PROCEEDINGS OF SCIENCE

# Recent topics in the analysis of neutrino mass-mixing parameters

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The parameters characterising neutrino masses, mixing and phases are subject to a worldwide experimental and theoretical research activity. We briefly review the status of such parameters in the standard three-neutrino framework, and discuss related open issues. We then consider an extension of the standard framework including heavy neutrinos in addition to the light ones, and discuss the implications for the determination of the effective Majorana mass parameter through neutrinoless double beta decay, in the hypothesis of negligible interference of light and heavy contributions. Far-future prospects in this field are mentioned.

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## 1. Introduction

In the last three decades, experimental discoveries of various neutrino oscillation phenomena have boosted a worldwide activity to determine the neutrino mass-mixing parameters with increasing details, both in the standard three-neutrino ($3\nu$) framework and in many possible nonstandard extensions. The literature on these topics is huge and, for a first orientation, the reader can usefully browse the neutrino reviews in [1] and the encyclopedical neutrino website in [2]. Herein we shall briefly discuss current $3\nu$ results as obtained in [3, 4], and one prospective example of future results including heavy neutrinos beyond $3\nu$, as discussed in [5] in the context of neutrinoless double beta decay. Far-future prospects in this field are mentioned. For the sake of simplicity, in this contribution we display some slides from the speaker's presentation in [6]; the original figures and results, as well as an extended discussion and bibliography, can be found in the publications [3–5].

## 2. The $3\nu$ framework and its mass-mixing parameters

The simplest ($3\nu$) framework that accommodates current evidence for neutrino oscillations includes the mixing of three neutrino flavor states ($\nu_e$, $\nu_\mu$, $\nu_\tau$) with three mass states ($\nu_1$, $\nu_2$, $\nu_3$), having masses ($m_1$, $m_2$, $m_3$). Figure 1 displays in the upper part the parameters of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix $U$, namely, the three mixing angles $\theta_{ij}$, the CP-violating phase $\delta$, and two additional phases iff neutrinos are Majorana; the latter are not tested in neutrino oscillations and may emerge only in neutrinoless double beta decay ($0\nu\beta\beta$). The lower part of the figure shows the mass spectrum in its two possible options: so-called normal ordering (NO) and inverted ordering (IO), associated to the $\pm$ sign of the squared mass splitting $\Delta m^2$. Oscillations probe $\pm\Delta m^2$ and the smaller splitting $\delta m^2$, but not the absolute masses.

