the (somewhat arbitrary) position of the Planck scale; and the effects of using scale-dependent parametrizations of the gravitational UV completion in the matter RGEs, resulting in a non-instantaneous decoupling of gravity from matter around the Planck scale.

In the gauge sector, the uncertainty induced by relaxing any of the simplifying assumptions never exceeds the 1% level. We can conclude that fixed-point predictions for the irrelevant gauge couplings of the SM and/or NP models are extremely robust, even when they are obtained in a heuristic, simplified approach to AS that is based on some approximations. A similar conclusion can be drawn for the Yukawa sector of the NP theory, if the predicted Yukawa couplings are of comparable size to the irrelevant gauge couplings. The uncertainties remain at bay, not exceeding $\sim 10\%$ at the fixed point if higher-order corrections are included.

Potentially more dangerous uncertainties could stem from considering the non-trivial scale dependence of the gravitational contributions to the matter beta functions, parameterized by functions $f_g(t)$ and $f_y(t)$, as in this case we lose the ability of determining the actual value of the Yukawa couplings at the fixed point. However, we have argued, based on both an analytical and numerical discussion, that in the range of variability of the gravitation parameters that can be realistically expected in the framework of the FRG, the resulting uncertainty is moderate. The situation is additionally helped by focusing of the RG trajectories in the sub-Planckian regime.

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