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Dimensional reduction of a $10D$, $\mathcal{N}=1$ E_8 over a modified flag manifold to Split NMSSM

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We examine the Standard Model extension that emerges from the dimensional reduction of a $10D$, $\mathcal{N}=1$, E_8 gauge theory over the $M_4 \times SU(3)/U(1)\times U(1)\times \mathbf{Z}_3$ space, resulting in a $4D$, $\mathcal{N}=1$, $SU(3)^3\times U(1)^2$ theory. Below the unification scale, a Split NMSSM effective theory is obtained. At the 2-loop level, the masses of the third-generation quark and light Higgs are consistent with experimental limits, and the neutralino LSP mass is predicted to be < 800 GeV.

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1. Introduction

Theoretical physicists have aimed to unify all four fundamental interactions for decades. This quest has been a central focus within the scientific community, leading to numerous innovative approaches in recent years. Notably, those involving additional dimensions have garnered substantial attention and acclaim. The concept of extra dimensions is a cornerstone of superstring theories. Among these, the heterotic string [1], defined in ten dimensions, is particularly promising due to its potential for experimental validation. The heterotic string's phenomenological aspects prominently feature in the resulting Grand Unified Theories (GUTs), which crucially contain the Standard Model (SM) gauge group. These GUTs arise from compactifying the ten-dimensional spacetime and subsequent dimensional reduction often on Calabi-Yau (CY) spaces. It is important to highlight an alternative framework that preceded superstrings, focusing on the dimensional reduction of higher-dimensional gauge theories, offering another avenue for exploring the unification of fundamental interactions. Forgacs-Manton and Scherk-Schwartz were pioneers in this area. Forgacs-Manton introduced Coset Space Dimensional Reduction (CSDR) [2–4], while Scherk-Schwartz worked on group manifold reduction [5]. These foundational works emphasized the dimensional reduction of higher-dimensional theories and their implications for unification, laying the foundation for the field.

A defining trait of higher-dimensional theories is their unification of the gauge and scalar sectors of a $4D$ theory. In particular, within the framework of CSDR, the kinetic terms of fermions in higher dimensions yield both kinetic and Yukawa terms in $4D$. Additionally, a $\mathcal{N}=1$ supersymmetric theory in certain higher dimensions can feature only one vector supermultiplet, unifying its $4D$ gauge and fermionic components. Two notable features of CSDR are: i) starting from a vector-like theory in $4n+2$ dimensions can result in chiral fermions in $4D$ [6, 7], and ii) $\mathcal{N}=1$ supersymmetry in $10D$ leads to softly broken supersymmetric theories in $4D$ when the coset space is non-symmetric, and to non-supersymmetric theories if the coset is symmetric [8–11].

In the heterotic string framework, the use of compact internal CY manifolds gained prominence because they preserve the supersymmetry of an initial $N=1$ supersymmetric gauge theory upon dimensional reduction to four dimensions [12]. However, the challenge of moduli stabilization led to the exploration of internal spaces with $SU(3)$-structure, particularly nearly-Kähler manifolds [13–19]. The $6D$ homogeneous nearly-Kähler manifolds that permit a connection with torsion are the non-symmetric coset spaces $G_2/SU(3)$, $Sp(4)/SU(2)\times U(1)_{non-max}$, $SU(3)/U(1)\times U(1)$, and the group manifold $SU(2)\times SU(2)$ [20–23]. In contrast to CY manifolds, CSDR can start from a $10D$, $N=1$ theory and, by utilizing a non-symmetric coset space, yield $4D$ theories with an inherently emerging soft supersymmetry breaking sector [8–11].

Here we review the dimensional reduction of a $\mathcal{N}=1$ E_8 theory over the modified flag manifold $SU(3)/U(1)\times U(1)\times \mathbf{Z}_3$, which is the non-symmetric coset space $SU(3)/U(1)\times U(1)$ with a freely acting discrete symmetry $\mathbf{Z}_3$. This setup facilitates the Wilson flux breaking mechanism, resulting in the $4D$ GUT $SU(3)^3\times U(1)^2$ [3, 8, 21, 24] (also [25]). The resulting theory is a softly broken $\mathcal{N}=1$ theory with small radii, aligning the compactification scale with the unification scale. The geometric origin of the soft terms makes all sfermions superheavy, causing them to decouple along with the additional fields from the trinification group (for older configurations with similar setups, see [26], [27], and [28]). Due to a specific choice of radii, we obtain the Split

Next-to-Minimal Supersymmetric Standard Model (NMSSM) [29, 30] (see also [31, 32]), where the lighter supersymmetric particles have masses < 1 TeV. For the original work, see [33].