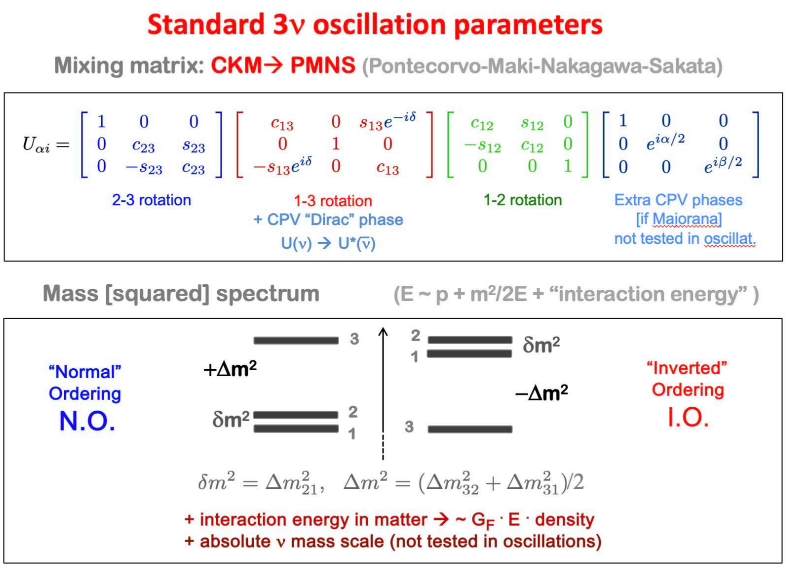


**Figure 1:** Parameters (masses, mixing, phases) of the standard $3\nu$ framework.

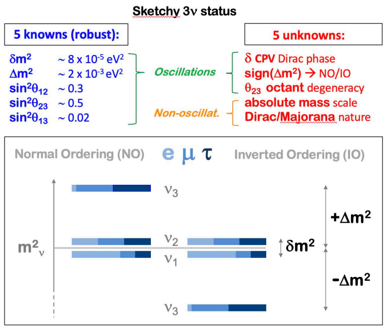


**Figure 2:** Overview of current knowns and unknowns in the standard $3\nu$ framework.

Figure 2 shows a sketchy status of what we (do not) know about the $3\nu$ framework. We know $\delta m^2$ and the absolute value of $\Delta m^2$, that determine the oscillation frequencies, as well as the three mixing angles $\theta_{ij}$, that determine the oscillation amplitudes. However, we do not know yet the CP phase ($\delta$) and the mass ordering (NO/IO), and we cannot yet distinguish two close solutions for $\theta_{23} \simeq \pi/4$ across the two octants. In addition, we do not know yet the absolute neutrino mass scale and its nature (Dirac or Majorana): these properties can only be tested by non-oscillation observables. The lower part of Fig. 2 shows the two NO/IO options, together with the approximate flavor content of each mass state. Theoretical models are trying to interpret this pattern.

Figure 3 refines the description of the $3\nu$ status with quantitative bounds (in terms of $N_\sigma = \sqrt{\Delta\chi^2}$) on the known parameters, as well as on the unknown ones ($\delta$, $\theta_{23}$ octant and mass ordering) as obtained by a global analysis of oscillation data [3]. The known parameters are determined with accuracy of order (few) percent. Concerning the unknowns, there are hints in favor of normal ordering, of $\sin\delta < 0$, and of the first octant of $\theta_{23}$, that need further investigation. Developments follow two axes: greater precision on known parameters, and discovery of the unknown ones, up to possible surprises beyond the standard $3\nu$ framework (such as light sterile neutrinos or new neutrino interactions, actively studied worldwide but not covered herein).

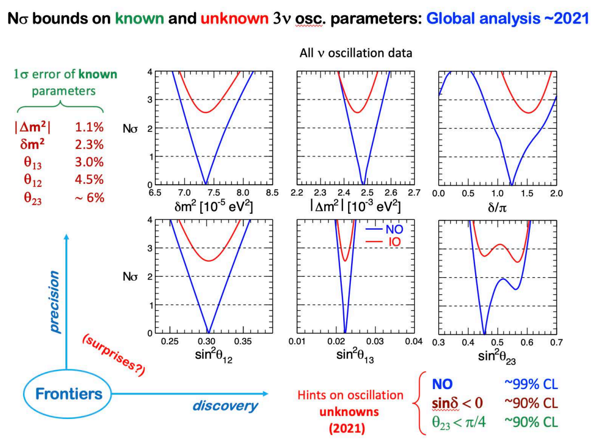


**Figure 3:** Quantitative constraints on $3\nu$ parameters from oscillation data. Adapted from [3].



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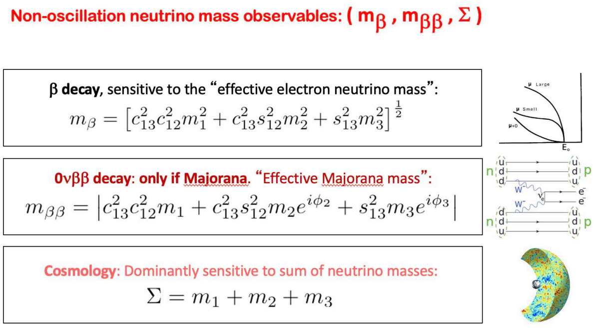


**Figure 4:** Overview of the three main non-oscillation observables ($m_\beta$, $m_{\beta\beta}$, $\Sigma$).

Open problems in $\nu$ oscillations include improved descriptions of $\nu$ interactions in nuclei of interest for detectors. Related progress is crucial to refine the known parameters and to make the unknown ones emerge. In general, "electroweak nuclear physics" issues need further understanding.