2. The Coset Space Dimensional Reduction

We begin by covering the basics of CSDR. A detailed examination of the geometric aspects of coset spaces and the fundamental principles of CSDR (including the reduction methodology and its constraints) can be found in [3]. Consider a D-dimensional space $M^4 \times S/R$, where $D = d + 4$ and d is the number of dimensions of S/R. The extra dimensions of $M^4 \times S/R$ are compactified on the coset space S/R, where S is a Lie group and R is its subgroup (with $d = dimS - dimR$). S acts as a symmetry group on the extra coordinates. The core idea of CSDR is that the transformations of fields under the action of the symmetry group S of S/R are compensated by gauge transformations. As a result, since the Lagrangian is gauge invariant, there is no dependence on the extra coordinates, effectively reducing the theory. It is important to note that fields defined in this manner are called symmetric.

For a Yang-Mills-Dirac theory with gauge group G defined on the D-dimensional manifold M^D and compactified on $M^4 \times S/R$, the action is:

$$S = \int d^4x d^d y \sqrt{-g} \left[-\frac{1}{4} \mathrm{Tr}(F_{MN} F_{K\Lambda}) g^{MK} g^{N\Lambda} + \frac{i}{2} \bar{\psi} \Gamma^M D_M \psi \right] \tag{1}$$

where the spinor ψ represents the fermions of the theory and belongs to the representation F of the gauge group.
The conditions that all fields of the theory that exist on the coset space are symmetric are given by:

$$\begin{aligned} A_\mu(x, y) =& g(s) A_\mu(x, s^{-1} y) g^{-1}(s) \\ A_a(x, y) =& g(s) J_a{}^b A_b(x, s^{-1} y) g^{-1}(s) + g(s) \partial_a g^{-1}(s) \\ \psi(x, y) =& f(s) \Omega \psi(x, s^{-1} y) f^{-1}(s) \,, \end{aligned} \tag{2}$$

where g, f are gauge transformations in the adjoint representation F of G, corresponding to the s transformation of S acting on S/R, J_a^b is the Jacobian for s and Ω is the Jacobian plus the local Lorentz rotation in tangent space. The fields A_μ and A_α refer to the components of the higher dimensional gauge field $A_M = (A_\mu, A_\alpha)$. The A_μ are the $4D$ gauge fields and the A_α are the extra-dimensional components, which "do not see" the Lorentz group, i.e. behave as $4D$ scalar fields. The above conditions imply constraints that the D-dimensional fields should obey.

The solutions of these constraints determine the gauge group and the surviving field content of the $4D$ theory (for details see e.g. [3]). The constraint referring to the $4D$ gauge fields, A_μ suggests that the surviving $4D$ gauge group is $H = C_G(R_G)$, i.e. the centralizer of R in G and A_μ do not depend on the coordinates of the coset. Concerning the $4D$ scalars, A_α, those that eventually survive are identified as follows:

$$G \supset R_G \times H \,, \qquad \mathrm{adj}G = (\mathrm{adj}R, 1) + (1, \mathrm{adj}H) + \sum (r_i, h_i) \,, \tag{3}$$

$$S \supset R \,, \qquad \mathrm{adj}S = \mathrm{adj}R + \sum s_i \,. \tag{4}$$

The scalars that survive in $4D$ are determined by the irreducible representations r_i and s_i of R. When r_i and s_i match, the representation h_i of H corresponds to a scalar multiplet. The remaning scalars do not satisfy the constraints and are projected out i.e. do not survive in $4D$.

In a similar way, the third constraint of (2), which is associated with the spinorial content of the theory, allows to determine the surviving $4D$ spinors, as the $4D$ spinors depend only on the coordinates of the $4D$ theory. The f_i representation of H (to which fermions are assigned) is determined by the decomposition of the representation F of G w.r.t. $R_G \times H$ and the spinorial representation of the local 'Lorentz group' of the tangent space, $SO(d)$, of the coset space S/R under R (after embedding R onto $SO(d)$):

$$G \supset R_G \times H\,, \qquad F = \sum (r_i, f_i), \tag{5}$$

$$SO(d) \supset R\,, \qquad \sigma_d = \sum \sigma_j\,. \tag{6}$$

For each pair of identical r_i and σ_i, a f_i spinor multiplet survives in $4D$.

Concerning fermions it is necessary to add few further remarks. If the higher-dimensional fermions are Dirac fermions, the surviving $4D$ fermions will not be chiral. However imposing the Weyl on an even D-dimensional spacetime yields chiral fermions in $4D$. Fermions accommodated in the adjoint representation in an initial theory defined in $D = 2n+2$ dimensions lead in $4D$ to two sets of chiral fermions with identical quantum numbers for the components of each set. If the Majorana condition is also imposed on the initial theory, the $4D$ theory does not feature the doubling of the spectrum. In $D = 4n+2$ dimensions both Weyl and Majorana conditions can be imposed.

3. Dimensional Reduction of E_8 over $SU(3)/U(1)^2$

We may now focus on a realistic implementation of the CSDR. We start from a $10D$, $\mathcal{N}=1$ supersymmetric E_8 with a vector supermultiplet and Weyl-Majorana fermions, which is reduced over the non-symmetric space $SU(3)/U(1)\times U(1)$ [3, 8, 25]. The $4D$ action is then given by:

$$S = C \int d^4x\, \mathrm{tr}\left[-\frac{1}{8}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}(D_\mu\phi_a)(D^\mu\phi^a)\right] + V(\phi) + \frac{i}{2}\bar{\psi}\Gamma^\mu D_\mu\psi - \frac{i}{2}\bar{\psi}\Gamma^a D_a\psi\,,$$

where the scalar potential is given by

$$V(\phi) = -\frac{1}{4}g^{ac}g^{bd}\mathrm{tr}\left(f_{ab}{}^{C}\phi_C - i[\phi_a,\phi_b])(f_{cd}{}^{D}\phi_D - i[\phi_c,\phi_d]\right)\,, \tag{7}$$

where C is the coset volume, D_μ the $4D$ covariant derivative, D_a the one of the coset, the metric of the coset is $g_{\alpha\beta} = \mathrm{diag}(R_1^2, R_1^2, R_2^2, R_2^2, R_3^2, R_3^2)$ and R_i are the coset radii.

The way $R = U(1)\times U(1)$ is embedded in $G = E_8$ determines the $4D$ gauge group. Our choice is that the two $U(1)$s are identified with the diagonal generators of $SU(3)$ (Cartan subalgebra) in the following maximal decomposition of E_8:

$$E_8 \supset SU(3)\times E_6\,. \tag{8}$$

The gauge group in $4D$ is the centralizer of R in G:

$$H = C_{E_8}(U(1)_A\times U(1)_B) = E_6 \times U(1)_A \times U(1)_B\,. \tag{9}$$

The surviving scalars and fermions are obtained by examining the decomposition of the vector and spinor representations of $SO(6)$, respectively, under $R = U(1)_A \times U(1)_B$ (following the methodology above).

Therefore the surviving $4D$ gauge fields fields are assigned in three $N = 1$ vector supermultiplets of the $E_6 \times U(1)_A \times U(1)_B$, while the matter fields into six chiral supermultiplets, of which three are E_6 singlets and the other three transform under $E_6 \times U(1)_A \times U(1)_B$. The unconstrained matter fields are:

$$A_i \sim 27_{(3,\frac{1}{2})},\;\; B_i \sim 27_{(-3,\frac{1}{2})},\;\; \Gamma_i \sim 27_{(0,-1)},\;\; A \sim 1_{(3,\frac{1}{2})},\;\; B \sim 1_{(-3,\frac{1}{2})},\;\; \Gamma \sim 1_{(0,-1)}$$

and the scalar potential -which is positive definite- becomes:

$$\begin{aligned}
V =&\frac{g^2}{2}\Bigg[\frac{2}{5}\left(\frac{1}{R_1^4}+\frac{1}{R_2^4}+\frac{1}{R_3^4}\right)+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\alpha^i\alpha_i+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\bar{\alpha}\alpha \\
&+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\beta^i\beta_i+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\bar{\beta}\beta+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\gamma^i\gamma_i+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\bar{\gamma}\gamma \\
&+\left[80\sqrt{2}\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)d_{ijk}\alpha^i\beta^j\gamma^k+80\sqrt{2}\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)\alpha\beta\gamma+h.c\right] \\
&+\frac{1}{6}\left(\alpha^i(G^\alpha)_i^j\alpha_j+\beta^i(G^\alpha)_i^j\beta_j+\gamma^i(G^\alpha)_i^j\gamma_j\right)^2+\frac{10}{6}\left(\alpha^i(3\delta_i^j)\alpha_j+\bar{\alpha}(3)\alpha+\beta^i(-3\delta_i^j)\beta_j+\bar{\beta}(-3)\beta\right)^2 \\
&+\frac{40}{6}\left(\alpha^i(\tfrac{1}{2}\delta_i^j)\alpha_j+\bar{\alpha}(\tfrac{1}{2})\alpha+\beta^i(\tfrac{1}{2}\delta_i^j)\beta_j+\bar{\beta}(\tfrac{1}{2})\beta+\gamma^i(-1\delta_i^j)\gamma^j+\bar{\gamma}(-1)\gamma\right)^2 \\
&+40\alpha^i\beta^j d_{ijk}d^{klm}\alpha_l\beta_m+40\beta^i\gamma^j d_{ijk}d^{klm}\beta_l\gamma_m+40\alpha^i\gamma^j d_{ijk}d^{klm}\alpha_l\gamma_m \\
&+40(\bar{\alpha}\bar{\beta})(\alpha\beta)+40(\bar{\beta}\bar{\gamma})(\beta\gamma)+40(\bar{\gamma}\bar{\alpha})(\gamma\alpha)\Bigg], \qquad (10)
\end{aligned}$$

where $\alpha^i, \alpha, \beta^i, \beta, \gamma^i, \gamma$ are the scalar components of A^i, B^i, Γ^i and A, B, Γ and d_{ijk} the fully symmetric E_6 invariant tensor. One can indentify $F-, D-$ and soft supersymmetry breaking terms in the potential of Eq. (10) The F-terms are identified in the last two lines and come from the superpotential:

$$\mathcal{W}(A^i, B^j, \Gamma^k, A, B, \Gamma) = \sqrt{40}d_{ijk}A^iB^j\Gamma^k + \sqrt{40}AB\Gamma\,, \qquad (11)$$