Nonoscillation observables probe absolute $\nu$ masses in different ways: the effective $\nu_e$ mass $m_\beta$ in $\beta$-decay endpoint searches, the effective Majorana mass $m_{\beta\beta}$ affecting $0\nu\beta\beta$ decay rates (if $\nu$'s are not Dirac), and the sum of $\nu$ masses $\Sigma$ acting as a source of gravity in cosmology, as summarized in Fig. 3. Since the parameters defining ($m_\beta$, $m_{\beta\beta}$, $\Sigma$) are constrained by oscillations (up to the mass ordering, absolute mass scale and Majorana phases), any pair of observables among ($m_\beta$, $m_{\beta\beta}$, $\Sigma$) displays positive correlations. Correlation bands are shown in Fig. 5 for NO (blue) and IO (red), together with current upper bounds on each of the ($m_\beta$, $m_{\beta\beta}$, $\Sigma$) parameters; see [3] for details. Altogether, these bounds (and especially $\Sigma$) cut a large fraction of the IO parameter space, but are not yet able to rule it out. In nonoscillation observables, open questions include: (1) critical discussions of the standard cosmological model and of related constraints on $\Sigma$ [3], and (2) reduction of large uncertainties on nuclear matrix elements (NME) affecting constraints on $m_{\beta\beta}$ [4].

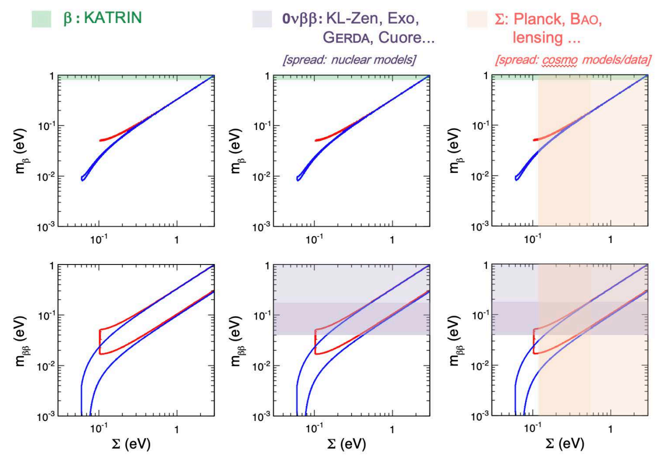


**Figure 5:** Overview of current bounds in the planes ($\Sigma$, $m_\beta$) and ($\Sigma$, $m_{\beta\beta}$) for NO (blue band) and IO (red band). Shaded bounds for $\Sigma$ and $m_{\beta\beta}$ range from conservative to aggressive constraints. Adapted from the discussion in [3].

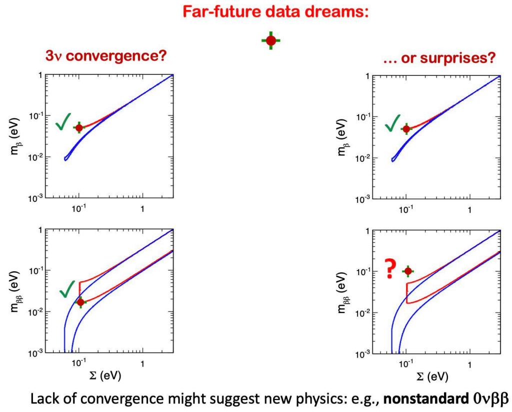


**Figure 6:** Examples of far-future nonoscillation data vs oscillation bands. Left: $3\nu$ consistency. Right: $3\nu$ mismatch.

Ongoing and future nonoscillation experiments will hopefully find signals rather than reduce constraints in the $(m_\beta,\, m_{\beta\beta},\, \Sigma)$ parameter space. In the far future, precise nonoscillation measurements might provide new decisive tests of the $3\nu$ framework, being either in agreement or in disagreement with oscillation bands, as shown in the left and right panels of Fig. 6, respectively. In particular, lack of convergence in the $(m_\beta,\, \Sigma)$ plane might be suggestive of new physics in $0\nu\beta\beta$ decay, beyond the exchange of light Majorana neutrinos.