The D-terms (lines 4-5 of Eq. (10)) have their usual structure and the remaining terms of Eq. (10) (except from the first term which is constant) are the soft scalar masses and soft trilinear terms. The gaugino mass is also of geometrical origin, although it behaves differently than the soft masses:

$$M = (1+3\tau)\frac{R_1^2+R_2^2+R_3^2}{8\sqrt{R_1^2R_2^2R_3^2}}\,, \qquad (12)$$

For a generic choice the gauginos would obtain compactification scale mass [3]. This is prevented by the appropriate choice of the contorsion τ (details can be found in [10]). Its value is chosen to be such that the following model features a electroweak (EW) scale unified gaugino mass.

4. Wilson Flux and the Surviving Theory

In the above, the 27 multiplet that contains the three $E_6 \times U(1)_A \times U(1)_B$ supermultiplets is insufficient to break E_6 to a GUT leading to the SM gauge group. To achieve further gauge breaking, we make use of the Wilson flux breaking mechanism [35–37]. What follows is a brief review of the basics of the Wilson flux mechanism and its application to the case at hand.

In the above, the dimensional reduction was performed over the simply connected manifold $B_0 = S/R$. However, the manifold can be multiply connected. This is achieved by considering $B = B_0/F^{S/R}$, where $F^{S/R}$ is a freely-acting discrete symmetry of B_0. For each element $g \in F^{S/R}$, there is a corresponding element U_g in the 4D gauge group H, which may be viewed as the Wilson loop:

$$U_g = \mathcal{P}exp\left(-i\oint_{\gamma_g} T^a A_M^a dx^M\right), \tag{13}$$

where A_M^a are the gauge fields, γ_g a contour representing the element g of $F^{S/R}$, T^a are the generators of the group and $\mathcal{P}$ denotes the path ordering. In the case where the considered manifold is simply connected, the vanishing of the field strength tensor implies that the gauge field can be set to zero through a gauge transformation. However, when γ_g is chosen to be non-contractible to a point, we have $U[\gamma] \neq 1$, and the gauge field cannot be gauged away. This means that the vacuum field strength does not lead to $U_g = 1$. As a result, a homomorphism of $F^{S/R}$ into H is induced with an image T^H, which is the subgroup of H generated by U_g. Furthermore, consider a field $f(x)$ defined on B_0. It is evident that $f(x)$ is equivalent to another field on B_0 that satisfies $f(g(x)) = f(x)$ for every $g \in F^{S/R}$. The presence of H generalizes this statement:

$$f(g(x)) = U_g f(x)\,. \tag{14}$$

Regarding the gauge symmetry that remains by the vacuum, in the vacuum state it is given that $A_\mu^a = 0$, and consider also a gauge transformation by the coordinate-dependent matrix $V(x)$ of H. In order to keep $A_\mu^a = 0$ and preserve the vacuum invariance, the matrix $V(x)$ must be chosen to be constant. Additionally, $f \to Vf$ is consistent with Eq. (14) only if

$$[V, U_g] = 0 \tag{15}$$

for every $g \in F^{S/R}$. Hence, the subgroup of H that remains unbroken is the centralizer of T^H in H. As for the matter fields that survive in the theory, meaning the matter fields that satisfy the condition in Eq. (14), they must be invariant under the combination:

$$F^{S/R} \oplus T^H.$$

The freely-acting discrete symmetries, $F^{S/R}$, of $B_0 = S/R$ are the center of S, $Z(S)$ and $W = W_S/W_R$, where it is understood that W_S and W_R are the Weyl groups of S and R, respectively. In this case we have

$$F^{S/R} = \mathbb{Z}_3 \subseteq W = S_3, \tag{16}$$

since the original coset was $B_0 = SU(3)/U(1) \times U(1)$.

The Wilson breaking projects the theory in a manner that the surviving fields are the ones that remain invariant under $\mathbf{Z}_3$ on their gauge and geometric indices. In our case, the $\mathbf{Z}_3$'s non-trivial action on the gauge indices of the fields is parameterized by the matrix [38]:

$$\gamma_3 = \mathrm{diag}\{\mathbf{1}_3, \omega\mathbf{1}_3, \omega^2\mathbf{1}_3\}\,, \tag{17}$$

where $\omega = e^{i\frac{2\pi}{3}}$ is the phase that acts on the gauge fields. The remaining gauge fields satisfy the condition:

$$[A_M, \gamma_3] = 0 \quad \Rightarrow A_M = \gamma_3 A_M \gamma_3^{-1} \tag{18}$$

and the new gauge symmetry is $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B$. The $U(1)$s are the R-symmetry of the theory, which is closely interrelated to supersymmetry. The matter counterpart of Eq. (18) is:

$$A^i = \gamma_3 A^i, \quad B^i = \omega\gamma_3 B^i, \quad \Gamma^i = \omega^2\gamma_3\Gamma^i, \qquad A = A, \quad B = \omega B, \quad \Gamma = \omega^2\Gamma\,. \tag{19}$$

By examining the decomposition of the 27 representation of E_6 under the $SU(3)_c \times SU(3)_L \times SU(3)_R$ gauge group, $(1,3,\bar{3}) \oplus (\bar{3},1,3) \oplus (3,\bar{3},1)$, one can obtain the representations of the trinification part of the gauge group in which the above fields are accommodated. Thus, the matter content of the projected theory is:

$$A_1 \equiv L \sim (\mathbf{1},\mathbf{3},\bar{\mathbf{3}})_{(3,1/2)}, \quad B_3 \equiv q^c \sim (\bar{\mathbf{3}},\mathbf{1},\mathbf{3})_{(-3,1/2)}, \quad \Gamma_2 \equiv Q \sim (\mathbf{3},\bar{\mathbf{3}},\mathbf{1})_{(0,-1)}$$
$$A \equiv \theta \sim (\mathbf{1},\mathbf{1},\mathbf{1})_{(3,1/2)}$$

where the former three are the remaining components of A^i, B^i, Γ^i. All together they form a 27 representation of E_6, which corresponds to the content representing one generation in the remaining theory. To end up with three generations, one can introduce non-trivial monopole charges in the $U(1)$s in R. This leads to three copies of the aforementioned fields, subsequntly resulting in three generations [39]. The trinification multiplets L, q^c, Q can be now assigned and written in the more standard way:

$$L^{(l)} = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & N \end{pmatrix}, \quad q^{c\,(l)} = \begin{pmatrix} d_R^{c1} & u_R^{c1} & D_R^{c1} \\ d_R^{c2} & u_R^{c2} & D_R^{c2} \\ d_R^{c3} & u_R^{c3} & D_R^{c3} \end{pmatrix}, \quad Q^{(l)} = \begin{pmatrix} d_L^1 & d_L^2 & d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix},$$

where $l = 1,2,3$ is the generation index. q^c and Q are quark multiplets, while L containts both the lepton and the Higgs sector. The quark multiplets also contain the vector-like down-type quarks $D^{(l)}$, which will eventually be $SU(2)_L$ singlets, while L also features the right-handed neutrinos $\nu_R^{c\,(l)}$ and the sterile neutrino-like fields $N^{(l)}$. It is useful to note that there are three generations of Higgs doublets. Finally, there are three trinification singlets, $\theta^{(l)}$.

It is useful to recall that if an effective $4D$ theory is renormalizable by power counting, then it is consistent to consider it a renormalizable theory [40]. In this context, we respect all the symmetries and the model structure derived from the higher-dimensional theory and its dimensional reduction. However, we treat all the parameters of the effective theory as free parameters, to the extent allowed by symmetries. Specifically, all kinetic terms and D-terms of the action have the gauge coupling g, as dictated by the gauge symmetry of the model. All superpotential terms must also share the same coupling to respect supersymmetry. The flexibility afforded by this treatment is evident in the soft sector, where each term is allowed its own coupling. Thus, the superpotential of the $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B$ effective theory is given by:

$$\mathcal{W}^{(l)} = C^{(l)} d^{abc} L_a^{(l)} q_b^{c\,(l)} Q_c^{(l)}\,, \tag{20}$$

since the B and Γ trinfication singlets were projected out. Similarly, the soft sector of the scalar potential is now:

$$
\begin{aligned}
V^{(l)}_{\text{soft}} =& \left(\frac{c^{(l)}_{L_1} R_1^2}{R_2^2 R_3^2} - \frac{c^{(l)}_{L_2}}{R_1^2}\right)\langle L^{(l)}|L^{(l)}\rangle + \left(\frac{c^{(l)}_{\theta_1} R_1^2}{R_2^2 R_3^2} - \frac{c^{(l)}_{\theta_1}}{R_1^2}\right)|\theta^{(l)}|^2 \\
&+ \left(\frac{c^{(l)}_{q_1^c} R_2^2}{R_1^2 R_3^2} - \frac{c^{(l)}_{q_2^c}}{R_2^2}\right)\langle q^{c(l)}|q^{c(l)}\rangle + \left(\frac{c^{(l)}_{Q_1} R_3^2}{R_1^2 R_2^2} - \frac{c^{(l)}_{Q_1}}{R_3^2}\right)\langle Q^{(l)}|Q^{(l)}\rangle \\
&+ \left(\frac{R_1}{R_2R_3} + \frac{R_2}{R_1R_3} + \frac{R_3}{R_1R_2}\right)(c^{(l)}_{\alpha} d^{abc} L^{(l)}_a q^{c(l)}_b Q^{(l)}_c + c^{(l)}_b d^{abc} L^{(l)}_a L^{(l)}_b L^{(l)}_c + h.c) \\
=& m^2_{L^{(l)}}\langle L^{(l)}|L^{(l)}\rangle + m^2_{q^{c(l)}}\langle q^{c(l)}|q^{c(l)}\rangle + m^2_{Q^{(l)}}\langle Q^{(l)}|Q^{(l)}\rangle + m^2_{\theta^{(l)}}|\theta^{(l)}|^2 \\
&+ (\alpha^{(l)abc} L^{(l)}_a q^{c(l)}_b Q^{(l)}_c + b^{(l)abc} L^{(l)}_a L^{(l)}_b L^{(l)}_c + h.c) ,
\end{aligned} \tag{21}
$$