Heavy Majorana neutrino ($N$) appear quite naturally besides the light ones ($\nu$) in various theoretical models. If their exchange in the $0\nu\beta\beta$ process is non-interfering, then the decay half-life in a nucleus labelled by $(Z,\, A) = i$ depends the incoherent sum of a light neutrino contribution $m^2_\nu$ and a heavy one $m^2_N$, weighted by the squares of the corresponding NME's for the nucleus, $M^2_{\nu,i}$ and $M^2_{N,i}$. A hypothetical future discrepancy, as the one displayed in Fig. 6 (right part), might be explained by a heavy neutrino contribution to $m_{\beta\beta}$ in addition to the light neutrino contribution. The next question would then be [5]: Can the two contributions be separated?

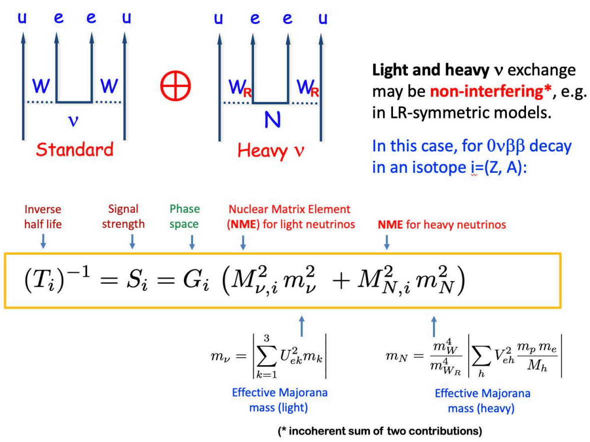


**Figure 7:** Upper part: diagrams for light ($\nu$) plus heavy ($N$) Majorana neutrino exchange in $0\nu\beta\beta$ decay, assuming no light-heavy interference. Lower part: decay half-life in terms of two effective mass parameters and related NME's.

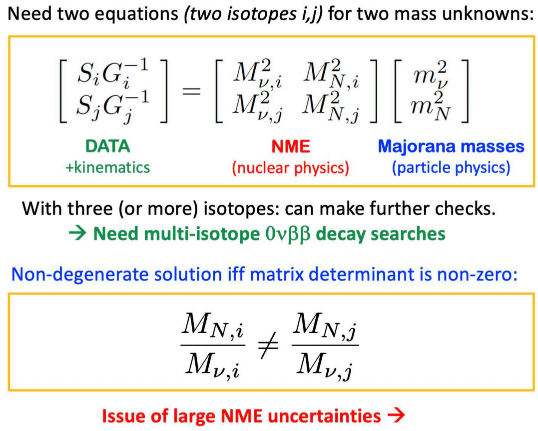


**Figure 8:** Algebraic conditions to separate non-interfering light and heavy neutrino contributions to $0\nu\beta\beta$ decay.

In principle, two $0\nu\beta\beta$ measurements in two different nuclei $i$ and $j$ are needed to constrain two quantities as $m_\nu$ and $m_N$. A separation is possible, provided that the degenerate case is avoided within theoretical (NME) and experimental (statistical) uncertainties. For non-interfering light and heavy neutrino, the (non)degeneracy condition simply reduces to a (non)vanishing determinant and to the (in)equality of heavy-to-light NME ratios in $i$ and $j$, as reported in Fig. 7. We have revisited the status of this algebraic condition in the light of current calculations of the NME's for $\nu$ and $N$ exchange, in isotopes of interest for future ton-scale projects [5]. Unfortunately, it appears that the NME ratios tend to be spread just around the degeneracy lines, represented by diagonal lines in Fig. 9. The separation of $\nu$ and $N$ contribution is thus possible only for NEM ratios sufficiently far from these lines, provided that the experimental errors are small enough. A phenomenological analysis of prospective data is needed to check this possibility case by case.

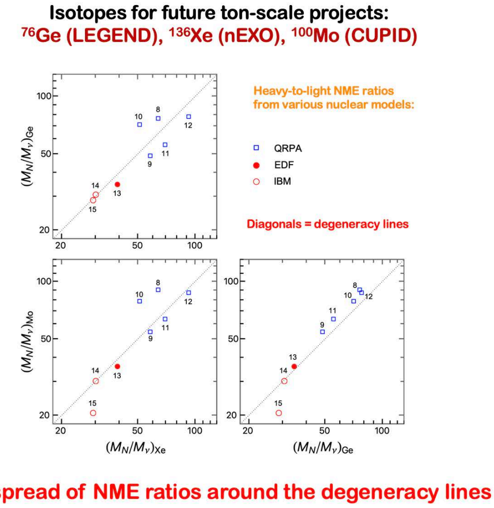


**Figure 9:** Heavy-to-light NME ratios for each pair of nuclei among the Ge, Xe, Mo isotopes of interest for ton-scale $0\nu\beta\beta$ decay projects. The points represent calculations in different models (QRPA, EDF, IBM). Adapted from [5].