where $c^{(l)}_i$ are free parameters of $O(1)$ and the above equation only involves the scalar components of the denoted superfields. It is evident that all sfermions, Higgs bosons and trinification singlet scalars acquire a soft mass parameter. Since supersymmetry is softly broken in the model, the associated R-symmetry can also be considered softly broken. Therefore, we introduce R-symmetry breaking terms L^3 to eventually generate a superheavy B-term, facilitating the standard rotation in the Higgs sector as featured in the Split NMSSM (see Sect. 7 below). These terms completely break R-symmetry, meaning any residual R-parity will also be broken once R-symmetry is broken.

5. Radii and GUT Breaking

In the case at hand the compactification scale, M_C is very high, thus Kaluza-Klein modes that occur from the dimensional reduction are irrelevant. Additionally, we consider $M_C = M_{GUT}$; this means that the coset radii are small:

$$R_l \sim \frac{1}{M_{GUT}} \,, \; l = 1, 2, 3 \,.$$

Since the trilinear soft terms and the soft scalar masses depend on the geometry of the coset, Eq. (21) suggests that they are $\sim O(M_{GUT})$. The choice $R_2 = R_3$ translates to $m^2_{q^{c(l)}} = m^2_{Q^{(l)}}$, but we employ a slightly different R_1. Together with appropriate selection of values for $c^{(l)}_{\theta_i}$ this leads to a cancellation among terms that dictate $m^2_{\theta^{(3)}}$ and guarantees that it is $\sim O(EW)$.

The breaking of the $SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B$ gauge group involves the vacuum expectation values (vevs):

$$
\langle L^{(1)}_s\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V_1 \end{pmatrix}, \quad \langle L^{(2)}_s\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V_2 & 0 & 0 \end{pmatrix}, \quad \langle L^{(3)}_s\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V_3 & 0 & V_4 \end{pmatrix},
$$

$$\langle \theta^{(1)}_s\rangle = V_5 \,, \quad \langle \theta^{(2)}_s\rangle = V_6 \,,$$

where with the s index we denote the respective scalar components of the fields. We proceed with the above-mentioned non-minimal vev content, which will prove useful in the low-energy model and we get the breaking:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B \xrightarrow{V_i} SU(3)_c \times SU(2)_L \times U(1)_Y \ .$$

6. Radiative terms

The superpotential (20) lacks the bilinear terms that would serve as μ-terms in the low-energy model at tree level. These terms would violate the two $U(1)$s. However, since R-symmetry is broken, trilinear terms among the Higgs doublets and the gauge singlets appear radiatively:

$$H_u^{(l)} H_d^{(l)} \overline{\theta}^{(l)} \ . \tag{22}$$

These terms feature a (natural) generation diagonality, which leads to an interesting phenomenology. Since these terms are effectively μ-like terms, the Higgs doublets of the two first generations acquire a superheavy μ term, since $\langle \theta_s^{(1,2)} \rangle \sim O(GUT)$, while the term of the third generation survives in the low-energy model. The missing lepton Yukawa terms get the same treatment. Both these terms also emerge via dim-5 operators [42]:

$$H_u^{(l)} H_d^{(l)} \theta^{(l)} \frac{K^{(l)}}{M} \quad , \quad L^{(l)} \overline{e}^{(l)} H_d^{(l)} \frac{K^{(l)}}{M} \ , \tag{23}$$

where $K^{(l)}$ can be any of the fields that acquire superheavy vevs, namely $N^{(1,3)}$, $\nu_R^{(2,3)}$, $\theta^{(1,2)}$ or any combination of them (provided the generation index is respected). An additional $\mathbb{Z}_2$ discrete symmetry in the lepton sector is needed to protect from dangerous radiative or higher-dimensional terms. The rest of the allowed terms lead to superheavy masses for all trinification singlet fields but $\theta_f^{(3)}$, in which case a cancellation among terms leads to a $\sim O(EW)$ mass.

7. The Split NMSSM Low-Energy Theory

The next step is to sort the particle content left under the surviving SM gauge group. The vector-like quarks $D^{(l)}$ along with $N^{(l)}$, $\nu_R^{(l)}$, $\theta^{(1,2)}$ and $H_{u,d}^{(1,2)}$ (this holds for fermion and scalar components).

As explained in Sect. 3, the torsion value is selected such that the gauginos get masses of a few TeV, while all sfermions get superheavy due to the geometric origin of the soft masses. The third generation soft Higgs mass parameters $m^2_{L^{(3)}} \equiv m^2_{H_{u,d}}$ are superheavy, while the last term of Eq. (21) contains a soft B-like term $\sim O(GUT)$:

$$b^{(3)} H_u^{(3)} \cdot H_d^{(3)} \equiv b H_u \cdot H_d \ . \tag{24}$$

Following the approach of [30], the Higgs doublets and the singlet field of the third generation $H_{u,d}^{(3)} \equiv H_{u,d}$ and $\theta^{(3)} \equiv S$ are light and survive down to the EW scale, with an interaction term from Eq. (22):

$$\lambda S H_u \cdot H_d \ , \tag{25}$$

where the family indices are now implicit, as we focus on the third generation from now on.

The scenario described above corresponds to the split NMSSM [29, 30] (we adopt the notation from [30]). The unification scale soft Higgs mass parameter makes the heavy Higgs scalars ($H_0, A_H, H^\pm$) superheavy, causing them to decouple. Consequently, the light scalar sector contains only the light Higgs boson h, the scalar S (which also acquires a vev), and its CP-odd counterpart A. The higher-energy theory imposes another constraint on the Yukawa sector of the model, as the structure of its superpotential implies matching top and bottom couplings at the unification scale. Therefore, a large value of $tan\beta$ is naturally expected.

We implemented the above in `SARAH` [43] and generated a `SPheno` code [44, 45] to produce the (light) particle spectrum. The 2-loop renormalisation group equations (RGEs) use the relations among couplings at the unification scale as boundary conditions and they are ran down to the EW level. We take into account threshold corrections originating from the decoupling superheavy particles, allowing for an extra 5% uncertainty on the Yukawa couplings boundary condition. For the analysis we use the on-shell values in case of the top quark and the $\overline{\text{MS}}$ in case of the bottom quark:

$$m_t = (172.69 \pm 0.30) \text{ GeV} \ , \quad m_b(m_b) = (4.18 \pm 0.03) \text{ GeV} \ , \tag{26}$$

as given in [46]. As expected, in order to satisfy these limits we have $70 < \tan\beta < 80$. β is the angle between H_u and H_d^*, which determines the light Higgs doublet at the high scale at which the second doublet is integrated out. Both, H and S, get vevs denoted by v_H and v_S, respectively. The combination $\mu = \lambda v_S/\sqrt{2}$ is the mass parameter for the higgsinos like fermions and has to be sufficiently large to be consistent with existing LHC searches.

The light Higgs boson mass is shown in Fig. 1 as a function of the unified gaugino mass M_U (left) and the trilinear coupling λ (right). Only points that satisfy the experimental limits of the top and bottom masses are included. For the unified gaugino mass we obtain $M_U < 1800$GeV, and the most points that satisfy the experimental limits on the Higgs boson mass [46],

$$m_h^{exp} = (125.25 \pm 0.17) \text{ GeV} \,, \tag{27}$$

are the ones that have $1600\text{GeV} < M_U < 1700$ GeV. We consider a theoretical uncertainty of 2 GeV [47].

The difference between the lightest chargino mass and the lightest neutralino, which is the lowest supersymmetric particle (LSP), is given in Fig. 2 w.r.t. the lightest chargino mass. For all points the Higgs mass is within the 2 GeV theoretical uncertainty of [47] and satisfy the lower exclusion bounds for the lightest chargino mass [46]. The ATLAS and CMS experiments of the LHC have searched for charginos and neutralinos and they have obtained bounds of up to 1.4 TeV, which, however, depend on the mass difference among the lighter chargino and the lightest neutralino and, to some extent, also on the details of the decays [48–51]. The points below the orange line feature a chargino mass of above 180 GeV and the mass difference to the lightest neutralino is below 30 GeV, impyling that these points pass the experimental bounds as these are higgsino-like states. For the other points a more detailed investigation is required which we postpone to a future work.

Fig. 3 features the predicted particle spectrum. The (mainly) CP-even singlet scalar is denoted as S, while its CP-odd counterpart as A. Interestingly, in the parameter region in which our model agrees with the observed Higgs boson mass, S is always heavier than ~ 300 GeV. The

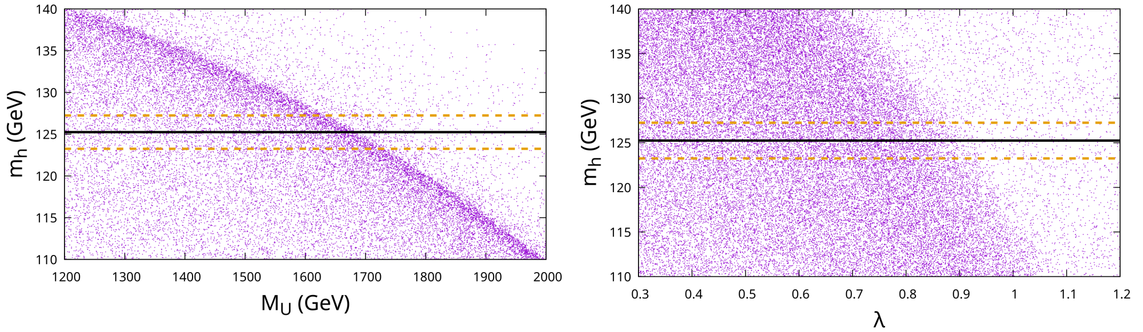

Figure 1: Left: the light Higgs boson mass as a function of the unified gaugino mass. Right: the light Higgs boson mass as a function of the trilinear parameter λ. In both plots the black line denotes the experimental value of the Higgs mass, $m_h = 125.25$ GeV, while the orange dashed lines denote the 2 GeV theoretical uncertainties.