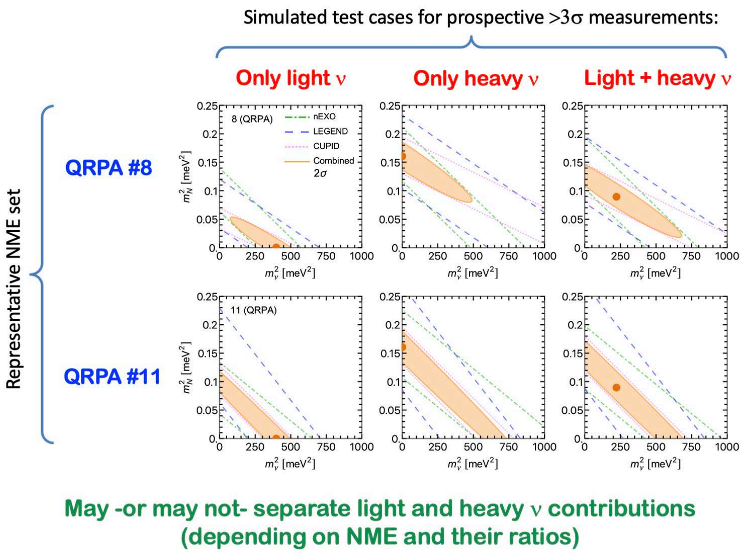


**Figure 10:** Prospective $2\sigma$ bounds on the light and heavy Majorana mass parameters, assuming $0\nu\beta\beta$ signals in the nEXO, LEGEND and CUPID projects, both separately and in combination, for 10 ton-year exposure and specific QRPA NME's. Left, central and right panels refer to, respectively: only $\nu$, only $N$, and both $\nu$ and $N$. Adapted from [5].

Figure 10 shows representative results for two sets of NME (labeled as QRPA n. 8 and 11 in Fig. 9) and for three hypotheses: (1) only $\nu$, (2) only $N$, and (3) both $\nu$ and $N$ contributions. Joint $2\sigma$ bounds on the pairs of parameters $(m^2_\nu, m^2_N)$ are derived assuming future $0\nu\beta\beta$ signals, both separately and in combination, in the nEXO, LEGEND, and CUPID projects with 10 ton-years exposure. The slanted allowed regions tend to reach both axes, as a result of the NME degeneracy; only in some cases the true hypothesis can be sufficiently well isolated. Attaching uncertainties to the NME would make the separation even more difficult, see [5] for details. Fortunately, a wide theoretical research program is being envisaged to improve the NME calculations, by using so-called *ab initio* methods calibrated with a variety of nuclear data. This program is necessary to match the great experimental investment in ton-scale projects. Once more, progress in "electroweak nuclear physics" appears to be mandatory for future $0\nu\beta\beta$ research, as well as for oscillation searches.

## 3. Conclusions

The standard $3\nu$ framework is being tested by a variety of oscillation experiments that will refine its known parameters and determine the unknown ones, as reviewed in this contribution. Nonoscillation data are also expected to find signals probing the absolute mass and the nature of neutrinos. In the future, a mismatch between different signals in the $3\nu$ parameter space might indicate possible new physics beyond $3\nu$. A nonstandard scenario with light and heavy neutrinos exchange in $0\nu\beta\beta$ decay has been studied as a representative example. For both current and future observations, theoretical improvements in describing the nuclear physics associated to neutrino interactions (from low energies relevant to decays to higher energies relevant to oscillations) are mandatory in all cases, standard and nonstandard.

## 4. Acknowledgments

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[6] Slides of this talk are available at the Indico website for the Corfu 2023 Workshop "Standard Model and Beyond": `https://indico.cern.ch/event/1311102`

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