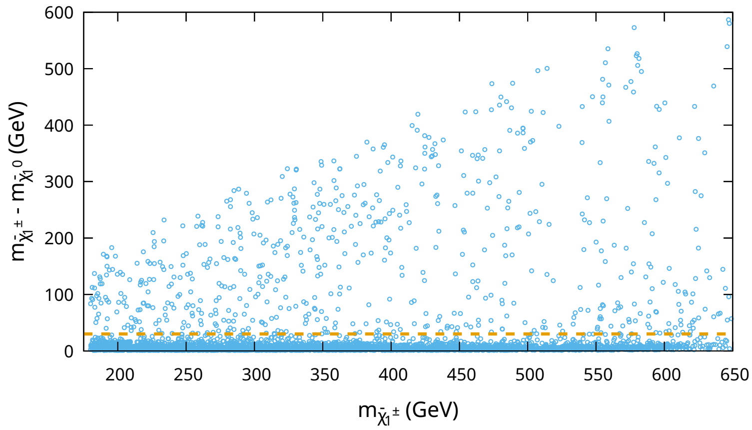

Figure 2: The plot shows the mass difference between the lightest chargino and the lightest neutralino, for points that satisfy the Higgs mass theoretical uncertainty of [47]. The orange dashed line denotes the 30 GeV mass difference limit.

singlet component of the Higgs boson h is sufficiently small to be consistent with the existing coupling measurements to vector bosons and fermions. They are not affected by existing searches as they can hardly be produced at the LHC because they are gauge singlets. $\tilde{\chi}_i^0, \tilde{\chi}_1^\pm$ and $\tilde{g}$ are the neutralinos, charginos and the gluinos, respectively. The points shown correspond to the ones below the orange line in Fig. 2 to ensure that they are compatible with existing searches at the LHC. Adding the other points wouldn't change the picture significantly and the most important change would be somewhat smaller values for mass of the lightest neutralino. Note, that the gluino is predicted to be heavier than 2 TeV in this model and, thus, this model can explain why so far no sign for supersymmetry has been found at the LHC. This also implies that this model will be difficult to probe in the coming LHC runs. The reach of the high luminosity LHC for the lightest chargino can go up to 200 GeV if the systematics are well under control [52]. Other possibilities are the combined production of a heavier neutralino together with the lightest chargino which we will investigate in an upcoming work. Last but not least we point out that the lightest neutralino is an admixture of the singlet fermion and a higgsino and thus it can be a cold dark matter candidate consistent with observations, which we will investigate together with details of the collider searches.

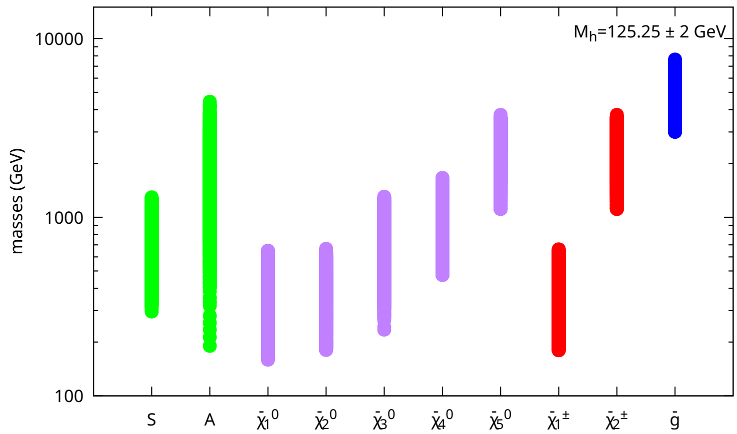

Figure 3: The plot shows the predicted particle spectrum. The green points are the CP-even and CP-odd singlet scalar masses; the purple points are the neutralino masses; the red ones are the chargino masses, followed by the blue points indicating the gluino masses.

8. Conclusions

We reviewed a theory that starts from a 10D, $\mathcal{N} = 1$, E_8 gauge theory dimensionally reduced over the manifold $SU(3)/U(1) \times U(1) \times \mathbb{Z}_3$. We are led first to a softly-broken $\mathcal{N} = 1$, $SU(3)^3 \times U(1)^2$, 4D effective theory and consequnently to the Split NMSSM. The top, bottom and light Higgs masses within the experimental limits, while the model predicts gluino masses beyond the reach of the high-luminosity LHC. However, the lighter charginos and neutralinos are below the TeV in most cases with a small mass splitting. The reach of the high-luminosity LHC for scenarios with a larger mass splitting will be investigated in a future work.

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